COMPUTER SIMULATION AND THE ACTUARY: A STUDY IN REALIZABLE POTENTIAL

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Abstract

This paper argues that computer simulation is an underappreciated and, therefore, underutilized casualty actuarial resource. In so contending, "Computer Simulation and the Actuary" discusses five applications of Monte Carlo computer simulation to everyday actuarial problems: establishing full credibility standards; testing the solidity of new, limited purpose insurance companies; pricing difficult or catastrophic exposures; customizing casualty insurance charges and excess loss premium factors; and developing loss reserve confidence intervals.

Illustrations of appropriate simulation solutions to each of these problems are provided.

OVERVIEW

Computer simulation refers to the process of accurately describing a complex system in a computer language, inputting this program into a computer, and allowing the machine to mimic ("simulate") the performance of the system described. For example, computers can easily be programmed to simulate accident year loss experience, given specific claim frequency and severity assumptions.

Historically, simulation has been afforded relatively little attention in the actuarial literature. Moreover, although this technique has been employed by actuaries confronting problems not soluble by more traditional means, primary emphasis has been placed upon non-simulation pricing and reserving procedures.

Reasons for this reluctance to rely more heavily upon simulation in addressing actuarial problems have included the lack of an adequate computer, the expense of the computer's operation, and occasionally the actuary's unfamiliarity with programming languages. Also, the need for simulation approaches has been somewhat mitigated by the publication in these *Proceedings* of elegant and impressive analytical solutions to most really difficult pricing problems.

The above obstacles are fast disappearing. Cost and access problems, for instance, are being overcome by the widespread introduction of microcomputer systems. "Conversational" programming languages, such as the BASIC language in which all simulations presented in this paper are written, are easy to learn and available on most systems. Moreover, special simulation languages, such as GPSS and SIMSCRIPT, give some simulation capability to the actuary without extensive programming knowledge.

More importantly, today's property/casualty insurance business faces problems which can be solved better and sooner with the assistance of computer simulation. The following sections present five such problems along with examples of appropriate simulation solutions. In presenting these illustrations, this paper argues that it is both inevitable and desirable that computer simulation will become an increasingly important weapon in the casualty actuary's arsenal.

Outline of This Paper

- Section I describes how computer simulation may be used to rediscover and expand upon classic "limited fluctuation" credibility notions. In so doing, this section provides a foundation for the more complex simulation applications presented in Sections II–IV.
- Section II illustrates a method for extending Section I's loss simulation procedure to test the solidity of a newly-formed insurance company.
- Section III incorporates computer simulation into the pricing of pneumoconiosis (coal miner's "black lung") exposures. The techniques described in this section can be utilized in the pricing of virtually any new, unique, or catastrophic exposure.
- Section IV uses the results of Section I's loss simulations to illustrate a procedure for developing insurance charges for casualty individual risk rating programs. This section then concludes with an example of a possible use of computer simulation in computing loss reserve confidence intervals.

I. EVALUATING FULL CREDIBILITY STANDARDS[1]

Computer simulation provides an alternative method for establishing the fundamental notions of credibility theory. In addition, a simulation-based approach imparts greater flexibility, and thereby a means for expanding upon some of the basic actuarial developments in this area.

[1] Section I discusses how computer simulation can be used to develop and apply *limited fluctuation* credibility theory. For the interested reader, Appendix E illustrates how a computer can also assist the actuary in explaining and applying Bühlmann/Hewitt's *greatest accuracy* credibility model.

The Classic Credibility Problem

A casualty actuary draws reasonable conclusions based upon data. More precisely, he translates these data into estimates of some future variable, such as next year's Workers' Compensation loss ratio or the amount of self-insurance funding required by a large commercial risk.

Inherent in the above process is the actuary's determination of the credence to be placed in the underlying data. In making this decision, he uses his experience to select a realistic volume of data which he will consider to be fully representative of the variable being estimated. In establishing this "full credibility standard," the actuary balances the conflicting objectives of stability and responsiveness.

Once this full credibility requirement has been established, the actuary next determines the maximum probable error in his estimate, given a fully credible volume of data. If this maximum error is unacceptably high, the full credibility criterion is revised upward. The *classic credibility problem* refers to this problem of determining the probable maximum error in an estimate developed from "fully credible" data.

This section begins by examining traditional actuarial solutions to this problem. Results obtained are then compared with corresponding figures developed using computer simulation. Finally, the relative advantages and limitations of the two approaches are compared.

The Basic "Limited Fluctuation" Credibility Model

The simplest and most popular model for evaluating the potential error implied by a particular full credibility standard assumes that an individual risk's claim frequency is Poisson distributed, and that all losses are of some fixed amount.[2] Under these conditions, the volatility in an estimate developed from a specified volume of loss experience is calculated by means of a relatively simple formula.[3]

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[2] L. H. Longley-Cook, *An Introduction to Credibility Theory* (hereafter cited as "Longley-Cook").

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[3] Assuming that the expected number of claims can be estimated without error, the formula becomes Confidence Bounds = $\pm P/E^{1/2}$, where P is the appropriate z-statistic obtained from a standard normal distribution table, and E is the expected number of claims.

For example, selecting 1,082 claims as one's full credibility standard implies that the actual losses arising out of a fully credible sample will fall within ±5% of expected levels 90% of the time, given the previous frequency and severity assumptions.[4]

An Alternative Development

Given these simple frequency and severity assumptions, an alternative means of estimating the statistical reliability of a selected full credibility standard is possible. As indicated earlier, this second approach involves computer simulation.

To illustrate, assume that:

- claim frequency is Poisson distributed, and therefore approximately normally distributed, with a mean of 1,000 claims;
- all claims cost $5,000.

Given these conditions, one can easily program a computer to simulate 1,000 random trials ("years") of claim experience. A histogram of one such set of 1,000 simulations is presented as Chart 1, on the following page.

This chart reveals that simulated losses fall between $4,725,000 and $5,260,000 in 900 of the 1,000 trials. That is, given 1,000 trials, simulated losses fall between 94.5% and 105.2% of expected losses ($5 million) 90% of the time. Under Longley-Cook's formula, the corresponding theoretical limits are $4,740,000 and $5,260,000. Not surprisingly, the analytical and simulation approaches produce similar results.

[4] Longley-Cook, page 200. In particular, $5\% = 0.05 = 1.645/(1{,}082)^{1/2}$.

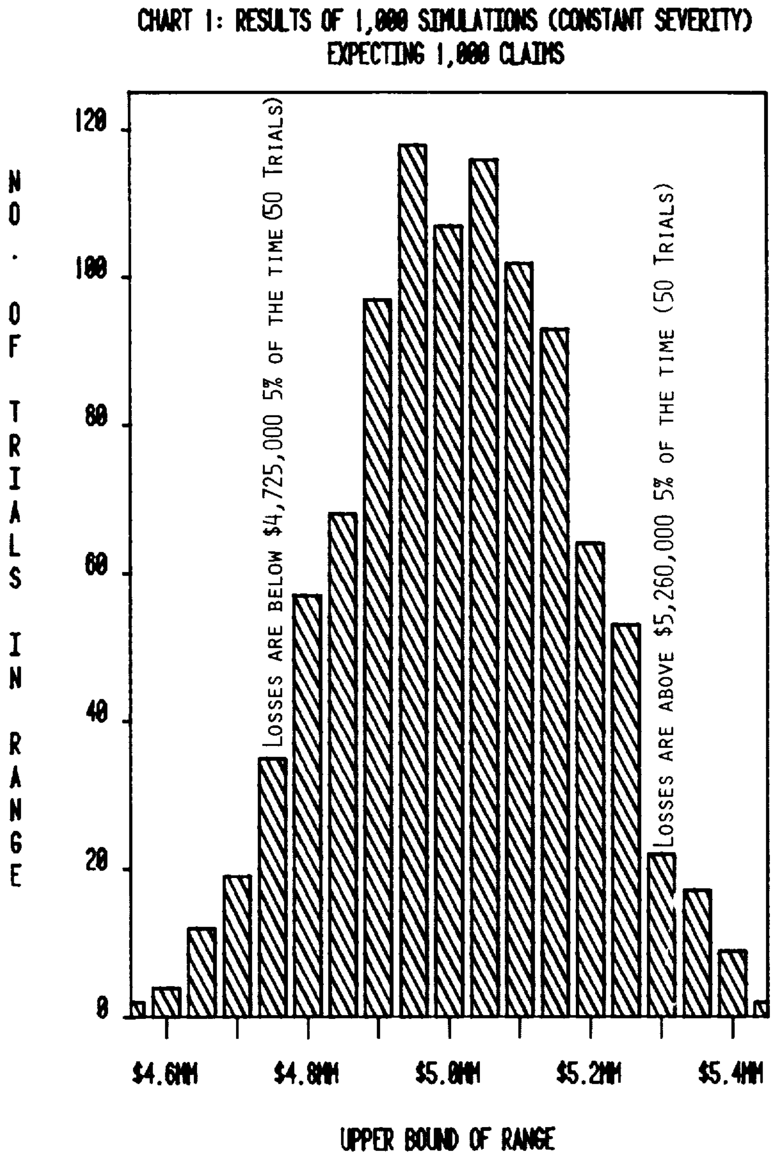
CHART 1: RESULTS OF 1,000 SIMULATIONS (CONSTANT SEVERITY)
EXPECTING 1,000 CLAIMS
NO. OF TRIALS IN RANGE
0
20
40
60
80
100
120
LOSSES ARE BELOW $4,725,000 5% OF THE TIME (50 TRIALS)
LOSSES ARE ABOVE $5,260,000 5% OF THE TIME (50 TRIALS)
$4.6MM
$4.8MM
$5.0MM
$5.2MM
$5.4MM
UPPER BOUND OF RANGE

Table 1 extends this comparison to include other probability ranges obtained from this run.

