# CREDIBILITY-WEIGHTED TREND FACTORS

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*Abstract*

The credibility of trend lines is important because trend lines cannot be extrapolated reliably far into the future. Credibility-weighted trend factors can be calculated if two or more alternative assumptions are considered. The effects of changes in the goodness of fit of the trend lines being considered can also be explored.

This paper approaches the problem by ad hoc blending of alternative sets of hypotheses. The appropriateness of the method is argued by analogy with Empirical Bayesian credibility formulas. A specific example is used throughout.

In this example, a particular pair of alternative assumptions is considered—that there is no trend and that there is linear trend. The results suggest that an increase in the $R^2$ of the linear trend line may imply an increase in the credibility of the trend line, reliance on a greater amount of trend, or a more reliable resulting estimate. Which of these or which combination of these in the case depends on the data at hand. A greater $R^2$ does not necessarily imply greater credibility for trend.

The methods shown in this paper can be extended to other sets of assumptions, and other questions about the appropriateness of trend assumptions can also be studied.

## *Introduction*

Trend lines are used in ratemaking in virtually all lines of insurance. The purpose of introducing a calculation of trend into a rate derivation is to arrive at an estimate of future loss costs that reflects the changes in loss costs over time.

Trend was introduced into workers' compensation ratemaking in the late 1970's. An example of a trend calculation by the National Council on Compensation Insurance (NCCI) is shown in Exhibit I. This is a particularly good example of the calculation of a trend factor for two reasons. First, the various subtotals that go directly into the calculation of the trend line are shown explicitly. Second, the trend factor finally derived is a credibility-weighted trend factor, and such factors are the subject of this paper.

*Problems with the Use of Trend Factors*

The academic training of actuaries gives them a general awareness that trend lines cannot be extrapolated reliably very far into the future. Here "very far into the future" is a vague notion, but it clearly has something to do with the length of the time series that is used in the trend calculation.

In the case of workers' compensation data, there has traditionally been some doubt as to whether an underlying trend exists at all. The use of payroll as a measure of exposure and the special handling of law amendments were intended to encompass the economic changes that would affect losses. As economic indices are used more often in other lines in the coming years, these lines, too, will generate times series data in which there is some *a priori* doubt about the assumption that there is any remaining trend.

This situation has led to a study of the credibility of trend factors. To what extent should the trend forecast be relied on, and to what extent the historical average? The answer depends on the situation at hand and on the length of the time series and the goodness of fit of the trend line. There is a practical problem in tying these considerations together.

The NCCI has adopted a framework for computing the credibility-weighted trend factor. This is illustrated in Exhibit I. This paper is not intended to be a review or criticism of the NCCI method. It is intended rather to illustrate an alternate approach.[1]

*Purpose*

If the actuary does not use credibility-weighted trend factors, or something equivalent, he must rely on a single assumption about the population from which his sample data was drawn. He might assume, for example, that all of the sample values are from a population with a mean (expected value) that is unchanging. Or he might assume that the sample values are from a population with a mean that is changing steadily over time. He might assume that the steady change is linear, quadratic, exponential or some other form. Whatever assumption he makes, he must use the indicated results of that one assumption. One purpose of this paper is to show that the actuary's options are not so limited. The paper proposes a method for combining the projections from two or more sets of assumptions, rather than having to choose between them.

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[1] Charles A. Hachmeister and G. C. Taylor have proposed other methods in papers in *Credibility: Theory and Applications*, P. M. Kahn, Ed., Academic Press, 1975.

Because of the reliance placed on the fraction of variance explained, $R^2$, in the application of trend factors derived by the regression analysis, this paper has a second purpose. It seeks to examine the implications of $R^2$ on (1) the credibility of the slope of the trend line, (2) the slope of the trend line and (3) the accuracy of the resulting forecast. By doing this for a particular application of the concepts of the first section, it intends to provide an example of how the effects of $R^2$ can be examined in other applications. This paper suggests some interesting conclusions. These are:

1. If only two alternative assumptions are considered—no trend and linear trend—and no *a priori* judgments are introduced, then the credibility-weighted trend factor declines asymptotically to zero as the length of the projection increases.
2. For these same two alternative assumptions, an increase in $R^2$ from one application to the next implies an increase in the credibility of the trend line, or reliance on a greater amount of trend, or a more reliable resulting estimate. A combination of these is also possible. Which of these three situations is really the case depends on the problem at hand. One cannot generally assume that a greater value of $R^2$ in one application than in another will imply greater credibility for trend.

