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LOSS RESERVES: PERFORMANCE STANDARDS

C. K. KHURY

INTRODUCTION

Loss reserves have a significant impact on the reported operating results as well as on the financial condition of an insurer. Actuarial literature to date has focused on developing loss reserving methods [1]. The matter of assessing the condition [2] of loss reserves, on the other hand, has received relatively little attention.

The scarcity of material in this area seems to have given observers of our industry some sort of license to make periodic pronouncements [3] about the adequacy of loss reserves. Security analysts, for example, have made such statements and distributed them throughout the investment community and the insurance industry. The strength of these statements seems to derive mainly from the ability to give them a wide distribution; a subsequent section of this paper suggests that such statements can be speculative and highly misleading. Concurrently and separately, there appears to be within regulatory circles, particularly the National Association of Insurance Commissioners, a movement towards requiring an actuarial opinion on the condition of annual statement loss reserves.

Both conditions noted here suggest that published loss reserves are not viewed with a great deal of confidence, either by investors or by regulators. The problems caused by this lack of confidence are numerous. Two prominent classes of problems deserve mention: the many applications for rate increases which have been denied, or cut back, because of disagreements about the condition of loss reserves; and, the capacity which has been lost to the insurance industry because of failure to attract and retain investor capital due to investor anxiety about the condition of loss reserves.

What is the root cause of this lack of acceptance of the published loss reserve estimates? The major premise of this paper is that the rationale underlying the reserving process is not well understood generally by the ultimate users of the reserve estimate.

There appears to be a need to crystallize the rationale underlying the loss reserving process. This paper is aimed in that direction. Specifically, three concepts will be developed:

- The concept of the loss reserve as a combination of a point estimate *and* a confidence interval.
- The concept of developing *actuarial assumptions* as a necessary step in the loss reserving process.
- The concept of *actuarial gain/loss* in reporting financial results.

Methodologies illustrating these concepts will be introduced along with a description of opportunities for further research.

THE LOSS RESERVE AS A POINT ESTIMATE AND A CONFIDENCE INTERVAL

In the ordinary course of events, an insurer's estimate of its unpaid claim liability (case reserves plus supplemental [4] reserves) is reported in the Annual Statement as a single numerical value. Schedules O and P provide a means for reporting updates of this estimate. In this way the Annual Statement provides for a *retrospective test* of the accuracy of previously published loss reserve estimates. The Annual Statement, however, does not disclose the *quality* of the original estimate in relation to subsequent updates. In other words, if the original estimate ultimately turns out to have missed the mark by some amount, positive or negative, there is nothing in the Annual Statement to tell us whether the variation is within "acceptable" bounds. In the absence of some standard(s) of expected variation, of course, the issue is largely academic. In fact, if one is interested in evaluating the loss reserving performance, then defining a band of expected variation becomes a necessary adjunct to the statement of a point estimate of the unpaid claim liability.

In order to develop the loss reserve confidence interval in the context of this paper, it will be necessary to digress and discuss the pure premium confidence interval.

Expense loadings aside, the ratemaking process is concerned with estimating an ultimate pure premium per unit of exposure during a prospective experience period. In other words, the pure premium is the present value of *expected* claim payments. The pure premium estimate is based mainly on prior claim experience which has been projected forward to the applicable policy period. This pure premium is normally stated as a point estimate, for practical reasons that are obvious. None the less, since this point estimate is just that, *an estimate,* it has associated with it a process variance [5] which is a function of the underlying frequency and severity distributions. This process variance exists whether explicitly stated or not and it is generally equal to the compound variance of the underlying frequency and severity of loss.

First, let us consider the frequency element of the pure premium. Suppose the occurrence of claims is distributed according to a known risk process, F, and suppose we have a body of recent experience which has produced an observed frequency $\hat{f}$. For a given probability p, one can generate a confidence interval around $\hat{f}$, of radius $R(F, p)$. In other words, the true ultimate frequency, f, will lie somewhere within the interval $[\hat{f} \pm R(F, p)]$ with probability p.

The severity element of the pure premium is amenable to the same treatment with:

Severity distribution	S
Observed severity	$\hat{s}$
Selected probability	p
Radius of the severity confidence interval associated with S and p	$R(S, p)$

Then for the given probability, the true ultimate severity, s, will be within the interval $[\hat{s} \pm R(S, p)]$ with probability p.