TABLE 1

CONFIDENCE BOUNDS AS A PERCENTAGE OF EXPECTED LOSS*

Probability	Based on 1,000 Simulations: Lower Bound	Based on 1,000 Simulations: Upper Bound	Theoretical Values
99%	−8.1%	+8.2%	±8.1%
98%	−7.4%	+7.7%	±7.4%
95%	−6.5%	+6.3%	±6.2%
90%	−5.5%	+5.2%	±5.2%
80%	−4.3%	+4.1%	±4.1%

* Assuming constant severity and an expected frequency of 1,000 claims.

This correspondence between theoretical and simulation results usually improves as the number of simulations increases. For example, extending the prior run to 5,000 random trials generated a 90% probability range of \$4,735,000–\$5,255,000, slightly closer to the corresponding theoretical values.

Simulated confidence ranges for several other full credibility standards are provided in Appendix A.

More Sophisticated Credibility Models

Since a complete and simple analytical solution to the previous problem exists, one may question the usefulness and necessity of a simulation alternative. Indeed, were frequency and severity to behave as postulated in the first model, simulation would be a needless and expensive approach to a simple problem.

Unfortunately, frequency and severity usually do not behave as postulated in the basic credibility model. In particular, seldom are all claims the same size.[5]

When variability in both the frequency and severity distributions is considered, the simulation solution is generally preferable to an analytical approach

[5] Nor is the Poisson frequency assumption necessarily appropriate in all instances. See L. Simon, "Fitting Negative Binomial Distributions by the Method of Maximum Likelihood," *PCAS* XLVIII, 1961, page 45.

to the classic credibility problem. The next two subsections illustrate why this is so.

The Poisson/lognormal Model

A number of models which reflect variability in the size-of-claim distribution are presented in the actuarial literature. Of these, the Poisson/lognormal model suggested by Longley-Cook[6] and generalized by Mayerson, Jones, and Bowers[7] is among the most often cited.

Rather than assuming all claims to be of equal size, this model assumes that claim sizes are distributed according to a lognormal distribution. This assumption significantly complicates the derivation of appropriate formulas for determining the potential error associated with a particular full credibility standard. However, both papers conclude that a lognormal severity distribution increases the error calculated according to the simple credibility model by a factor of approximately $(1.0 + CV^2)^{1/2}$, where "CV" is the coefficient of variation[8] of the severity distribution.

For example, under the basic (constant claim size) model, choosing 1,000 claims as one's full credibility standard implies that the error in one's fully credible estimate will be less than 5.2% in nine of ten instances. By assuming a lognormal severity distribution with a coefficient of variation of 3.0, the error increases to approximately 16.4% (16.4% = 5.2% × $10^{1/2}$).

These results are easily confirmed by computer simulation. To illustrate, Chart 2 displays the distribution of 1,000 random trials developed assuming that:

- claim frequency is once again Poisson distributed with a mean frequency of 1,000 claims,
- claim sizes are lognormally distributed with a mean of $5,000 and a coefficient of variation of 3.0,
- the number of claims (frequency) does not influence their average cost (severity), and
- the cost of a particular claim is independent of the cost of prior claims.

[6] Longley-Cook, page 220.

[7] A. L. Mayerson, D. A. Jones, and N. L. Bowers, Jr., "On the Credibility of the Pure Premium," *PCAS* LV, 1968, page 175 (hereafter cited as "Mayerson et al"). This paper's full credibility formulas can also apply for non-lognormal severity processes.

[8] A coefficient of variation is the standard deviation of a distribution divided by its mean.

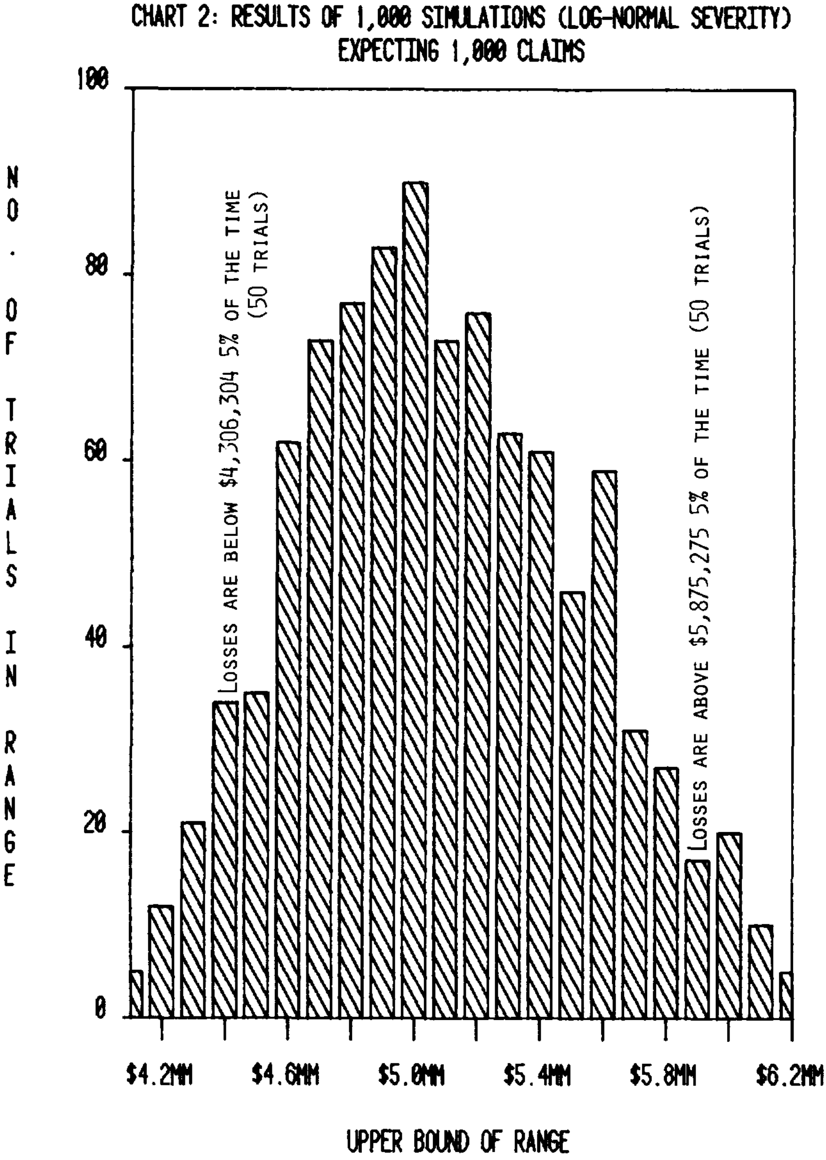
CHART 2: RESULTS OF 1,000 SIMULATIONS (LOG-NORMAL SEVERITY)
EXPECTING 1,000 CLAIMS
NO. OF TRIALS IN RANGE
100
80
60
40
20
0
LOSSES ARE BELOW $4,306,304 5% OF THE TIME
(50 TRIALS)
LOSSES ARE ABOVE $5,875,275 5% OF THE TIME (50 TRIALS)
$4.2MM
$4.6MM
$5.0MM
$5.4MM
$5.8MM
$6.2MM
UPPER BOUND OF RANGE

The various confidence bounds read from this chart are compared with their corresponding theoretical approximations in Table 2.

TABLE 2

CONFIDENCE BOUNDS AS A PERCENTAGE OF EXPECTED LOSS*

	Based on 1,000 Simulations		Theoretical Values[9]
Probability	Lower Bound	Upper Bound	Approx. Bounds
99%	−21.5%	+27.5%	±25.7%
98%	−19.7%	+24.6%	±23.3%
95%	−16.2%	+20.2%	±19.6%
90%	−13.9%	+17.5%	±16.4%
80%	−10.9%	+13.4%	±12.8%

* Assuming an expected claim frequency of 1,000 claims and a lognormal severity distribution with a *CV* of 3.0.

Unlike the analytical derivation, the simulated results in Chart 2 reflect the slight skewness of the resulting pure premium distribution. This skewness is also evident in Table 2, wherein lower confidence bounds are closer to expected loss levels than their corresponding upper bounds.

Also in contrast to the traditional derivation, the procedure used to simulate confidence ranges under a lognormal severity assumption is essentially identical to the simulation technique employed in the first model. The ease with which simulation accommodates this added complication suggests that this technique might be employed to address problems which are not readily answerable by analytical methods.

[9] Determined according to the formula Bound $= \pm P \times 10^{1/2}/1{,}000^{1/2}$, where P is the appropriate z-statistic from a standard normal distribution table.

Another Model

This subsection discusses a credibility problem which does not lend itself to an easy or general analytical solution. Specifically, the following comments outline a solution to the classic credibility problem in a situation where the pure premium distribution is a product of a "compound" severity process.[10]

For example, situations sometimes arise wherein losses up to a certain level (say, $25,000) appear to be the product of a number of influences, whereas losses above this level seem to be influenced by totally different elements. In Workers' Compensation, for instance, smaller indemnity losses might be viewed as the product of the injured worker's wage, the state benefit level, and projected future movements in wage levels. Losses above a certain level, on the other hand, tend to be influenced mainly by such factors as quality of attorney and the liberalness of the Workers' Compensation administration in that particular state.

Under such situations, the size-of-loss distribution is really a "compound distribution," in the sense that it is a weighted average of two different severity distributions—a "primary" and an "excess" loss distribution. Intuitively, one suspects that a pure premium distribution resulting from a compound severity distribution is more volatile than the corresponding distribution developed under a simple size-of-loss assumption. The following paragraphs test this intuitive notion.