*Derivation of Credibility-Weighted Trend Factors*

The purpose of this section is to show that it is not necessary to make a single assumption about the trend in order to estimate the value of a time series at some time in the future. This is shown by deriving a trend line by assuming that: (1) either there is no trend, or (2) there is a linear trend. The steps shown here could be extended to allow three or more assumptions to be reflected in the computation. Two assumptions are used to simplify the mathematics.

The projection for the value at time $X$ depends on the assumption about trend that is being used. If the assumption that there is no trend is being used, the estimate of the value at any time in the future would be the average of the historical values, i.e.,

$$\hat{Y}(X) = \overline{Y} = (\Sigma Y_i)/n \qquad (1)$$

for all $X$.

(There is no discussion of maximum likelihood or minimum variance in this statement or those which follow. This would be a useful addition to this work. Also, it should be clear that all of the summations are for $i = 1, \ldots, n$.)

If the assumption is that there is linear trend, the estimate of the value at some time $X$ would be

$$\hat{Y}(X) = \overline{Y} + \frac{\Sigma(X_i - \overline{X})(Y_i - \overline{Y})}{\Sigma(X_i - \overline{X})^2} \cdot (X - \overline{X}), \tag{2}$$

where $\overline{X} = (\Sigma X_i)/n$.

In the problem we are dealing with, we do not wish to choose between these estimates because that would be the same as choosing between the alternative assumptions. Instead, we wish to regard each estimate as a valid estimate based on the data at hand.

If each estimate is a valid estimate based on the data on hand, then we have no preconceived way of improving any of the estimates. We know of no correction terms which can be added *a priori* to improve either of the estimates. In other words, for each estimate

$E$ [estimate of $Y$] $= Y$.

In statistical terms, each estimate is unbiased.

In most of our experience with estimators we are accustomed to the idea that only one of several alternative models can be unbiased. For example, if the model of linear trend is unbiased, the model of no trend must be biased. The formula omits the term for the trend component. How then, can each of the estimates be unbiased, as stated above? The answer is that we are not dealing with models in the formulation above. We are dealing only with empirical evidence and what can be learned from it. And given only the *data* at hand, each estimate is unbiased.[2]

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[2] Consider a set of alternative states of the world, $\theta$. Each value, $\theta_i$, is associated with a particular model being valid. We do not know which value of $\theta$ exists for our problem, since we have only empirical evidence about the problem. The discussion above states that

$E[\overline{Y}|\theta_1] = Y$ and

$$E\left[\overline{Y} + \frac{\Sigma(X_i - \overline{X})(Y_i - \overline{Y})}{\Sigma(X_i - \overline{X})^2} \cdot (X - \overline{X})|\theta_2\right] = Y.$$

This does not imply that $E\left[\frac{\Sigma(X_i - \overline{X})(Y_i - \overline{Y})}{\Sigma(X_i - \overline{X})^2} \cdot (X - \overline{X})\right] = 0.$

The mathematics of the approach parallels that of empirical Bayes methods of Hans Bühlmann, *Mathematical Methods in Risk Theory*, Springer-Verlag, New York, New York, 1970, pp. 93–110.

A theorem of statistics states that if two estimators are unbiased and independent, then the minimum variance estimator is the weighted average of the two estimators with weights inversely proportional to the variances of the two (c.f., D. A. S. Fraser, *Probability and Statistics*, Duxbury Press, 1976, p. 382). This theorem can be applied to $\hat{Y}(X) - \bar{Y}$, which is zero in the first case and

$$\frac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2} \,.\, (X - \bar{X}) \tag{3}$$

in the second case.

We have changed the definition of the problem now and ought to check that we are still solving the problem we want to solve. The new problem is to estimate the amount by which the time series will exceed its historical average (as it is known now) at some time in the future. This is not quite the same problem, but it certainly encompasses our reasons for using trend lines.