The pure premium confidence interval can now be constructed several different ways depending on the desired degree of precision. The endpoints of the simplest and most "liberal" interval are the products of the endpoints of the frequency and severity confidence intervals. On the other hand, the smallest and most "economical" confidence interval is based on the compound distribution of F and S. Often an explicit form for this distribution is not available. Mayerson, Jones, and Bowers [6], and Hewitt [7] have described procedures for determining the expected process variance under certain conditions, which can be extended to derive a confidence interval about the pure premium given a probability. In any event, between the two extremes lie many choices, with attendant varying degrees of economy, of confidence intervals which can be associated with the point estimate of the pure premium. The selected pure premium confidence interval will be a key input in the development of a confidence interval for the loss reserve point estimate.

Viewing the reserve situation from the threshold (January 1) of a given accident year, say 1978, suppose a pure premium, P, and a confidence interval of radius $R(P, p)$, corresponding to a selected probability p, have been developed. Also let the number of exposure units which are expected to be earned during 1978 be given by N. On January 1, 1978, the ultimate total incurred loss cost for accident year 1978 is expected to be in the interval $[NP \pm N^{1/2} \cdot R(P, p)]$ with probability p. The radius $[N^{1/2} \cdot R(P, p)]$ is associated with process variance. That is, if the *a priori* pure premium were known, the final result might still differ from the *a priori* level by as much as $[N^{1/2} \cdot R(P, p)]$ with probability p. In reality, however, there is still the uncertainty associated with parameter selection in the course of constructing P. In other words, the *a priori* frequency and severity do not exist, but have to be estimated. Thus, the aggregate expected variation of $(N \cdot P)$ is $[N^{1/2} \cdot R(P, p)]$ plus something to recognize *parameter* variance. The author has arbitrarily chosen $N^{1/2}$ as the factor by which the *a priori* radius has to be expanded. In other words, the ultimate total incurred loss will be in the interval $[NP \pm N \cdot R\ (P, p)]$ with probability p. The radius of this interval is $N^{1/2}[N^{1/2} \cdot R(P, p)] = [N \cdot R(P, p)]$.

Moving to January 1, 1979, the question of the rate (with its underlying pure premium) is now a matter of history. In other words, all policies written to become effective in 1978 at the pure premium P have been written, and all resultant earned exposures have been determined. Recalling that P is the sum of all present values of claims arising out of the N exposure units earned in 1978, the estimated value of P may be stated as follows:

$$P(1978, 1978) = \sum_{i=0}^{n} Pd(1978, 1978 + i, 1978), \text{ where:}$$

$P(x, y)$ = The pure premium for accident year x as calculated (estimated) on January 1, y. Thus, $P(1978, 1978)$ is the 1978 accident year pure premium as estimated on January 1, 1978. $P(1978, 1980)$ is the 1978 accident year pure premium estimated (recalculated) on January 1, 1980.

$Pd(x, y, z)$ = The present value (on January 1, x) of all claim payments made on behalf of accident year x, during year y as estimated on January 1, z. Thus, $Pd(1978, 1980, 1978)$ is the present value (on January 1, 1978) of all claim payments to be paid on behalf of accident year 1978 during 1980 as estimated on January 1, 1978. $Pd(1978, 1981, 1979)$ is the present value of all claim payments to be paid on behalf of accident year 1978 during 1981 as estimated on January 1, 1979.

n = The number of years needed to close out an accident year x counting from December 31, x.

Thus, on January 1, 1979, one is in fact able to compare the estimate $Pd(1978, 1978, 1978)$ with actual experience, that is, with $Pd(1978, 1978, 1979)$. One can construct Table 1 (letting $n = 3$ for this example).