The form of most theoretical pure premium distributions resulting from known frequency processes and compound severity distributions tends to be formidable. Thus, explicit analytical solutions to the classic credibility problem are generally not available in such situations. Simulation, on the other hand, does not discriminate on the basis of complexity; hence, the simulation solutions obtained for earlier, simpler situations can be extended to take into account this added consideration.

[10] For a discussion of several compound theoretical distributions, see C. C. Hewitt, Jr. and B. Lefkowitz, "Methods for Fitting Distributions to Insurance Loss Data," *PCAS* LXVI, 1979, page 139.

To illustrate this flexibility, Table 3 presents results of 1,000 random simulations which assume that:

- claim frequency is Poisson distributed with a mean of 1,000 claims,
- 95% of all claims are lognormally distributed with an average claim size of $5,000 and a coefficient of variation of 3.0,
- the remaining claims ("above $25,000") are Pareto distributed,[11]
- claim frequency is independent of claim severity, and
- the size of a loss is not influenced by the size of prior losses.

TABLE 3

CONFIDENCE BOUNDS* DEVELOPED WITH AND WITHOUT PARETO "TAIL"

	Without Tail (per Table 2)		With Tail	
Probability	Lower Bound	Upper Bound	Lower Bound	Upper Bound
99%	−21.5%	+27.5%	−30.5%	+37.7%
98%	−19.7%	+24.6%	−28.6%	+34.8%
95%	−16.2%	+20.2%	−24.4%	+29.1%
90%	−13.9%	+17.5%	−20.7%	+23.3%
80%	−10.9%	+13.4%	−16.2%	+18.4%

* Expressed as a percentage of expected loss.

This table clearly confirms our earlier supposition that a severity tail can add considerable volatility to a pure premium distribution.

Post-mortem: Section I

Section I describes how computer simulation may be used to develop approximate solutions to the classic credibility problem. In this process, it has become apparent that a simulation solution, unlike its analytical counterpart, is essentially pictorial. Specifically, each solution presented in Section I involved the computer's producing a histogram, from which this writer simply read his answer.

[11] That is, Y is distributed according to the formula $f(y) = 1.25y^{-2.25}$. In this formulation, "y" represents "normalized" losses in excess of $25,000; that is, Y = LOSS/25,000. Note that Y is never less than 1.

This intuitiveness carries with it obvious advantages for anyone charged with the difficult task of explaining the foundations of credibility theory to lay participants in the insurance process. Indeed, in such cases, one simulated picture may well be worth 1,082 words.

More significantly, however, the techniques used to generate Tables 1–3 can be modified to reflect almost any combination of theoretical or empirical frequency/severity assumptions. This inherent flexibility makes computer simulation an invaluable tool for applying and expanding traditional limited fluctuation credibility concepts, as well as for solving other difficult actuarial problems.

Illustrations of two such applications are provided in Sections II and III of this paper.

II. TESTING THE CAPITAL STRUCTURE OF A NEW INSURANCE COMPANY

Section I illustrates how a computer can be used to simulate a body of losses under assumed frequency and severity conditions. Simulating the ability of a new insurance company's capital structure[12] to meet its prospective loss obligations is a logical extension of this technique.

Accordingly, Section II uses simulation to test the capital structure of a hypothetical, limited-purpose "captive" insurance company. Again, results obtained under this approach are compared with those suggested by more traditional actuarial procedures.

The Company

The Consulting Actuaries' Reciprocal Exchange ("CARE") is being formed to provide a consistent and fairly priced market for Casualty Actuaries' Errors & Omissions coverage. Thus far, the steering committee examining the feasibility of this endeavor has agreed upon the following operational guidelines:

- the company will be domiciled offshore;
- the company will be a mutual insurance company, whose members will include consulting actuaries with three years of acceptable claim experience;
- the company will sell occurrence-basis Errors & Omissions policies;

[12] As used in this paper, "capital structure" includes the company's initial capitalization, as well as any other elements affecting its ability to pay losses. Such elements include retained earnings to date, applicable reinsurance arrangements, policyholder assessment provisions, and, of course, the company's underlying rate level.

- the company will be "capitalized" by means of a per-member capitalization fee, payable at a member's first policy inception;
- each member will be subject to a "solvency assessment," payable in the event that serious or sustained underwriting losses jeopardize the continued operation of the company on a sound basis;
- CARE will purchase only quota-share reinsurance because of reinsurer reluctance to participate on an excess-of-loss basis;
- to protect its solvency, the company will arrange to quota-share a substantial percentage of its exposure during its early years of operation.

In addition to the above seven constraints, the committee agrees upon the following preferences:

- a small initial capitalization fee, ideally $500 or $750;
- for marketing reasons, a maximum call provision of one year's premium;
- minimal use of quota-share reinsurance, since each dollar ceded costs CARE several cents.[13]

The committee retains an independent consultant to recommend the appropriate per-actuary rate, the maximum per-member assessment, a per-member capitalization fee, and the optimal amount of quota-share reinsurance which the company should purchase. The remainder of Section II illustrates how the consultant might use computer simulation to address these last three issues.

The Model: Underlying Assumptions

Of the four issues raised in the introduction, the first item—determining the proper per-actuary rate—is routinely accomplished by traditional actuarial means. For purposes of this example, assume that the consultant reviews the most recent three-year loss history of 1,000 prospective members, and thereby recommends a uniform annual rate of $1,750 for $3 million of occurrence-basis protection.

The consultant next turns his attention to the more complex and equally important questions concerning the proper assessment percentage, the initial capital contribution, and the percentage of business which the company should cede. Since these elements interact to jointly influence CARE's solidity, the consultant constructs a computer model to simultaneously address these three issues.

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[13] Expenses incurred less ceding commission allowance. CARE costs per dollar ceded are 7.5¢ (15¢ − 7.5¢) during first year, 4.5¢ during year 2, and 2.5¢ thereafter, per underlying assumptions 7 and 10.

The principal assumptions underlying this model are presented below.

1. *Distribution of the number of claims:* Since the base of insureds appears to be relatively homogeneous with a low expected frequency, the model assumes a Poisson claim frequency process.
2. *Distribution of claim amounts:* No credible Actuaries' Errors & Omissions size-of-loss information is available for this review. Fortunately, considerable size-of-loss data for other professional liability sublines are readily and generally available. Based on this information, the consultant hypothesizes a joint lognormal/Pareto severity distribution, as described below:

Claim-size Range	% of Claims	Distribution	Parameter
Below $500,000	98%	Lognormal	CV is 3.5
Above $500,000	2%	Pareto	Constant is 1.30

3. *Expected number of claims and average claim size:* Recall that a loss history was reviewed by the consultant. He estimated an average claim frequency of 2.5 claims/100 actuaries and a basic-limits[14] average claim size of approximately $45,000. Due to the underlying "parameter variance" in any distribution of sample means, these estimates are themselves subject to a certain amount of chance error. Accordingly, the consultant adjusts his simulation model to take into account the inherent error in his frequency and severity estimates.
   Specifically, the model assumes that these frequency and severity averages are normally distributed with standard errors of 0.2 claims/100 actuaries and $6,000, respectively. The means which underlie any particular trial's frequency and severity distributions reflect the consultant's initial estimates adjusted for this parameter error.
4. *Number of first-year participants:* Marketing intelligence estimates first-year participation of 1,000 actuaries, a level expected to continue through year five. Annual membership growth of 10% is projected for each of years six through ten.
5. *Frequency and severity trend:* No upward or downward trend in claim frequency is assumed. However, annual increases in E&O claim sizes are anticipated.

[14] "Basic-limits" losses are limited to $500,000 per occurrence.

Specifically, a 12% severity increase is assumed for year one. During subsequent years, the annual *change* in the claim inflation rate is assumed to be normally distributed with an average change of 0 and a standard deviation of 1 point.

6. *Collectibility of assessments:* Recognizing that the company would not be able to collect all assessments in the event a call is required, the model assumes an effective collection rate of 75%.
7. *Operating costs:* Administrative, underwriting, unallocated loss expense, and premium tax costs total 15% of premium during the first year, 12% in year 2, and 10% thereafter.
8. *Common inception date and policy term:* All CARE policies are to be written for one year, effective January 1.
9. *Rate level changes:* The $1,750 per-member rates will continue through the third year. Thereafter, annual 10% premium increases are assumed.[15]
10. *Ceding reinsurance commission:* CARE will receive a 7.5% commission on all quota-share reinsurance which it cedes.
11. *Payout of incurred losses:* Payout of a given policy-year's E&O losses is assumed to occur over five years, in 30/25/20/15/10 proportions.
12. *Interest earned on reserves, capital, and surplus:* The company's investable funds are assumed to earn interest at an annual rate of 10%.
13. *Federal income taxation:* Full (46%) corporate income taxation is assumed. To simplify computations, this taxation is assumed to occur during the year in which the corresponding income is earned.

[15] In practice, loss-sensitive pricing would probably be assumed.

The Model: Results

The consultant next uses his simulation model to carry out a first-level screening of the following twelve CARE operating scenarios:

Scenario	Quota-share Percentage	Per-member Capitalization Fee	Maximum Annual Policyholder Assessment (as a % of annual premium)
1	15%	$500	50%
2	15%	$500	100%
3	15%	$750	50%
4	15%	$750	100%
5	25%	$500	50%
6	25%	$500	100%
7	25%	$750	50%
8	25%	$750	100%
9	50%	$500	50%
10	50%	$500	100%
11	50%	$750	50%
12	50%	$750	100%

For each of these twelve scenarios, the model simulates 50 random "trials." Each trial consists of ten years' operating experience; for each year, net operating income is developed, and changes in CARE's policyholder surplus are recorded. To illustrate this technique, results of the first trial of Scenario 5 are presented in Table 4, on the following page.