To apply the theorem we need to know only the variance associated with each estimate. The variance in the first estimate is the population sample variance,

$$V_A = \frac{\Sigma(Y_i - \bar{Y})^2}{n - 1} \tag{4}$$

The variance of the second, trended estimate is

$$V_T = \frac{\Sigma(Y_i - \bar{Y})^2}{n - 1}\left[\frac{1}{n} + \frac{(X - \bar{X})^2}{\Sigma(X_i - \bar{X})^2}\right] \tag{5}$$

The desired estimator of $\hat{Y}(X) - \bar{Y}$ is, therefore,

$$= \frac{\dfrac{1}{V_A}\cdot 0 + \dfrac{1}{V_T}\cdot\dfrac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2}\cdot(X - \bar{X})}{\dfrac{1}{V_A} + \dfrac{1}{V_T}} \tag{6}$$

$$= \frac{V_A\cdot\dfrac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2}\cdot(X + \bar{X})}{V_A + V_T}$$

$$= \frac{V_A\cdot\dfrac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\Sigma(X_i - \bar{X})^2}\cdot(X + \bar{X})}{V_A + V_A\cdot\left[\dfrac{1}{n} + \dfrac{(X - \bar{X})^2}{\Sigma(X_i - \bar{X})^2}\right]}$$

$$= \frac{\Sigma(X_i - \bar{X})(Y_i - \bar{Y})}{\frac{n+1}{n} \Sigma(X_i - \bar{X})^2 + (X - \bar{X})^2} \cdot (X - \bar{X} \qquad (7)$$

This is similar to the trend estimate. The difference is in the denominator of the slope, which now includes the term $1/n \cdot \Sigma(X_i - \bar{X})^2 + (X - \bar{X})^2$. This is a quadratically increasing function of $X - \bar{X}$, so the credibility-weighted trend line is a declining function of $X$ as $X$ moves away from $\bar{X}$. In fact, this estimate of $\hat{Y}(X) - \bar{Y}$ tends to zero as $X - \bar{X}$ gets very large, which means the credibility of the trend goes to zero as the extrapolation is taken far into the future.

Exhibit I provides the data for a numerical example. (We shall ignore the problems caused by autocorrelation in the observed values for loss ratios; they are beyond the scope of this paper.) The key values can be taken from Exhibit I as follows:

$$\begin{aligned}
n &= 9 \\
\bar{X} &= 2 \\
\Sigma(X_i - \bar{X})(Y_i - \bar{Y}) &= \Sigma X_i Y_i - (\Sigma X_i)(\Sigma Y_i)/n \\
&= 11.354 - 18 - 5.334/9 \\
&= .686 \\
\Sigma(X_i - \bar{X})^2 &= \Sigma X_i^2 - (\Sigma X_i)^2/n \\
&= 51 - 18^2/9 \\
&= 15
\end{aligned}$$

The slope of the trend line, assuming a linear trend exists, is .686/15, or .0457. The height of the revised trend line, without assuming that a trend line exists (but assuming that if it does not there is no change in the expected value of the loss ratio over time), is

$$\hat{Y} - \bar{Y} = \frac{.686}{\frac{50}{3} + (X - \bar{X})^2} \cdot (X - \bar{X})$$

Extrapolated values of the time series of loss ratios are shown in Exhibit II.

For this set of data and this set of alternative assumptions, the credibility-weighted trend line is well below the linear regression trend line. This is because of the set of alternative assumptions used.

The trend, if any, could be exponential or quadratic, and considering these possibilities would raise the credibility-weighted trend line. *A priori* considerations could also lead one to give greater weight to the linear trend line. This paper does not advocate the use of the two-assumption formula in equation (7), but uses it to illustrate a general approach for determining credibility-weighted trend factors by averaging several separate projections using weights inversely proportional to each projection's variance.

There is another reason for the low trend line: the linear trend line is based on only nine data points. It is therefore not reliably estimated from the data alone.

*The Effects of $R^2$ on the Credibility-Weighted Trend Factors*

One would expect that the better the fit of the linear regression, the more credible the trend factors would be. This turns out to be the case, but only in a limited way. This section shows that for a given number of historical observations:

- If the slope of the trend line and the variance of the observations are held constant, an increase in $R^2$ implies an increase in the credibility of the trend line.
- If the variance of the independent variable and the variance of the observations (the dependent variable) are held constant, an increase in $R^2$ increases the slope of the trend line but not necessarily its credibility.
- If the variance of the independent variable and the slope are held constant, an increase in $R^2$ does not affect the credibility of the trend line. It does, however, increase the credibility of any forecasts based on the credibility-weighted trend line, the trend line or the simple average.