TABLE 1

Projected	Actual
$Pd(1978, 1978, 1978)$	$Pd(1978, 1978, 1979)$
$Pd(1978, 1979, 1978)$	
$Pd(1978, 1980, 1978)$	
$Pd(1978, 1981, 1978)$	

On January 1, 1978, with little information about 1978, the range of the ultimate incurred loss was estimated to fall in the range $[N(P \pm R(P, p))]$ with probability p. On January 1, 1979, actual information about accident year 1978 becomes available; most of the claims have been reported and a portion of the severity has been incurred (the degree of f and s realized depends on the nature of the subject line of business). Recall that the issue at hand is "what kind of a confidence interval can be attached to the loss reserve estimate as of January 1, 1979?" [8]

The loss reserve for accident year 1978, valued as of January 1, 1979, can be viewed as the *newly* estimated

$$\sum_{i=1}^{3} Pd(1978, 1978 + i, 1979)$$

In other words, the reserving process on January 1, 1979 is equivalent to computing $P(1978, 1979)$ based on all information available on January 1, 1978, *plus* all the new information acquired during 1978. It should be quite safe to assume that the quality of $P(1978, 1979)$ is no worse (and is probably better) than $P(1978, 1978)$. In other words, $P(1978, 1979)$ is closer than $P(1978, 1978)$ to the mark:

$$|P(1978, 1979) - P(1978, 1982)| \leq |P(1978, 1978) - P(1978, 1982)|$$

Now the perhaps obvious transition can be made from pricing to its sister process, reserving. The process of estimating $P(1978, 1979)$ is reduced to estimating $\Sigma_{i=1}^{3} Pd(1978, 1978 + i, 1979)$, since $Pd(1978, 1978, 1979)$ is already a known quantity. Thus, the comparison table (Table 1), shown earlier, can be extended into Table 2.

TABLE 2

Increments as of:

January 1, 1978	January 1, 1979
$Pd(1978, 1978, 1978)$	$Pd(1978, 1978, 1979)$ = History
$Pd(1978, 1979, 1978)$	$Pd(1978, 1979, 1979)$ = New Estimate
$Pd(1978, 1980, 1978)$	$Pd(1978, 1980, 1979)$ = New Estimate
$Pd(1978, 1981, 1978)$	$Pd(1978, 1981, 1979)$ = New Estimate
$P(1978, 1978)$	$P(1978, 1979)$ = New Estimate

The radius of the confidence interval associated with $P(1978, 1978)$ was given by $[N \cdot R(P, p)]$. The radius of the confidence interval associated with $P(1978, 1979)$ must be no greater than $[N \cdot R(P, p)]$. This is true because *more* information is available on January 1, 1979 for computing $P(1978, 1979)$ than was available on January 1, 1978 for computing $P(1978, 1978)$; both values represent attempts at hitting the same unknown, but fixed, bull's eye: $P(1978, 1982)$.

TABLE 3

Increments as of January 1,

1978	1979	1980	1981	1982
$Pd(1978, 1978, 1978)$	$Pd(1978, 1978, 1979)$	$Pd(1978, 1978, 1980)$	$Pd(1978, 1978, 1981)$	$Pd(1978, 1978, 1982)$
$Pd(1978, 1979, 1978)$	$Pd(1978, 1979, 1979)$	$Pd(1978, 1979, 1980)$	$Pd(1978, 1979, 1981)$	$Pd(1978, 1979, 1982)$
$Pd(1978, 1980, 1978)$	$Pd(1978, 1980, 1979)$	$Pd(1978, 1980, 1980)$	$Pd(1978, 1980, 1981)$	$Pd(1978, 1980, 1982)$
$Pd(1978, 1981, 1978)$	$Pd(1978, 1981, 1979)$	$Pd(1978, 1981, 1980)$	$Pd(1978, 1981, 1981)$	$Pd(1978, 1981, 1982)$
$P(1978, 1978)$	$P(1978, 1979)$	$P(1978, 1980)$	$P(1978, 1981)$	$P(1978, 1982)$

Extending Table 2 to an ultimate basis produces Table 3. The boxed amounts to the right of the dotted line are accident year 1978's loss reserves as they enter financial statements at successive year-ends. The values to the left of the dotted line are boxed for emphasis only, as they are values *implied* by the rates in use by the insurer and, as such, do not appear in any financial statements. Indexing $[N \cdot R(P, p)]$ in the same manner as P produces the following associations:

