DISCUSSION BY DALE NELSON

"In any case, I am convinced that God does not play dice."
—A. Einstein, 1926.

In a 1970 paper,[1] Donald Weber presented a stochastic model of the automobile accident process. In that paper, Weber took age and gender differences into account in a deterministic fashion by means of some ad hoc rational functions of time. The present paper, on the other hand, deals with these differences explicitly though a stochastic model, by using a Markov process to describe how an individual's accident likelihood varies over time. The latter approach is much more satisfying since it recognizes that accident likelihoods do vary among individuals with otherwise identical risk characteristics.[2] In the light of current controversies over risk classification, this paper undoubtedly will be an important contribution to the literature, and it is certainly a timely and thought-provoking one.

Emilio Venezian's paper is not a particularly easy paper to read. One's inclination is either to be caught up in the intricacies of the model—and the attendant need to make the underlying assumptions more "realistic"—or to be turned off by the formidable mathematics involved, the latter being a characteristic of stochastic processes generally. My comments will steer clear of either extreme, and instead I will try to consider some of the implications of these kinds of models.

Individual Models

There are two ways that these models can be used to describe the accident process. The first, which is the way Venezian has developed his model, is to use the model to explain the expected behavior of an individual over time. The Poisson model is the most familiar of these models; in its simplest form it assumes that the accident likelihood is constant over time. This obviously is

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[1] Donald C. Weber, "A Stochastic Approach to Automobile Compensation," *PCAS* LVII (1970), pp. 27–63.

[2] That is, identical in the sense that there are no clearly identifiable differences between individuals in a particular risk group. See Michael A. Walters, "Risk Classification Standards," *Pricing Property and Casualty Insurance Products*, 1980 Discussion Paper Program, Casualty Actuarial Society.

simplistic, but it does provide a useful model. The Polya model, which Venezian discusses briefly and which leads to one formulation of the negative binomial distribution, assumes that the individual's accident likelihood increases linearly with the number of prior accidents. In contrast, if the likelihood is assumed to decrease, rather than increase, the binomial model results.[3] A variety of other individual models are cited in the Ashton[4] and Seal[5] references.

All of these generalizations of the Poisson model are derived from a basic premise that an individual's future accident likelihood is somehow dependent on his/her prior record. Venezian's model, however, is premised on the assumption that the individual's future likelihood is dependent only on the person's present likelihood. Specifically, individuals are in one of two states—either they are "good" or they are "bad"—at any moment of time, with a fixed probability of moving to the other state at the next moment of time. Fitting this model to some California Driving Record data, the evaluated parameters indicate that the "bad" drivers have an accident likelihood which is about 4.5 times that of the "good" drivers; that there is, roughly, a .17 chance at any instant of time of someone in the "bad" state moving to the "good" state and a .03 chance of someone in the "good" state moving to the "bad" one; and that, under the assumption that everyone starts out in the "bad" state, girls start to move to the "good" state a couple years ahead of the boys. Because of the difference between the two transition probabilities mentioned above, there is a long-term drift toward the "good" state. After several years, you can expect about 85% (.85 = .17/(.03 + .17)) of the individuals to be in the "good" state and 15% to be in the "bad" state at any moment of time. Thus the model, at once, seems to account for the observed differences between males and females and between different age groups, and also allows for some "unexplainable" heterogeneity within these groups.

Or does it? Let's go back to the Markov assumption. It says that the probability of being in the "good" or "bad" state at the next moment of time is dependent only on the individual's current state. In other words, from the point of view of this model, there is no difference between an 18 year old male

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[3] Winifred D. Ashton, *The Theory of Road Traffic Flow*, Methuen's Statistical Monograph Series, John Wiley and Sons, Inc., New York, 1966, pp. 148–168.

[4] *Ibid.*

[5] Hilary L. Seal, *Stochastic Theory of a Risk Business*, John Wiley and Sons, Inc., New York, 1969, pp. 12–29.

and a 55 year old female, for example, if it is known that presently both are in the same state (and, of course, are identical in all other respects, i.e., drive the same amount, drive in the same locale, etc.). How many underwriters—or actuaries, for that matter—will accept that conclusion? I know one group—the critics of risk classification—that would readily accept the conclusion, since it seemingly argues against the classification of individuals on the basis of personal characteristics.

Although not usually stated in so many words, underlying any classification plan is a belief that accident likelihoods are definite quantities and that changes in these likelihoods over time occur, if not deterministically, at least in some causal, generally non-random fashion. This is not the case under the author's model. But to the extent a person's accident likelihood is determined stochastically, I'm not sure classification is appropriate. Would you classify individuals as to whether they were more or less likely to have an accident if the mechanism involved were the toss of one of two biased coins and the choice of the coin, in turn, were the result of tossing yet another biased coin?