We must begin by deriving the credibility of the trend that is implicit in the credibility-weighted trend line. Equation (6) shows that the credibility of the trend estimate is

$$Z = \frac{\dfrac{1}{V_T}}{\dfrac{1}{V_A} + \dfrac{1}{V_T}}$$

This is what one would expect from the statistical theorem. This can be repressed in terms of the data as:

$$Z = \frac{V_A}{V_A + V_A\left[\dfrac{1}{n} + \dfrac{(X - \overline{X})^2}{\Sigma(X_i - \overline{X})^2}\right]}$$

$$= \frac{1}{1 + \frac{1}{n} + \frac{(X - \overline{X})^2}{\Sigma(X_i - \overline{X})^2}}$$

$$= \frac{n}{n + 1 + \frac{(X - \overline{X})^2}{\Sigma(X_i - \overline{X})^2/n}}$$

This is the familiar form for credibility. The number of points in the time series plays the role of exposure, $n$, and the "exposure constant" $K$ is a function of the length of the extrapolation and the spread of the independent observations about their mean.

In terms of the data from which it is calculated, $R^2$ can be expressed as

$$R^2 = \frac{[\Sigma(X_i - \overline{X})(Y_i - \overline{Y})]^2}{\Sigma(X_i - \overline{X})^2\Sigma(Y_i - \overline{Y})^2}$$

An abbreviated notation will make the relationships clearer. Let

$$SS_{XY} = \Sigma(X_i - \overline{X})(Y_i - \overline{Y})$$

$$SS_X = \Sigma(X_i - \overline{X})^2$$

$$SS_Y = \Sigma(Y_i - \overline{Y})^2$$

Then

$$R^2 = \frac{SS^2_{XY}}{SS_X SS_Y}$$

$$Z = \frac{n}{n + 1 + \frac{(X - \overline{X})^2}{SS_X/n}}$$

The credibility-weighted trend factor is

$$\frac{SS_{XY}}{\frac{n+1}{n} SS_X + (X - \overline{X})^2} \cdot (X - \overline{X})$$

The trend factor itself is

$$\frac{SS_{XY}}{SS_X} \cdot (X - \overline{X})$$

and the slope of the trend line is $SS_{XY}/SS_X$.

If the slope of the trend line, $SS_{XY}/SS_X$, and the variance of the observations, $SS_Y/n-1$, are both held constant, then an increase in $R^2$ implies an increase in $SS_{XY}/n-1$. Since $SS_{XY}/SS_X$ is constant, this implies an increase in $SS_X/n-1$. An increase in $SS_X/n-1$ implies an increase in $Z$, and the first point is established.

If the variance of the independent variable, $SS_X/n-1$, is held constant, $z$ is a function of $n$ and $(X - \overline{X})$ only. If $SS_X/n-1$ and the variance of the observations, $SS_Y/n-1$, are held constant, an increase in $R^2$ implies an increase in $SS_{XY}/n-1$, and hence of the trend factor itself. This establishes the second point.

If the variance of the independent variable, $SS_X/n-1$, and the slope, $SS_{XY}/SS_X$, are held constant, an increase in $R^2$ implies a decrease in $SS_Y/n-1$. This does not affect either the trend or the credibility of the trend. The variance of the credibility-weighted estimate is (see Fraser, op. cit):

$$\frac{1}{\dfrac{1}{V_A} + \dfrac{1}{V_T}} = \frac{V_A \cdot V_T}{V_A + V_T}$$

$$= \frac{V_A \cdot V_A \cdot \left[\dfrac{1}{n} + \dfrac{(X - \overline{X})^2}{SS_X}\right]}{V_A + V_A \cdot \left[\dfrac{1}{n} + \dfrac{(X - \overline{X})^2}{SS_X}\right]}$$

$$= [SS_Y/(n - 1)] \frac{\dfrac{1}{n} + \dfrac{(X - \overline{X})^2}{SS_X}}{1 + \dfrac{1}{n} + \dfrac{(X - \overline{X})^2}{SS_X}}$$

Therefore, a decrease in $SS_Y/n-1$ implies a decrease in the variance of the credibility-weighted estimate. The rest of the third point can be demonstrated using a similar analysis.

In summary, for a given number of observations, an increase in $R^2$ implies an increase in the credibility of the trend line, or reliance on a greater amount of trend, or a more reliable resulting estimate. A combination of these is also possible. Which of these three situations is really the case depends on the problem at hand.

These conclusions rest on the choice of alternative assumptions that was made. That choice was (1) that there is no trend, or (2) that there is linear trend. And the phrase "more reliable" is only valid in its least-squares sense. Still, these conclusions point up the fact that a greater $R^2$ does not necessarily imply greater credibility for trend.