TABLE 4

RESULTS OF FIRST TRIAL, FIFTH SCENARIO
(All dollar figures are in thousands)

	YEAR				
	1	2	3	4	5
Net Premium Earned	$1,313	$1,313	$1,313	$1,444	$1,588
Reins. Commission	$ 33	$ 33	$ 33	$ 36	$ 40
Investment Income	$ 81	$ 152	$ 210	$ 260	$ 321
Net Losses Incurred	$ 458	$1,266	$ 610	$1,572	$3,104
Expenses Incurred	$ 263	$ 210	$ 175	$ 193	$ 212
Fed. Income Taxes	$ 325	$ 10	$ 354	$ −11	$−629
Oper'tg Surplus @ Start	$ 500	$ 881	$ 893	$1,308	$1,295
Call Funds Required	$ 0	$ 0	$ 0	$ 0	$ 0
Oper'tg Surplus @ End	$ 881	$ 893	$1,308	$1,295	$ 557
Claim-cost inflation	12.0%	12.5%	12.0%	11.0%	11.3%
No. of members	1,000	1,000	1,000	1,000	1,000

	YEAR				
	6	7	8	9	10
Net Premium Earned	$1,922	$2,325	$2,813	$3,404	$4,118
Reins. Commission	$ 48	$ 58	$ 70	$ 85	$ 103
Investment Income	$ 358	$ 384	$ 438	$ 520	$ 627
Net Losses Incurred	$1,090	$1,961	$1,852	$2,799	$1,400
Expenses Incurred	$ 256	$ 310	$ 375	$ 454	$ 549
Fed. Income Taxes	$ 452	$ 228	$ 503	$ 348	$1,334
Oper'tg Surplus @ Start	$ 607*	$1,192*	$1,521*	$2,178*	$2,660*
Call Funds Required	$ 0	$ 0	$ 0	$ 0	$ 0
Oper'tg Surplus @ End	$1,137	$1,460	$2,112	$2,586	$4,225
Claim-cost inflation	11.7%	11.3%	11.4%	11.9%	10.6%
No. of members	1,100	1,210	1,331	1,464	1,611

* Includes $500/member assessment from new members.

Average annual surplus growth: 23.8%

Finally, each scenario is evaluated in terms of:

- the likelihood of CARE's avoiding a "capital call" (assessment),[16]
- the expected 10-year profitability of the operation as measured by surplus growth, and
- the consistency of CARE's year-to-year surplus growth.

This preliminary screening is carried out in Table 5.

TABLE 5

SUMMARY OF RESULTS OF TEN-YEAR OPERATING SIMULATION

	Adequacy of Call Provision		Profitability	
Scenario	No. of trials (out of 50) in which "call" is required	No. of trials in which "call" funds are *not* sufficient to offset surplus impairment(s)	Median 10-yr. surplus growth	Range in average surplus growth (Low/High)*
(1)	(2)	(3)	(4)	(5)
1 (15/500/50)	22	5	22.9%	Co. fails / 30.7%
2 (15/500/100)	23	None	23.1%	12.6% / 29.1%
3 (15/750/50)	12	6	19.7%	Co. fails / 24.9%
4 (15/750/100)	20	2	18.2%	None / 25.6%
5 (25/500/50)	20	8	21.0%	Co. fails / 27.0%
6 (25/500/100)	17	2	20.3%	None / 29.1%
7 (25/750/50)	18	6	16.2%	Co. fails / 22.9%
8 (25/750/100)	13	None	18.9%	10.1% / 23.8%
9 (50/500/50)	13	1	17.0%	3.9% / 24.9%
10 (50/500/100)	18	1	18.3%	11.9% / 24.2%
11 (50/750/50)	7	1	16.1%	6.4% / 19.2%
12 (50/750/100)	7	None	16.5%	2.0% / 19.9%

* Range represents the fifth lowest and fifth highest annual surplus growth rates recorded during the fifty trials.

[16] In this illustration, a call is required only in the event that CARE's policyholder surplus is exhausted. In practice, the company would empower its management to issue a call whenever surplus drops by some predetermined percentage (25–50%) during a specified period.

Recommendations

Given the previous criteria and the results presented in Table 5, the consultant narrows his field of possible recommendations to Scenarios 2, 6, 9, and 10. He cites the following reasons.

- Column (3) clearly establishes that a 50% call provision, in the absence of at least 50% quota-share reinsurance, does not provide sufficient contingent capitalization to assure the company's solidity. This observation eliminates Scenarios 1, 3, 5, and 7 from further consideration.
- Table 5 also demonstrates that a $750 per-member capitalization fee does not significantly improve CARE's operating integrity. On the other hand, higher initial capitalization reduces the company's premium/surplus leverage; Column (4) quantifies the negative impact of this added capitalization on CARE's annual surplus growth. Thus, Scenarios 4, 8, 11, and 12 are eliminated as possible candidates.

The consultant next reviews these findings with CARE's steering committee. During this review, the committee re-emphasizes its desire to avoid extensive reinsurance; accordingly, Scenarios 9 and 10 are dismissed. Moreover, the group asks the consultant to:

- extend his simulation analysis to 1,000 trials for each of the remaining two options, and
- analyze each of the remaining options under both the proposed $1,750/actuary base rate scenario, as well as under a $2,000 base rate assumption.

After reviewing this additional input, the committee adopts the sixth option (25% reinsurance, a $500 per-member capitalization charge, and a 100% call provision) along with a $2,000/actuary base rate.

Post-Mortem: Section II

This section extends the loss simulation techniques presented in Section I to include a consideration of inflation, reinsurance, corresponding premium movements, and cash flow. The simulation model which results from this extension provides an intuitively appealing method for testing the solidity of a new or existing casualty insurance company.

The reader will note that this approach to gauging an insurer's solidity bears little resemblance to traditional solvency testing procedures. The two, in fact, differ not only in form, but in what they are actually testing.

The first difference involves the form of the two procedures. Most traditional solvency tests, such as the NAIC Insurance Information Regulatory System ratios and the A. M. Best insurance company rating system, are designed for widespread application. In fact, many of these tests are conducted annually for all or most U.S. property/casualty carriers. It is doubtful that any organization could conduct meaningful solvency simulations on such a scale.

Beyond structural differences, however, the two approaches differ in what they attempt to measure. Specifically, traditional tests attempt to identify companies which are *already* experiencing surplus difficulty. The simulation approach suggested in this section, on the other hand, focuses primarily on the likelihood that a company may become insolvent. Also, simulation is designed to highlight steps which would reduce this probability.

Since traditional and simulation approaches measure different things, a direct comparison of the two is not meaningful. What is clear, however, is that a combination of the two methods produces a far better system than either approach alone provides. In particular, traditional ratio analysis is cost-effective for most large, established carriers, but is of little value to new or limited purpose insurance companies. For this latter group, simulation generally produces far more useful information.

A specific and needed application of a simulation approach to solvency testing concerns "captive" insurance companies. These carriers, which are increasing in number by approximately 100 to 150 per year, often find it difficult to convince the established reinsurance market of their legitimacy as insurance operations. In turn, this failure to gain market acceptance can seriously limit a captive's effectiveness, particularly during reinsurance negotiations and in its efforts to procure a book of "quality" non-related business.

For a captive in this position, a simulation analysis along the lines suggested in this section would either convincingly confirm the company's operating integrity, or provide information with which the carrier could judiciously strengthen its capital structure. In either case, the company would almost certainly improve its image and stature in the insurance market.

III. SIMULATION AS AN AID IN PRICING NEW, UNIQUE, OR CATASTROPHIC EXPOSURES

Computer simulation can also be used to improve pricing of exposures for which historical information is unavailable or not indicative of future experience. For example, the pricing of endemic disease exposures, such as coal miner's "black lung," textile worker's "brown lung," and asbestosis, can be improved with the aid of computer simulation.

To illustrate the use of simulation in pricing these exposures, Section III compares the current actuarial formula for pricing black lung (pneumoconiosis) coverage with an alternative procedure incorporating computer simulation. The advantages of the latter approach are highlighted.

Background

Title IV of The Federal Coal Mine Health and Safety Act of 1969 extended Workers' Compensation benefits to underground coal miners totally disabled by pneumoconiosis, a respiratory disease associated with dust levels in coal mines. The Act also provided benefits for the families of miners who died from the disease.

Understandably, pricing this coverage has proven to be a problem for the major Workers' Compensation ratemaking organizations. A partial listing of conditions complicating black lung pricing includes the facts that:

- there exists very little data on claim emergence patterns, even ten years after introduction of coverage;
- coverage is limited to death, permanent total disability, and medical benefits; therefore, the average undiscounted cost of black lung claims is currently estimated to be $200,000–400,000;
- the program provides for a dual benefit structure; affected miners qualify for the higher of Federal or state benefits;
- the Act's coverage continually changes, often retroactively;
- most importantly, the Act contains a (rebuttable) presumption that any miner with a respiratory impairment and a specified number of years of service in the mines is disabled from work-related black lung disability, and thereby entitled to black lung benefits.

Current Pricing Procedures

Given the previous considerations, a black lung pricing formula based entirely upon historical experience is neither possible nor appropriate. Thus, the National Council on Compensation Insurance has adopted a procedure which utilizes available actuarial and government statistics to estimate the appropriate *expected* pure premiums for coal mining classes.[17]

[17] See Appendix B for a fuller discussion of the current NCCI formula.