Aside from this consideration, I have some other difficulties with the model as outlined in the paper; these comments are more along the lines of making the assumptions more realistic. For instance, it is hard for me to visualize how persons can constantly be jumping back and forth between the "good" and "bad" states. This difficulty can be overcome by hypothesizing a larger number of states—perhaps even a continuum—and a more gradual drifting up and down the scale. Also, since a learning curve probably is involved—from both driving experience and driving record—the Markov assumptions perhaps could be relaxed to permit time-varying transition probabilities. However, this still leaves the process as being essentially a stochastic one—that is, one in which an individual's accident likelihood is as much a chance event as whether or not he/she actually has an accident. And, as discussed above, this seems to suggest the inappropriateness of risk classification or underwriting based on individual characteristics.

Cohort Models

There is a way of using the model, however, that seems much more satisfactory, and which overcomes this apparent conflict with risk classification. That would be to regard it as a description of the expected behavior over time of a group of individuals with similar risk characteristics. Thus the model no longer describes the behavior of individuals, but simply provides the expected distribution of these individuals among the "good" and "bad" states at various points in time. With this interpretation, the model says nothing about individual behavior, and in particular it does not imply that the movement between states is stochastic.

I think it is in the context of this interpretation of the model that much of the author's discussion has been written and, in any event, seems to hold.

The time-varying mix of business between "good" and "bad" states clearly implies the need for the fine-tuning of merit rating relativities. In fact, the model suggests that merit rating is of very little value for youthful risks, and that the use of such plans could best be limited to adults. Further, to the extent that underwriting is intended to identify the "good" risks, the model suggests that little would be accomplished by selective underwriting of youthful risks. Where both merit rating and underwriting are useful, the model implies an interesting but not surprising interaction between the two. Namely, the more successful the underwriting effort is at identifying "good" risks, the less important is the role of merit rating, and vice versa.

The validity of these implications is, of course, dependent on the validity of the model. However, as the author suggests, we probably will never be able to determine with any certainty which of several alternative models is the correct one—or even which of several possible explanations or interpretations of a particular model makes the most sense. The random element of the accident process is so large, and the available statistical estimation techniques are so robust, that empirical data "fit" a wide variety of models. For example, as has been frequently pointed out,[6,7] it generally is difficult to distinguish between a two state process such as the author is using and one which includes several (or possibly even a continuum of) states. These practical difficulties become virtually insurmountable as you complicate the models to make them more realistic (e.g., to measure the risk of insured cars, rather than individual operators; to reflect inflation, changes in driving patterns, changes in the types of cars on the road; etc.)

All this suggests that the value of these models is more in the qualitative insights that they can provide, than in any precise, verifiable predictions. As such they are more akin to economic models than to the types of models used in the physical sciences. This also means that to understand or test these models, you cannot rely on comparing predicted results with empirical evidence. You must know the underlying assumptions, and ask whether these make sense. In the present case, this also means that, as actuaries, we probably should know more about stochastic processes—and their mathematical development—than most of us do. The latter is tough going, but stimulating; I suspect the examination syllabus eventually will include a great deal more on these processes, simply as a matter of necessity.

The author is to be congratulated for a thought-provoking and timely contribution.

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[6] *Ibid.*

[7] Robert A. Giambo, "SRI and the Gamma Poisson: A Review of the Stanford Research Institute Report," presented at the Casualty Actuarial Society Risk Classification Conference, March 30–31, 1981. In his discussion, Mr. Giambo presented a simple example showing that the expected accident distribution for a class of "good" and "bad" drivers is, for all practical purposes, indistinguishable from one in which the accident likelihoods have a gamma density. This example and similar ones, which can be constructed easily, also shed light and cast doubt on some of the recent findings regarding class heterogeneity and overlap (e.g., Division of Insurance, Commonwealth of Massachusetts, *Automobile Insurance Risk Classification: Equity and Accuracy*, 1978; Department of Insurance, State of New Jersey, *Final Determination — Analysis and Report. In re: Hearing on Automobile Insurance Classifications and Related Methodologies*, 1981). In particular, these examples suggest that the relative variance of the accident likelihood distribution is not an adequate measurement of homogeneity, and that the degree of overlap between classes is greatly dependent on the assumptions regarding the underlying distribution of accident likelihoods. For instance, while there usually is a good deal of overlap between two different gamma densities, representing the distribution of accident likelihoods for two different classes, there may be little or no overlap if the two classes are each represented by "good-bad" distributions.

Similarly, the homogeneity depicted by a gamma density is vastly different than that represented by a "good-bad" distribution, even though their relative variances may be the same. Or, two different "good-bad" distributions may be equally homogeneous — in the sense of having a similar dispersion of accident likelihoods—yet have different relative variances.