Valuation	Radius of Confidence Interval
$P(1978, 1978)$	$N \cdot R(P(1978, 1978), p) = N \cdot R(P, p)$
$P(1978, 1979)$	$N \cdot R(P(1978, 1979), p)$
$P(1978, 1980)$	$N \cdot R(P(1978, 1980), p)$
$P(1978, 1981)$	$N \cdot R(P(1978, 1981), p)$
$P(1978, 1982)$	$N \cdot R(P(1978, 1982), p) = 0$

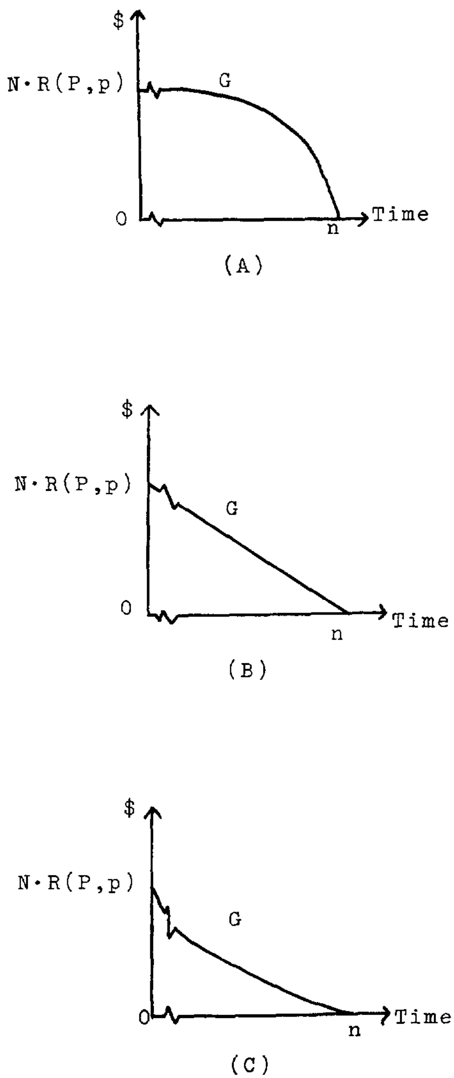
This illustrates the conclusion that the confidence interval associated with $P(1978, 1978 + i)$ must have a radius between 0 and $[N \cdot R(P, p)]$. The appropriate radius value (between 0 and $[N \cdot R(P, p)]$) can be determined by a function, $G(i)$, which satisfies the following conditions:

$G(i)$ exists on the interval $[0, n]$
$G(0) = [N \cdot R(P, p)]$
$G(n + 1) = 0$
$G(i) \geq G(j)$ whenever $i \leq j$

The choice of G should reflect the degree of conservatism the practitioner may wish to introduce into the reserving process in recognition of the volatility [9] of the subject line of business.

Figure 1 displays several possible forms of the function G. The form shown on Graph A in Figure 1 may be suitable for medical malpractice; Graph B may be suitable for workers' compensation; Graph C may be suitable for automobile property damage liability.

FIGURE 1



A numerical application of this process is provided below for illustrative purposes:

$$
\begin{aligned}
\text{Accident Year} &= 1978 \\
N &= 10{,}000 \\
n &= 4 \text{ (accident year will be closed out on 12/31/82)} \\
P(1978, 1978) &= \$80 \\
p &= .85 \\
R(P, p) &= R(80, .85) = \$9 \\
N \cdot R(P, p) &= \$90{,}000 \\
Pd(1978, 1978, 1978) &= \$400{,}000 \\
Pd(1978, 1979, 1978) &= \$200{,}000 \\
Pd(1978, 1980, 1978) &= \$100{,}000 \\
Pd(1978, 1981, 1978) &= \$60{,}000 \\
Pd(1978, 1982, 1978) &= \$40{,}000 \\
G(i) &= [90{,}000/(i + 1)^2] - 500i \\
i &= 0, 1, 2, 3, 4, 5 = n + 1
\end{aligned}
$$

Thus, the reserve amounts (projected as of January 1, 1978) and the radii of their projected confidence intervals are given by:

(1) Projected Valuation To Be Made As Of December 31	(2) i	(3) Expected Estimated Reserve	(4) Projected Confidence Interval $G(i)$	(5) % Swing (4) ÷ (3)
1978	1	\$400,000	\$22,000	5.5%
1979	2	200,000	9,000	4.5
1980	3	100,000	4,125	4.1
1981	4	40,000	1,600	4.0
1982	5	0	0	N/A

ACTUARIAL ASSUMPTIONS

At this point a brief digression from the property and casualty lines is in order. Consider the life insurance company statement. Policyholder reserves are by far the largest single liability. The derivation of these reserves is a mechanical process based on actuarial assumptions and well defined formulae. For each kind of policy that is in force, for example, actuarial assumptions are made (mortality, morbidity, interest, etc.), and the reserve is produced mechanically via various actuarial formulae.