This writer believes that the current expected loss black lung pricing formula is flawed, in that it ignores the interaction among variables affecting black lung costs. Instead, expected loss pricing focuses directly on the end products of the loss determination process—the number of claims filed and the average cost of these claims. In so doing, current procedures may exaggerate any underlying conservative biases on the part of the pricer, and thus result in his unintentionally overstating required pure premiums.

Simulation overcomes this problem by forcing the pricer to specify the assumed interaction among variables, and by displaying a range of possible outcomes consistent with these assumptions. As will be demonstrated, using a simulation based pricing formula narrows the range of reasonably foreseeable outcomes confronting the pricer, thus allowing him to select a saleable and reasonable pure premium which actual experience should not exceed in more than a specified percentage of instances.

The following illustration highlights these advantages.

The Model: Underlying Assumptions

Many factors interact to determine the discounted indemnity costs to be paid under the current black lung benefit system. The following eight items are among the more important of these influences. They provide the basis of the assumptions underlying the simulation model presented in this section. Appendix C contains a detailed discussion of each of these eight assumptions.

IMPORTANT FACTORS AFFECTING BLACK LUNG LOSSES

1. Frequency of retiring miners' filing of claims
2. Success rate among retiring miners who file claims
3. Miner mortality
4. Age of claimants
5. Number and nature of dependents
6. Wage inflation rate (for Federal benefit escalator)
7. Loss discounting percentage
8. Current and projected indemnity benefits

The Model: Results

Given the above assumptions, a computer program was written to simulate *one policy-year's black lung indemnity costs arising out of the retirement of 1,000 miners.* Table 6 presents results of 50 random trials carried out with losses discounted at 3.5%.

TABLE 6

Results of 50 Random Trials (Policy Years) of Black Lung Experience

	Wage Inflation				Miners Receiving Both State and Federal Benefits			
Trial	Yr. 1	High	Low	Losses Disctd. at	No. w/o Surv	No. with Surv	Total State Indemnity	Federal Supplement
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	8.0%	8.2%	1.4%	3.5%	29	39	$11,793,424	$ 422,171
2	8.0%	18.9%	6.2%	3.5%	108	106	$35,417,949	$28,329,620
3	8.0%	13.8%	1.5%	3.5%	61	59	$19,609,308	$ 802,263
4	8.0%	12.7%	0.0%	3.5%	197	181	$61,742,029	$ 5,382,631
5	8.0%	10.0%	0.0%	3.5%	29	38	$11,314,850	$ 315,453
6	8.0%	9.0%	0.0%	3.5%	79	64	$23,066,573	$ 1,260,823
7	8.0%	13.5%	0.1%	3.5%	58	41	$15,699,009	$ 3,997,199
8	8.0%	11.8%	3.8%	3.5%	83	72	$25,439,610	$ 3,073,195
9	8.0%	26.8%	6.6%	3.5%	88	80	$27,594,427	$ 7,417,803
10	8.0%	17.2%	8.0%	3.5%	121	101	$36,389,192	$24,345,390
11	8.0%	14.4%	0.9%	3.5%	48	39	$14,145,943	$ 335,548
12	8.0%	12.2%	5.8%	3.5%	90	100	$31,561,550	$ 7,606,673
13	8.0%	14.6%	6.0%	3.5%	74	86	$26,779,215	$ 3,657,434
14	8.0%	11.4%	0.0%	3.5%	110	111	$36,922,111	$ 5,088,262
15	8.0%	13.5%	0.0%	3.5%	78	96	$29,132,473	$ 4,098,705
16	8.0%	9.3%	1.6%	3.5%	73	90	$28,195,854	$ 249,448
17	8.0%	9.1%	0.0%	3.5%	25	25	$ 8,275,362	$ 273,957
18	8.0%	12.6%	4.8%	3.5%	168	173	$56,694,638	$11,261,580
19	8.0%	11.2%	2.3%	3.5%	36	51	$14,811,720	$ 706,279
20	8.0%	10.5%	2.8%	3.5%	51	74	$21,753,482	$ 2,595,748
21	8.0%	8.0%	0.0%	3.5%	100	110	$34,918,094	$ 6,552
22	8.0%	18.9%	8.0%	3.5%	66	35	$25,523,520	$ 9,524,768
23	8.0%	8.9%	0.0%	3.5%	69	74	$24,098,558	$ 0
24	8.0%	10.3%	1.7%	3.5%	84	67	$24,296,278	$ 2,303,286
25	8.0%	11.5%	6.0%	3.5%	142	141	$46,506,422	$ 5,975,460
26	8.0%	17.1%	6.7%	3.5%	146	137	$46,696,035	$ 5,743,238
27	8.0%	26.0%	8.0%	3.5%	104	104	$34,418,369	$14,616,021
28	8.0%	8.0%	0.0%	3.5%	58	41	$16,037,961	$ 22,121
29	8.0%	8.0%	0.0%	3.5%	82	83	$27,268,924	$ 143,290
30	8.0%	22.2%	5.8%	3.5%	56	72	$21,795,755	$ 5,302,253
31	8.0%	8.0%	2.2%	3.5%	79	82	$26,838,126	$ 272,082
32	8.0%	16.6%	3.1%	3.5%	48	38	$13,928,681	$ 732,148
33	8.0%	9.4%	1.5%	3.5%	61	60	$20,385,387	$ 1,403,695
34	8.0%	16.5%	8.0%	3.5%	74	84	$26,310,580	$ 4,360,145
35	8.0%	19.7%	4.2%	3.5%	62	57	$19,698,029	$ 3,586,797
36	8.0%	18.9%	6.3%	3.5%	117	113	$37,901,940	$ 5,632,761
37	8.0%	11.0%	3.2%	3.5%	73	69	$23,615,719	$ 2,907,338
38	8.0%	9.8%	0.0%	3.5%	28	32	$ 9,823,467	$ 4
39	8.0%	8.2%	0.0%	3.5%	34	59	$16,262,965	$ 106,774
40	8.0%	9.4%	2.6%	3.5%	81	81	$26,730,032	$ 1,070,919
41	8.0%	8.2%	0.0%	3.5%	91	81	$27,882,454	$ 0
42	8.0%	10.6%	0.0%	3.5%	38	48	$14,293,282	$ 772,804
43	8.0%	14.4%	2.0%	3.5%	86	97	$30,891,675	$12,989,713
44	8.0%	10.4%	3.9%	3.5%	86	84	$28,007,285	$ 2,108,072
45	8.0%	11.3%	0.0%	3.5%	45	36	$13,091,872	$ 2,222,252
46	8.0%	19.1%	8.0%	3.5%	93	79	$27,905,591	$ 8,345,419
47	8.0%	8.0%	0.3%	3.5%	85	79	$27,062,875	$ 31,161
48	8.0%	8.2%	0.0%	3.5%	103	96	$32,772,717	$ 613,959
49	8.0%	10.3%	0.0%	3.5%	71	55	$20,324,758	$ 772,486
50	8.0%	10.1%	0.0%	3.5%	24	29	$ 8,844,989	$ 152,660
Avg.					77	77	$25,809,421	$ 4,058,767

TABLE 6 cont.

RESULTS OF 50 RANDOM TRIALS (POLICY YEARS) OF BLACK LUNG EXPERIENCE

	Miners Receiving Only State Awards			Miners Receiving Only Federal Awards			All Miners	
Trial	No. w/o Surv.	No. with Surv.	Total State Indemnity	No. w/o Surv.	No. with Surv.	Total Federal Indemnity	No. Total Benef.	Total Cost
(1)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
1	156	146	$50,043,093	34	24	$ 6,675,629	428	$ 68,934,318
2	207	228	$73,168,306	11	10	$10,893,322	670	$147,809,200
3	165	132	$48,210,590	46	63	$13,818,380	526	$ 82,440,543
4	220	244	$77,521,904	14	8	$ 2,927,979	864	$147,574,546
5	151	155	$50,829,437	6	6	$ 1,437,476	385	$ 63,897,218
6	136	107	$39,214,016	19	25	$ 5,592,972	430	$ 69,134,384
7	156	133	$46,776,478	17	26	$ 8,440,706	431	$ 74,913,394
8	197	198	$65,582,734	22	25	$ 7,341,589	597	$101,437,130
9	197	185	$62,730,284	21	30	$ 9,926,297	601	$107,668,811
10	211	228	$73,061,427	2	4	$ 2,189,764	667	$135,985,774
11	189	190	$62,707,417	2	5	$ 991,287	473	$ 78,180,199
12	237	211	$72,991,772	8	13	$ 4,317,375	659	$116,477,371
13	162	141	$49,930,546	6	4	$ 1,572,640	473	$ 81,939,836
14	152	166	$53,086,514	106	88	$29,380,007	733	$124,476,896
15	183	183	$60,631,021	19	28	$ 7,733,378	587	$101,595,576
16	137	137	$44,960,660	69	73	$13,707,441	579	$ 87,113,404
17	111	101	$34,891,522	21	20	$ 4,788,235	303	$ 48,229,077
18	188	206	$65,710,368	48	34	$13,892,526	817	$147,559,112
19	227	197	$69,215,503	24	10	$ 3,609,897	545	$ 88,343,400
20	133	158	$49,170,926	56	55	$17,293,975	527	$ 90,814,132
21	234	212	$73,594,906	5	13	$ 1,681,190	674	$110,200,746
22	164	142	$49,827,723	7	7	$ 3,387,674	471	$ 88,263,686
23	250	274	$87,529,053	19	23	$ 3,524,010	709	$115,151,620
24	202	178	$62,469,631	77	55	$18,508,876	663	$107,578,072
25	269	286	$92,486,717	14	9	$ 3,454,796	861	$148,423,394
26	286	267	$90,594,835	8	9	$ 2,785,405	853	$145,819,512
27	238	208	$73,089,956	14	3	$ 3,890,234	671	$126,014,583
28	191	228	$70,586,766	18	12	$ 2,783,397	548	$ 89,430,247
29	127	131	$43,516,001	63	60	$12,591,221	546	$ 83,519,436
30	102	95	$32,258,242	43	47	$17,927,388	415	$ 77,283,640
31	163	145	$50,366,895	6	6	$ 1,236,084	481	$ 78,713,188
32	202	225	$71,403,735	18	21	$ 4,826,069	552	$ 90,890,635
33	157	152	$50,954,859	40	39	$10,417,018	509	$ 83,160,960
34	148	156	$50,412,515	7	3	$ 1,711,021	472	$ 82,794,263
35	303	254	$90,997,005	24	29	$ 8,850,339	729	$123,132,170
36	223	251	$78,745,314	32	31	$10,373,152	767	$132,653,168
37	161	165	$54,453,021	11	8	$ 2,692,884	487	$ 83,668,964
38	158	150	$50,694,697	15	11	$ 2,209,892	394	$ 62,728,060
39	201	201	$66,826,740	49	30	$ 8,045,087	574	$ 91,241,567
40	209	188	$65,022,151	26	26	$ 6,541,827	611	$ 99,364,928
41	167	201	$62,416,777	28	23	$ 4,738,808	591	$ 95,038,040
42	109	107	$35,797,900	38	31	$ 8,963,530	371	$ 59,827,517
43	182	194	$62,673,852	51	77	$33,699,587	687	$140,254,826
44	182	201	$63,325,629	25	18	$ 5,669,090	596	$ 99,110,077
45	185	201	$64,591,462	5	3	$ 1,311,648	475	$ 81,217,237
46	106	110	$35,917,009	23	24	$11,627,224	435	$ 83,795,244
47	151	217	$63,196,180	28	27	$ 5,020,107	587	$ 95,310,325
48	231	224	$75,000,411	8	9	$ 1,888,053	671	$110,275,142
49	139	115	$41,716,405	25	16	$ 4,522,414	421	$ 67,336,064
50	158	143	$48,791,959	19	10	$ 2,793,983	383	$ 60,583,591
Avg.	182	181	$60,193,857	25	24	$ 7,484,058	569	$ 97,546,104