*Summary*

The credibility of trend lines is important because trend lines cannot be extrapolated reliably far into the future. Credibility-weighted trend factors can be calculated if two or more alternative assumptions are considered. The effects of changes in the goodness of fit of the trend lines being considered can also be explored.

The methods shown in this paper can be extended to other sets of assumptions. Other questions about the factors that contribute to the appropriateness of trend assumptions can also be studied.

If a particular pair of alternative assumptions is considered—that there is no trend and that there is linear trend—an increase in the $R^2$ of the linear trend line may imply an increase in the credibility of the trend line, reliance on a greater amount of trend, or a more reliable resulting estimate. Which of these or which combination of these is the case depends on the data at hand. A greater $R^2$ does not necessarily imply greater credibility for trend.

# EXHIBIT I

## NATIONAL COUNCIL ON COMPENSATION INSURANCE

NATIONAL COUNCIL ON COMPENSATION INSURANCE

CALCULATION OF TREND FACTOR

| (0) Temporal Rank | (1) Calendar Year | (2) Index (x) For (1) | (3) Standard Earned Premium | (4) Incurred Losses Incl. Loss Adj. | (5) Factor To Adjust Premium | (6) Factor To Adjust Losses | (7) Earned Premium On Level (3)x(5) | (8) Incurred Losses On Level (4)x(6) | (9) Loss Ratio (y) (8)÷(7) | (10) Loss Ratio Rank | (11) $x^2$ $(2)^2$ | (12) xy (2)x(9) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1973 | 0.0 | 101,757,432 | 78,532,264 | 1.606 | 1.075 | 163,422,436 | 84,422,184 | .517 | 1 | 0.00 | 0.000 |
| | 1973-74 | 0.5 | 109,194,609 | 82,846,099 | 1.536 | 1.070 | 167,722,919 | 88,645,326 | .529 | | 0.25 | 0.265 |
| 2 | 1974 | 1.0 | 110,495,110 | 82,016,281 | 1.477 | 1.069 | 163,201,277 | 87,675,404 | .537 | 2 | 1.00 | 0.537 |
| | 1974-75 | 1.5 | 113,385,164 | 86,052,451 | 1.413 | 1.059 | 160,213,237 | 91,129,546 | .569 | | 2.25 | 0.854 |
| 3 | 1975 | 2.0 | 116,229,990 | 90,187,533 | 1.341 | 1.050 | 155,864,417 | 94,696,910 | .608 | 4 | 4.00 | 1.216 |
| | 1975-76 | 2.5 | 125,236,306 | 89,761,215 | 1.270 | 1.048 | 159,050,109 | 94,069,753 | .591 | | 6.25 | 1.478 |
| 4 | 1976 | 3.0 | 141,708,133 | 97,926,553 | 1.205 | 1.042 | 170,758,300 | 102,039,468 | .598 | 3 | 9.00 | 1.794 |
| | 1976-77 | 3.5 | 162,912,010 | 117,405,688 | 1.132 | 1.038 | 184,416,395 | 121,867,104 | .661 | | 12.25 | 2.314 |
| 5 | 1977 | 4.0 | 180,000,246 | 134,741,173 | 1.060 | 1.025 | 190,800,161 | 138,109,702 | .724 | 5 | 16.00 | 2.896 |
| TOTAL | | 18.0 | xx | xx | xx | xx | xx | xx | 5.334 | xx | 51.00 | 11.354 |

13. $D = \Sigma((0) - (10))^2 = 0 + 0 + 1 + 1 + 0$ .................... 2

14. Mid-point of Experience in Filing from 7-1-73 .................... 4-1-77 or 3.7500

15. Mid-point of Period during which proposed rates effective .................... 9-1-79 or 6.1667

16. Annual Increment in Loss Ratio = $B = [9\,\Sigma(12) - \Sigma(2)\,\Sigma(9)] \div [9\,\Sigma(11) - (\Sigma(2))^2]$ .................... .0457

17. Loss Ratio at Base = $A = [\Sigma(9) - (16)\,\Sigma(2)] \div 9$ .................... .5013

18. Probability of $D \leq (13)$ .................... .0417

19. Credibility $[100\,(.5 - 2 \times (18))^{1/2}] \div .7$ .................... 92%

20. Trend Factor prior to credibility $[(17) + (16)(15)] \div [(17) + (16)(14)]$ .................... 1.163

21. Credibility weighted Trend Factor $[(19) \times (20)] + [(1.00 - (19)) \times 1.000]$ .................... 1.150

# EXHIBIT II

## CREDIBILITY-WEIGHTED TREND LINE