The same is true in the valuation of pensions. To arrive at funding estimates, actuarial assumptions are made (mortality, morbidity, interest, employee turnover, etc.), and the funding estimate is produced mechanically. When one examines the process of estimating funding requirements, it quickly becomes apparent that this process is indeed very similar to loss reserving! In pension funding, both "frequency" (number of retired lives) and "severity" (the duration of an average retirement) are subject to frequent shifts. The pension actuary attempts to recognize these movements by reviewing and updating the plan's actuarial assumptions annually and making adjustments to the funding requirements based on those changes.

In many property and casualty lines, there normally exists a good deal of historical experience that can lend itself to a similar approach to the loss reserve estimation process. Consider, for example, two components of the pure premium, frequency and severity.

Frequency

Historical development (incidence) patterns of frequency over time can be arrayed so as to develop model frequency assumptions. For example, consider a given line of business, with a history of five completed accident years. Also, assume that all claims are reported within three years of occurrence.

Incidence of Frequency

Development Year	Accident Year 1	2	3	4	5
1	$f(1, 1)$	$f(2, 1)$	$f(3, 1)$	$f(4, 1)$	$f(5, 1)$
2	$f(1, 2)$	$f(2, 2)$	$f(3, 2)$	$f(4, 2)$	$f(5, 2)$
3	$f(1, 3)$	$f(2, 3)$	$f(3, 3)$	$f(4, 3)$	$f(5, 3)$

Several frequency models can be extracted from this history depending on environmental factors [10] as well as on the actuary's points of emphasis. One approach is to develop a frequency time index by dividing $f(k, i)$ by $f(k, 3)$ for every k and i:

Frequency Time Indices

Development Year	Accident Year 1	. . .	Accident Year 5	Model Frequency Time Index
1	$f(1, 1)/f(1, 3)$	. . .	$f(5, 1)/f(5, 3)$	$f(., 1)$
2	$f(1, 2)/f(1, 3)$	. . .	$f(5, 2)/f(5, 3)$	$f(., 2)$
3	$f(1, 3)/f(1, 3)$	. . .	$f(5, 3)/f(5, 3)$	$f(., 3)$

At this point, one should note that the extrapolation of model frequency time indices $f(., 1), f(., 2)$, and $f(., 3)$ can be accomplished by taking the arithmetic mean for each of the development years; or by weighting the indices by an arithmetic series, by a geometric series, or by exposure units; or by using other approaches. The result, in any case, is the same: a frequency model has, in fact, been produced. Denote the general model by:

$$[F(L, m, T, n): f(., 1), f(., 2), \ldots, f(., n)]$$

This model is for line of business L, based on m years of experience ending with year T, and requires n years to develop f to a fully reported basis. For example, for line of business L, one might have:

$$[F(L, 5, 1974, 4): .80, .88, .95, 1.00]$$

Severity

The same construction applies to the severity element, producing the following general model:

$$[S(L, m', T', n'): s(., 1), s(., 2), \ldots, s(., n')]$$

Two other prominent factors need to be fixed as actuarial assumptions: interest and inflation. Also, in utilizing these assumptions, the need to develop a claim payout (cash flow) model will have to be met.

Interest

Since reserves represent funds held by the insurer, they will earn interest, regardless of to whom these funds (and, therefore, the interest) belong. An interest assumption, therefore, is needed to recognize future interest income on these loss reserves. There are those who believe that reserves should not be discounted for interest; for them an interest assumption of zero is suitable. An assumption must be made nevertheless. In this paper, interest will be treated as an assumption, i, which may take on any non-negative value. Because interest can vary from year to year, the assumption may be varied. Accordingly, denote the interest rate expected to prevail during year t by $i(t)$.