Similar simulations were carried out under 0% and 7% loss discounting assumptions. Given the results of these runs and assuming both a $15,000 average miner's salary and a 1.5% retirement rate, the pure premiums in Table 7 were obtained.

TABLE 7

COMPARISON OF DISCOUNTED BLACK LUNG INDEMNITY PURE PREMIUMS

Loss Discounting Percentage	Based on Average Simulated Loss Level[18]	Based on 80th Percentile Loss Level	Based on Lowest Simulated Loss Level	Based on Highest Simulated Loss Level
0% (No disct.)	$15.72	$20.00	$7.23	$30.76
3.5% (NCCI)	9.75	12.31	4.82	14.84
7%	7.30	9.13	3.78	11.03

Interpretation and Significance of Results

Table 7 may be interpreted as follows. Consider the pure premiums displayed in the second (3.5% discount) row. When loaded 16% for expenses, these figures translate into black lung indemnity rates which range from $5.75 to nearly $18.00. From the viewpoint of the pricing actuary, however, the range of *selectable* rates runs from $11.60 (based upon the mean simulated loss level) to more conservative (80th percentile) estimates in the area of $14.50.

In this manner, incorporating computer simulation into black lung pricing enables the actuary to significantly narrow his range of potential loss (and thus rate) levels.

Post-Mortem: Section III

For practical and philosophical reasons, this paper does not suggest that computer simulation should diminish the current role of "traditional" insurance pricing formulas. However, using simulation to complement these formulas in the pricing of new or certain difficult exposures offers several obvious advantages.

In particular, by reflecting the often offsetting interactions among the many factors influencing such a coverage's ultimate cost, simulation usually enables the actuary to significantly narrow his range of potential prices. For instance,

[18] Pure premium (3.5% discount) = $97,546,104/10 million (units of $100 payroll). Similar computations apply for other discounting assumptions.

the $11.60–14.50 range of probable outcomes developed in the previous example is probably much smaller than the spread which one might intuitively expect, given earlier comments on the nature of this coverage and the problems encountered in its pricing.

More importantly, the simulation approach presented in this section requires an initial, detailed delineation of elements which affect the program's cost. Each of these go-in assumptions can readily be tested and appropriately modified as soon as meaningful experience becomes available. As a result, accurate pricing may occur more quickly by incorporating a simulation analysis into traditional expected loss pricing formulas.

IV. OTHER APPLICATIONS

Sections I–III present three applications of computer simulation to insurance pricing problems:

- in re-establishing and extending the fundamental notions of credibility theory,
- in assessing the solidity of an existing or contemplated property/casualty insurance company, and
- in pricing catastrophic exposures or hazards for which no relevant historical information is available.

Simulation can also assist the actuary in two other areas of historical and current concern: customizing individual risk insurance charges[19] and developing loss reserve confidence intervals. A brief discussion of these applications follows.

Customizing Insurance Charges and Excess Loss Premium Factors

The current "Table M" and "Table L" provide the insurance charges underlying most casualty retrospective rating plans, policyholder dividend schemes, and certain types of casualty premium allocation programs.

Historically, massive data requirements and other logistic problems have precluded regular periodic overhauls of the Tables or development of separate insurance charges for the non-Compensation casualty lines. As a result, the insurance charges currently used in most states (Table M) are grounded in the Workers' Compensation policy year experience of the early 1960's. Moreover, this same table often provides the charges used in automobile, burglary, and general liability retrospective rating programs.

[19] An *insurance charge at entry (or loss) ratio r* represents the proportion of a risk's losses which can be expected to fall above entry (loss) ratio *r*.

Simulation, on the other hand, is not constrained by these logistic complexities. In addition, the loss simulation techniques described in Section I of this report can easily be extended to include a calculation of insurance charges.[20]

To illustrate, consider a body of automobile exposures, each with an expected claim frequency of 500 claims and an average claim size of $500. Further, assume a Poisson/lognormal pure premium process with an underlying coefficient of variation of 3.25 for severity. Given these assumptions, loss experience of 1,000 trials (risks) is simulated as described in Section I. From these results, a table of insurance charges and excess loss premium factors[21] is generated by the standard formula.[22] Moreover, this same procedure is carried out with individual claims limited to $2,500.

The resulting insurance charges and ELPF's at selected entry ratios are compared with the corresponding 1977 Table M charges in Table 8.

TABLE 8

COMPARISON OF TABLE M CHARGES WITH SIMULATED VALUES

Entry Ratio	1977 Table M Charge (EL Group 19)	Simulated P.D. Insurance Charge (Unlim. Losses)	Simulated P.D. Charge with Losses Limited to $2,500	Indicated $2,500 ELPF ((4) − (3))
(1)	(2)	(3)	(4)	(5)
0	1.000	1.000	1.000	0
0.25	.750	.766	.766	0
0.50	.526	.532	.532	0
1.00	.190	.089	.217	.128
1.25	.105	.010	.217	.207
2.00	.029	0	.217	.217

[20] Viewing each trial as the experience of a single risk, the formula is

$$\text{Charge at entry ratio } r = \sum_{\substack{\text{trials where}\\ \text{losses exceed}\\ r^*(\text{Expected Losses})}} \{\text{Losses} - r^*(\text{Expected Losses})\} \Big/ \sum_{\substack{\text{all}\\ \text{trials}}} \text{Losses}.$$

[21] An *excess loss premium factor* ("ELPF") is the charge made for limiting a retrospectively rated risk's rateable losses to a per-occurrence amount, such as $25,000 or $50,000.

[22] See note 20.

The simulation approach to developing insurance charges offers three advantages over traditional means of developing Tables M or L:

1. The simulated factors are "customized" to reflect the specific frequency and severity characteristics of this particular insurance;
2. Updating (for inflation, etc.) is routine;
3. The table can be recast to reflect any desired level of loss limitation, thereby avoiding the classic problem of overlapping insurance charges and ELPF's.

Improving Loss Reserve Confidence Interval Calculations

A recent paper in these *Proceedings*[23] suggested a procedure for developing loss reserve confidence intervals from the corresponding pure premium confidence intervals underlying a given policy year's initial pricing. As illustrated in Section I of this report, simulation provides a means of improving the accuracy in one's estimate of these underlying pure premium ranges.

A description of how simulation might be used in the computation of loss reserve confidence intervals is provided as Appendix D.

V. CONCLUSION

The preceding pages discuss five specific areas—credibility theory, solvency testing, pricing new or difficult exposures, estimating individual risk rating charges, and developing confidence intervals around loss reserve estimates—in which computer simulation presents an opportunity for us to take a step toward overcoming traditional pricing and reserving obstacles. The recent introduction of inexpensive and highly efficient microcomputers provides the corresponding method and motive. Given method, opportunity, and motive, therefore, this writer believes that computer simulation will become a prominent (dominant?) actuarial tool during the 1980's.

This paper will be successful to the extent that it encourages other members of this Society to come forward with additional uses of computer simulation, or to offer improvements upon the applications suggested in this paper.

[23] C. K. Khury, "Loss Reserves: Performance Standards," *PCAS* LXVII, 1980, page 1.

