A NOTE ON BASIC LIMITS TREND FACTORS

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DISCUSSION BY JOHN P. ROBERTSON

Mr. Finger's paper makes the excellent point that the frequently noted fact that excess losses tend to rise faster than overall loss costs has a converse, namely, that basic limits costs tend to rise more slowly than overall loss costs. He then proceeds to give a method for modeling these changes and gives examples using the lognormal distribution.

There is one point, however, that needs to be clarified before the methods of the paper are used. This is that the *ART* computed by the method of the Appendix is not quite the same as the *ART* defined on page 109:

$$ART(R,i) = \frac{1}{i} \frac{(1 + i) \cdot X(R/(1 + i)) - X(R)}{X(R)}$$

The *ART* defined on page 109 is a "linear" *ART*. That is, if the factor for the total limits cost change (*TLCC*) is $1 + i$, then the factor for the basic limits cost change (*BLCC*) is given by $BLCC = 1 + ART \cdot i$. To see that this *ART* is not the one computed in the Appendix, consider the following example.

Start with a lognormal loss distribution with $CV = 0.4$ and ratio of basic limits to the total mean of 10. Then let total costs change by a factor of 100. This then makes the ratio of the basic limit to the total mean become 0.1. Since the $CV = 0.4$, there are very few claims that exceed ten times the mean or fall below one tenth of the mean. Thus, at first, one was paying practically the total amount of all claims since very few claims exceeded the basic limit. After the cost change, one pays one tenth of the total amount of losses, since one pays the basic limit on almost all claims and the basic limit is one tenth the mean claim cost. Basic limits costs have, therefore, increased by a factor of 10 $(=(100M \cdot 0.1) \div (M \cdot 1))$. The "linear" *ART* then satisfies $1 + ART \cdot 99 = 10$, or $ART = 1/11 = 0.09$. This can also be obtained by taking $(1/99) \cdot (100 \cdot (0.1) - 1) \div (1)$ per the second formula on page 109 ($X(10) = 1$, $X(0.1) = 0.1$, $i = 99$, $1 + i = 100$).

On the other hand, using the method of the Appendix, one computes the ART to be $(2.297 - 0) \div (2.303 - (-2.303)) = 0.50$. What is this ART?

I have learned through correspondence with Mr. Finger that the ART of the Appendix is an "exponential" one and satisfies $(TLCC)^{ART} = BLCC$. Note that $(100)^{0.50} = 10$. Call the linear one ART_L and the exponential one ART_E. Then $(1 + i)^{ART_E} = 1 + ART_L \cdot i = BLCC$, where $1 + i = TLCC$. Thus, given either ART, the other can be computed.

Regardless of ART used, we have

$$BLCC = \frac{TLCC \cdot X(R/TLCC)}{X(R)}$$

That is, the basic limits cost change is the total limits cost change times the change in the percentage of losses that are below the basic limit. R is the ratio of the basic limit to the unlimited mean before the cost change. This holds regardless of the form of the size of loss distribution.

I determined that Table II of the Appendix shows $\ln(R/X(R))$ and $\ln(R)$ for various R and CV for lognormal distributions. These can then be used to compute the ART_E by the method cited, since

$$ART_E = \frac{\ln(R/X(R)) - \ln((R/TLCC)/X(R/TLCC))}{\ln(R) - \ln(R/TLCC))} \quad \text{(By the rules given in the appendix)}$$

$$= \frac{\ln(TLCC \cdot X(R/TLCC)/X(R))}{\ln(TLCC)}$$

$$= \log_{TLCC}(TLCC \cdot X(R/TLCC)/X(R))$$

This implies $TLCC^{ART_E} = \dfrac{TLCC \cdot X(R/TLCC)}{X(R)} = BLCC$

which is the required relationship.

Table II should be labeled as showing

$$\int_{-\infty}^{\ln A} f(e^z)dz$$

This can be shown to be exactly equal to $\ln(A/X(A))$. The proof of this equivalence does not depend on the properties of the lognormal, but rather applies generally to all distributions. I will be happy to send this proof to anyone who requests it.

In order to follow example 1, it is useful to note that if $1 + i$ is the average *annual* total cost change, over n years, with *overall* total cost change $(1 + i)^n$, then the overall basic limit cost change is $(1 + i)^{nART_E}$. This gives an *annual* basic limits cost change of $(1 + i)^{ART_E}$ which is approximately $1 + ART_E \cdot i$. This is the reasoning that allows the ART_E to be applied to annual cost changes instead of overall cost changes.

Another item deserving of mention is the author's definition of B as a function of one variable, i.e. as $B(A/M)$ on page 107. This reviewer finds a definition of B as a two variable function, $B(A,M)$, more reasonable. The definition of B as a one variable function obscures the relationship $B(A,(1 + i)M) = (1 + i)B(A/(1 + i),M)$ which is needed for the derivation of the formula for ART_L on pages 108 and 109. By noting that $X(R) = B(RM,M)/T(M)$ and that this definition of $X(R)$ does not depend on the choice of M, the author's proof follows.

In summary, Mr. Finger has provided a mechanism for comparing basic limits cost changes to total limits cost changes. He points out that such changes can be modeled with the lognormal distribution, and in many cases it is possible to obtain useful results from such a model even when the shape of the lognormal cannot be determined exactly. This reviewer hopes that clarification of the above technical detail will help readers understand Mr. Finger's paper.