Inflation

It has been suggested often that loss reserves need not be discounted for interest because inflation acts as negative interest. This view may have validity as long as inflation and interest rates are identical. In all other cases, both factors need to be recognized separately. Denote the inflation assumption expected to prevail during year t by $j(t)$.

Payout Model

P can be constructed [11] in much the same way as S was, producing the following model increments:

$[A(L, m', T', n')\colon a(., 1), a(., 2), \ldots, a(., n')]$

where $a(., 2)$ represents the portion of an individual accident year that will be paid during the second year of development. Note the similarity of index construction to that underlying S.

Given that inflation acts on loss reserves in the same manner as "negative interest," the combined interest/inflation assumption may be constructed for year t, as $[i(t) - j(t)]$ and be denoted by $Z(t)$.

Given these assumptions: frequency, severity, payout model, interest, and inflation; the loss reserve estimate can be derived mechanically. For example, consider accident year 1978 as of December 31, 1978:

- Exposure units earned ...... N
- Observed frequency .... $f(78, 1)$
- Observed severity .... $s(78, 1)$
- Observed payments .... $a(78, 1)$,

given the model assumptions developed earlier:

$F\colon f(., 1), f(., 2), \ldots, f(., n)$
$S\colon s(., 1), s(., 2), \ldots, s(., n')$
$A\colon a(., 1), a(., 2), \ldots, a(., n')$
$Z\colon z(79), z(80), \ldots, z(77 + n')$

Now the reserve can be generated directly as follows:

1. Determine the ultimate reserve assuming $z(t) = 0$ for all t:

$$B = (N)[f(78, 1)f(., n)/f(., 1)][s(78, 1)s(., n')/s(., 1)] - a(78, 1)$$

2. Subdivide B into its payment increments using the payout model A:

$$B_1 = (B)[a(., 2) - a(., 1)]/[a(., n') - a(., 1)]$$
$$B_2 = (B)[a(., 3) - a(., 2)]/[a(., n') - a(., 1)]$$
$$B_3 = (B)[a(., 4) - a(., 3)]/[a(., n') - a(., 1)]$$
$$\vdots$$
$$B_{n'-1} = (B)[a(., n') - a(., n' - 1)]/[a(., n') - a(., 1)]$$

3. Adjust the reserve for Z (assuming all the increments of B are paid on December 31 of the subject year [12]) and generate the present value of the final reserve for accident year 1978 as of December 31, 1978:

Final Discounted Reserve =

$$[1/(1 + z(79))]B_1 +$$
$$[1/(1 + z(79))(1 + z(80))]B_2 +$$
$$[1/(1 + z(79))(1 + z(80))(1 + z(81))]B_3 + \ldots =$$
$$\sum_{q=1}^{n'-1} [1/(1 + z(79))(1 + z(80)) \ldots (1 + z(78 + q))]B_q$$

Under the arrangement described above, the pressure points underlying the reserving process are completely exposed; the focus is on the *assumptions* underlying the computations. Perhaps it is now clear why a security analyst should not assess the state of loss reserves based solely on the published reserve: he does not have access to a key part of the *prospective* reserve computation, namely, the actuarial assumptions. He is normally working with *retrospective* returns, which assess the adequacy of *past* reserves.

Knowledge of the adequacy level of past reserves, by itself, provides no information about the adequacy of current reserves. Knowledge of the assumptions underlying current reserves is needed before valid conclusions can be drawn about their condition. Viewed in this light, pronouncements about the adequacy of reserves by anyone not having access to the underlying assumptions are essentially numerology and have no foundation in fact. In this sense, statements by security analysts about the condition of loss reserves may generally be described as speculative and uninformed.

One last point: if the reserving process for the property and casualty lines becomes fully predicated on actuarial assumptions, it will have pulled alongside the life insurance reserving process. The significance of this observation lies in the fact that security analysts do not usually publish statements evaluating the adequacy of life insurance company reserves. They know neither the assumptions nor the formulae.

ACTUARIAL GAIN/LOSS

This section presumes that the estimate of the ultimate unpaid claim liability has been set as of December 31, t. During calendar year $(t + 1)$, the actual experience corresponding to this estimate can generate two effects on the financial results of an insurer:

- The effect of the difference between expected and actual claim payments. That is $|\hat{a} - a|$, actual development.
- The effect of any restatement of the remaining unpaid claim liability arising from changes in the underlying assumptions. That is, change in expected development.