APPENDIX A

Comparison of Simulated and Theoretical Confidence Intervals for Several Sample Sizes

		Constant Severity			Lognormal Severity*		
			Simulated Values (1,000 Trials)			Simulated Values (1,000 Trials)	
Expected Number of Claims	Conf. Range	Theor.	Lower	Upper	Theor.	Lower	Upper
500	99%	±11.5%	−13.2%	+11.8%	±36.4%	−25.2%	+44.8%
	98%	±10.4	−11.6	+11.0	±32.9	−25.0	+38.5
	95%	±8.8	−9.0	+8.2	±27.7	−21.3	+32.0
	90%	±7.4	−7.8	+7.2	±23.3	−18.5	+25.3
	80%	±5.7	−5.8	+5.8	±18.1	−15.5	+18.2
1,500	99%	±6.6%	−7.4%	+5.9%	±21.0%	−19.3%	+21.5%
	98%	±6.0	−6.7	+5.7	±19.0	−17.2	+19.3
	95%	±5.1	−5.5	+4.8	±16.0	−13.8	+16.3
	90%	±4.2	−4.5	+4.0	±13.4	−11.7	+13.5
	80%	±3.3	−3.5	+3.0	±10.5	−9.1	+10.6
2,500	99%	±5.2%	−5.2%	+5.1%	±16.3%	−13.1%	+20.0%
	98%	±4.7	−4.6	+4.5	±14.7	−12.0	+16.8
	95%	±3.9	−3.8	+3.7	±12.4	−10.2	+13.1
	90%	±3.3	−3.3	+3.1	±10.4	−8.3	+10.8
	80%	±2.6	−2.6	+2.6	±8.1	−6.8	+8.4

* Coefficient of variation is 3.0.

APPENDIX B

CURRENT NCCI BLACK LUNG PRICING PROCEDURES[24]

In developing black lung pure premiums, the National Council on Compensation Insurance:

1. Determines the average black lung indemnity cost using standard mortality assumptions (U.S. Life Total Population Tables, 1959–61), discounting assumptions (3.5%/year), and a black lung dependency distribution developed by the National Council;
2. Loads (1) for expenses (12.3% plus premium taxes, as of May, 1980);
3. Estimates the percentage of insured miners filing successful Compensation claims;
4. Multiplies (2) by (3) to obtain the expected cost per 100 miners;
5. Separately computes a medical pure premium;
6. Loads (5%?) for contingencies (e.g., mine closedowns which result in an unforeseeable outbreak of claims);
7. Converts (6) to a rate per $100 of payroll.[25]

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[24] As described in Roy H. Kallop's *Black Lung Ratemaking*, a presentation to an industry symposium on black lung, St. Regis Hotel, New York City (May 19, 1980).

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[25] The ratemaking formula described in Kallop's paper actually computes rates in two parts—one part paying for new claims, the second amortizing the cost of additional liabilities imposed by the black lung legislation effective March 1, 1978. This paper focuses exclusively on the National Council's calculation of premiums to pay for new claims.

APPENDIX C

EIGHT ASSUMPTIONS UNDERLYING BLACK LUNG LOSS SIMULATION

1. *Frequency of retiring miners' filing of claims.* Currently, a retiring miner has little reason not to file a black lung claim. Thus, a high but unknown percentage of retiring workers will probably file claims.
   The simulation presented in this paper assumes the following filing rates along with their respective likelihoods of occurrence.

Retiring Miners Filing Black Lung Claims	Likelihood of Occurring
5 of 10	25%
7 of 10	50%
9 of 10	25%

   Also, we assume that all claimants will file both state and Federal claims.
2. *Success rate among retiring miners who file claims.* The rate of successful claimants varies substantially by state. In the Federal area, currently high success ratios are expected to fall during the coming years. Thus, the following rates are assumed.

State Claims		Federal Claims	
Approval Rate	Assumed Likelihood	Approval Rate	Assumed Likelihood
5 of 10	25%	4 of 20	25%
7 of 10	50%	7 of 20	50%
9 of 10	25%	10 of 20	25%

   One consequence of this assumption should be noted. As mentioned earlier, a miner who successfully pursues a state and Federal black lung claim receives the higher of the state or Federal awards. Currently, most states' weekly benefits are higher than the Federal benefit; however, Federal amounts are annually escalated for inflation. Thus, miners qualifying for both types of benefits receive state benefits until Federal amounts exceed state levels, at which time the miner or his survivor receives a Federal supplement equal to the benefit difference.

It follows that the percentage of miners qualifying for both state and Federal awards is an important consideration in pricing black lung coverage. Assuming that a claimant's success or failure in pursuing a state claim does not affect the disposition of his Federal award, a sufficient number of simulations should produce a distribution of beneficiaries along the following lines.

Type of Benefits Received	% of Miners Filing Claims
Both State and Federal	25%
State only	45%
Federal only	10%
No benefits awarded	20%

3. *Mortality rates*. A 1977 study of miner mortality[26] revealed a significantly higher incidence of lung-related diseases in retiring underground coal miners. Accordingly, the following mortality assumptions are used in this illustration.

Mortality Assumption[27]	Probability
Retiring miner's life expectancy is five years less than that of a "typical" retiree	55%
Retiring miner's life expectancy is typical	35%
Retiring miner's expectancy is five years greater than that of a "typical" retiree	10%

4. *Age of claimants*. The simulation assumes that only retiring miners (pension age 57 or 62) file claims.
5. *Number of dependents*. As discussed earlier, benefits are paid to survivors of a deceased miner who was totally disabled from pneumoconiosis at the time of his death. Accordingly, the model presented in this paper

[26] National Institute for Occupational Safety and Health, *Mortality Among Coal Miners Covered by the UMWA Health and Retirement Funds*, March, 1977.

[27] "Typical" mortality as per the U.S. total population mortality table, 1969–71. Other mortality tables could, of course, be used.

assumes the following dependency distribution.

Number of Dependents	% of Retiring Miners
0	50%
1	50%

Also, instead of computing joint survivorship probabilities, the simulation program assigns an effective pension age of 57 years to miners with dependents, and 62 years to all other miners.

6. *Wage inflation (for Federal benefit escalator).* The simulation arbitrarily assumes the current year's wage inflation rate to be 8%. Also, to illustrate the flexibility of this approach to pricing, the *change* in annual wage inflation rates is assumed to be normally distributed with an average change of 0 and a standard deviation of 1 point. Negative inflation rates are not allowed.
7. *Loss discounting percentages.* In view of the dramatic impact of this assumption on the program's ultimate cost, separate simulations were carried out for discounting assumptions of 0% (losses not discounted), 3.5% (the current National Council assumption), and 7%.
8. *Black lung indemnity benefits.* This simulation attempts to price for black lung indemnity payments; a similar approach could, of course, be employed for pricing the medical component.
   All black lung beneficiaries are assumed to qualify for the following hypothetical state or Federal indemnity payments.

Maximum State Benefit		Maximum Federal Benefit	
With Dependent	No Dependent	With Dependent	No Dependent
$300/week	$225/week	$440/month	$340/month

APPENDIX D

ILLUSTRATION OF LOSS RESERVE CONFIDENCE INTERVAL CALCULATION USING COMPUTER SIMULATION

Note: All calculations presented in this appendix assume a Poisson frequency process (expecting 1,000 claims) and a lognormal claim size distribution with a *CV* of 3.0. For 1976, a $5,000 average claim size was anticipated; for 1977, the corresponding figure was \$5,500; for 1978, \$6,050; for 1979, \$6,655; and for 1980, \$7,320.

Step 1. Begin by modifying Table 2 to account for the *error in the initial estimate* of the expected frequency and claim cost. This "parameter error" increases the confidence ranges in Table 2 as indicated below.

SIMULATED CONFIDENCE BOUNDS AS A PERCENTAGE OF EXPECTED LOSSES

	Per Table 2—Reflects Process Variance Only		Adjusted to Reflect Parameter Error	
Conf. Range	Lower Bound	Upper Bound	Lower Bound	Upper Bound
99%	−21.5%	+27.5%	−30.2%	+38.0%
98%	−19.7	+24.6	−27.8	+32.3
95%	−16.2	+20.2	−24.3	+28.5
90%	−13.9	+17.5	−20.7	+23.2
80%	−10.9	+13.4	−17.3	+17.9

Step 2. Use the above results to compute the 90% confidence ranges about the go-in pure premium for each accident year in which losses remain outstanding.

Accident Year	Expected Losses at Policy Inception	Approximate 90% Confidence Limit at Inception (above table)	
		Lower Bound	Upper Bound
1976	\$5,000,000	−\$1,035,000	+\$1,160,000
1977	\$5,500,000	−\$1,138,500	+\$1,276,000
1978	\$6,055,000	−\$1,253,500	+\$1,404,000
1979	\$6,655,000	−\$1,377,500	+\$1,544,000
1980	\$7,320,500	−\$1,515,500	+\$1,698,000

Step 3. Complete the calculation by assuming that

- loss reserve estimates improve in direct proportion to the time elapsed since policy inception, and
- all losses are settled within five years from date of occurrence.

Accident Year	90% Confidence Interval as of 1/1/81	
	Lower Bound	Upper Bound
1976	0	0
1977	−$228,000	+$255,000
1978	−$501,500	+$561,500
1979	−$826,500	+$926,500
1980	−$1,212,500	+$1,358,500
Total All Years	−$2,768,500	+$3,101,500

APPENDIX E

A COMPUTER APPROACH TO "GREATEST ACCURACY" CREDIBILITY THEORY

Section I of this paper illustrates how computer simulation can be used to develop and apply "limited fluctuation" credibility theory, as described by Longley-Cook, Mayerson, and Carlson.[28] This approach to establishing full credibility standards is basically a matter of developing confidence intervals, an application for which computer simulation is ideally suited.

Of course, a second credibility system—the "greatest accuracy" theory of A. L. Bailey,[29] Hewitt,[30] and others—has also gained wide acceptance within this Society. Under this second approach, credibility weights produce the best linear fit of observed pure premium data to conditional expectations of the pure premium over all possible data outcomes. Since confidence intervals are not involved in this formulation, the usefulness of a computer in developing greatest accuracy credibility factors is not readily apparent.

[28] Longley-Cook, page 196; Mayerson et al, page 175; T. O. Carlson, "Observations on Casualty Insurance Rate-Making Theory in the United States," *PCAS* LI, 1964, page 282.

[29] A. L. Bailey, "A Generalized Theory of Credibility," *PCAS* XXXII, 1945, page 13.

[30] C. C. Hewitt, Jr., "Credibility for Severity," *PCAS* LVII, 1970, page 148.

This Appendix illustrates how (non-simulation) computer techniques can also be used to explain and possibly expand upon the uses of the greatest accuracy credibility model.

"Greatest Accuracy" Credibility Theory Restated

As described in these *Proceedings*, the greatest accuracy credibility factor, z, is a number between 0 and 1.00 which minimizes the "mean square error," $M(z)$, given by

$$M(z) = \int_D \{E(H|d) - z \cdot d - (1.0 - z) \cdot E(H)\}^2 \cdot f(d)dd$$

$$\simeq \sum_d \{E(H|d) - z \cdot d - (1.0 - z) \cdot E(H)\}^2 \cdot \Pr(d).$$

"H" is here the prior estimate of an underlying parameter and "d" is actual observed data. $E(X|y)$, $E(X)$, $\Pr(x)$, and $f(x)$ have their usual interpretations.

Since this problem involves selecting a weight, z, which minimizes $M(z)$ over a large range of values, a computer approximation of z is feasible. An illustration follows.

The Problem

Assume that you are analyzing a body of 100 exposures with the following characteristics.

- You expect 0.25 claims per exposure (25 claims in your sample).
- A given exposure's expected frequency may be 0.15, 0.20, 0.25, 0.30, or 0.35 claims, with equal likelihood.
- All exposures have the same (but unknown) underlying frequency.
- The frequency process is Poisson.
- All claims cost $5,000.

Since severity is assumed constant, the following analysis deals exclusively with claim frequency. Clearly, the conclusions apply equally to a consideration of the pure premium.

Ignoring severity, $E(H) = 25$ claims. Moreover, the various values of $E(H|d)$ (d a value of D) can be determined as follows.

d	h	$\Pr(d\|h)$	$\Pr(h)$	$\Pr(d)$	$\Pr(h\|d)$*	$E(H\|d)$**
0 claims	15 claims	0	.20		0.99326	
	20 claims	0	.20		0.00669	
	25 claims	0	.20	0	0.00005	15.03392 claims
	30 claims	0	.20		0	
	35 claims	0	.20		0	
⋮						
20 claims	15 claims	.04181	.20		.21122	
	20 claims	.08884	.20		.44878	
	25 claims	.05192	.20	.03959	.26228	21.08226 claims
	30 claims	.01341	.20		.06775	
	35 claims	.00197	.20		.00996	
⋮						
25 claims	15 claims	.00498	.20		.02535	
	20 claims	.04459	.20		.22696	
	25 claims	.07952	.20	.03929	.40480	25.73962 claims
	30 claims	.05112	.20		.26019	
	35 claims	.01625	.20		.08270	
⋮						

* $\Pr(h|d) = \{\Pr(d|h) \cdot \Pr(h)\}/\Pr(d)$.

** $E(H|d) = \sum_h h \cdot \Pr(h|d)$.

Unfortunately, manually carrying out these calculations for all possible values of D is tedious and impractical. However, this routine is easily handled by a computer. With the assistance of an appropriately programmed machine, for example, the results in Table E1 were obtained for $d = 0, 1, 2, 3, \ldots,$ 59 claims.

TABLE E1

$E(H|d)$ AND $\Pr(d)$ GIVEN 0–59 CLAIMS

d (# of Claims)	$E(H\|d)$	$\Pr(d)$	d (# of Claims)	$E(H\|d)$	$\Pr(d)$
0 claims	15.03392	0	30 claims	29.63554	.03530
1	15.04528	0	31	30.25558	.03375
2	15.06044	.00001	32	30.81878	.03191
3	15.08069	.00003	33	31.32651	.02980
4	15.10772	.00013	34	31.78125	.02746
5	15.14379	.00040	35	32.18624	.02493
6	15.19187	.00101	36	32.54518	.02229
7	15.25587	.00218	37	32.86203	.01961
8	15.34087	.00416	38	33.14079	.01696
9	15.45337	.00709	39	33.38534	.01441
10	15.60155	.01096	40	33.59939	.01203
11	15.79532	.01555	41	33.78634	.00985
12	16.04625	.02046	42	33.94936	.00793
13	16.36697	.02526	43	34.09129	.00626
14	16.76991	.02953	44	34.21469	.00485
15	17.26539	.03301	45	34.32185	.00369
16	17.85911	.03563	46	34.41480	.00275
17	18.54995	.03743	47	34.49534	.00201
18	19.32882	.03857	48	34.56505	.00145
19	20.17966	.03924	49	34.62535	.00102
20	21.08226	.03959	50	34.67745	.00071
21	22.01589	.03974	51	34.72243	.00048
22	22.96208	.03977	52	34.76125	.00032
23	23.90576	.03971	53	34.79471	.00021
24	24.83492	.03955	54	34.82355	.00014
25	25.73962	.03929	55	34.84838	.00009
26	26.61130	.03890	56	34.86976	.00005
27	27.44238	.03834	57	34.88815	.00003
28	28.22649	.03757	58	34.90397	.00002
29	28.95864	.03657	59	34.91756	.00001

Having developed $E(H|d)$ and $\Pr(d)$ for all reasonably foreseeable outcomes of D, the computation of $M(z)$ becomes routine, if lamentably tedious. Again, however, a computer accomplishes the necessary calculations in a matter of microseconds. Since z is selected to minimize $M(z)$, it is easily seen from Table E2 that the appropriate greatest accuracy credibility is 0.67.

TABLE E2

DISPLAY OF z AND $M(z)$

z	$M(z)$	z	$M(z)$
0.00	34.70944	0.51	3.21716
0.01	33.71701	0.52	2.98960
0.02	32.73957	0.53	2.77704
0.03	31.77713	0.54	2.57948
0.04	30.82969	0.55	2.39691
0.05	29.89724	0.56	2.22934
0.06	28.97980	0.57	2.07677
0.07	28.07735	0.58	1.93920
0.08	27.18989	0.59	1.81662
0.09	26.31744	0.60	1.70904
0.10	25.45998	0.61	1.61646
0.11	24.61752	0.62	1.53888
0.12	23.79006	0.63	1.47629
0.13	22.97760	0.64	1.42870
0.14	22.18013	0.65	1.39611
0.15	21.39766	0.66	1.37852
0.16	20.63019	0.67	1.37593
0.17	19.87772	0.68	1.38833
0.18	19.14024	0.69	1.41573
0.19	18.41776	0.70	1.45812
0.20	17.71028	0.71	1.51552
.		.	
.		.	
.		.	
0.48	3.98982	0.98	8.73426
0.49	3.71727	0.99	9.21158
0.50	3.45972	1.00	9.70391

Comparison with Theoretical Results

As expected, the previously derived greatest accuracy credibility factor ($z = 0.67$) can be verified analytically. Specifically, it is easily shown that this outcome would have resulted from Bühlmann's $z = N/(N + K)$ formulation, where K is the mean of process variance divided by the variance of the hypothetical means.[31]

- Mean of (Poisson) process variances = 0.25 claims^2/exposure.
- Variance of hypothetical means $= E(H^2) - (E(H))^2$
  $= 0.0675 - 0.0625 = 0.005$.
- Thus $K = 0.25/0.005 = 50$.
- Hence, for 100 exposures, $z = N/(N + K) = 100/150 = 0.67$, as per the previous development.

Advantages of a Computer-Based Approach

Section I suggested two advantages of a computer-based approach to determining limited fluctuation credibility standards:

- the computer approach is more intuitive, and therefore more easily presented and explained to non-actuarial users, and
- computer simulation provides a means of extending previous analytically derived results.

To a lesser extent, these same advantages can be realized by using a computer to develop greatest accuracy credibility factors.

Clearly, the computer approach is more intuitive than its analytical counterpart. In this writer's opinion, for example, tables along the lines of Table E2 aid considerably in explaining greatest accuracy factors.

Moreover, using a computer provides additional flexibility in the development of credibility formulas.

- By rerunning the necessary computer programs, the sensitivity of credibility factors to small changes in one's prior distribution assumption can readily be determined.
- Variations on greatest accuracy credibility formulas are easily accomplished. In particular, the previous procedure lends itself quite nicely to the development of credibility weights which minimize the mean square

[31] H. Bühlmann, *Mathematical Methods in Risk Theory* (1970), page 102.

error, $M(z)$, over a limited range of possible outcomes, instead of over all possible outcomes.

For instance, suppose that one wishes to determine the credibility factor which minimizes the mean square error of the preceding illustration over the range of outcomes 0, 1, 2, . . . , 25 claims. It can be verified by computer that the appropriate factor is $z = .71$.

The computer-based procedure outlined in this Appendix requires the derivation of $E(H|d)$. This additional information is usually helpful, if not directly applicable in all instances.

Conclusion

While *simulation* may not have direct application to greatest accuracy credibility theory, a computer can be used to explain and present these concepts. Moreover, while the theory behind the greatest accuracy credibility model is probably more advanced than its limited fluctuation counterpart, a computer may open the door to new and expanded applications of greatest accuracy theory.