The financial results for calendar year $(t + 1)$ are composed of the results for the most recent calendar/accident year, $(t + 1)$, and of the results generated by the two factors noted above in connection with the development of prior years' loss reserves. Because of this composition, the interpretation of current financial results is generally not favored with a great deal of clarity [13]. There appears to be a need to spell out [14] the composition of current financial results, distinguishing between those generated by current operations and those generated by loss development. In response to this need, this section contains one way in which this split can be effected and displayed in the annual statement.

Consider Exhibit I. While the construction is largely self-explanatory, the following comments may be helpful:

Line 1. $(t + 1)$ is the only year generating premium income during the subject year (hence the zero under "all other").

Line 2. From the moment a premium dollar is received, it generates investment income until it is fully earned. The total investment income generated by the premiums earned during $(t + 1)$ represents another source of premium-related income. As in the case of line 1, only calendar/accident year $(t + 1)$ generates this category of income.

Line 3. As the premium dollar is earned, the pure premium gradually becomes an incurred loss—partly paid, partly in case reserves, and partly in supplemental reserves. Until the pure premium is fully paid, it generates investment income. The investment income generated by the unpaid pure premium during the year $(t + 1)$ represents a source of income for both categories of experience periods: calendar/accident year $(t + 1)$ and all other accident years.

Line 10. The arithmetic is clear. The amount under the "all other" category represents the impact on current operations of loss reserve development, and it is proposed as the actuarial gain/loss realized during $(t + 1)$ as a result of loss reserve development. As mentioned earlier, this amount is composed of two segments due to:

$|\hat{a} - a|$, and
Changes in the December 31, t, reserve assumptions.

The exhibit might be even more striking if the actuarial gain/loss were split into its two components [15] and displayed in a footnote. In this way the impact of changes in assumptions would be plainly in view.

Although Exhibit I shows only one accident year split, there is no reason why it could not be extended to make use of several splits; the concepts are the same, and the actuarial gain/loss would be more precisely charged back to the appropriate accident period.

DISCUSSION, PROBLEMS, AND OPPORTUNITIES

Given the three concepts advanced here, the loss reserving process tends to take on a slightly different look. Exhibit II describes the input/output flowchart of the process. Of all the process steps, perhaps the fifth is the one requiring comment.

The chief executive might, with one stroke of the pen, unilaterally change the reserve estimate. While the right to do so is not at issue here, two consequences of such action should be spelled out:

- All rates which utilize the revised loss reserve estimate will be inadequate or redundant depending on which way the judgment is made.
- The accountability for the loss reserving performance will have shifted upward to the chief executive.

The first consequence has the greatest potential for immediate damage. Whether the rates are either inadequate or excessive, the "system" is out of synchronization. The ratemaking and reserving processes are joined together by many of the attributes joining the proverbial "chicken and egg" cycle. Because of this relationship, any change in the loss reserve estimate produced by the actuary should be made with the utmost care and with full awareness of its impact on the ratemaking operation.

The second consequence would emerge most prominently if and when the Annual Statement had to be certified. Can the actuary certify the judgment of the chief executive? There is a suggestion here that, if the Annual Statement has to be certified by an actuary, then the fifth step should be omitted from the reserving process. If the ratemaking consequence is not sufficient to remove this step, perhaps a certification requirement would be.

If the loss reserving process is fully predicated on actuarial assumptions as described here, then monitoring the performance of those making the assumption selections becomes a rather simple task. This can best be illustrated by the run-off chart illustrated in Exhibit III. The track record is plainly spelled out in terms of how the original assumptions fared. As a collorary to this application, one is able to test the ratemaking performance as well by inserting an additional column (in box) headed January 1, t. The assumptions in this column would be those underlying the original rate. In this manner the full interdependence of the ratemaking and reserving processes is further magnified.

Although the proposals advanced here stand alone, there still remain numerous opportunities for further research that would enhance the proposed procedures:

- The derivation of confidence intervals for the pure premium for different *classes* of business.
- The composition of confidence intervals for the loss reserve of *several* lines/classes of business.
- The development of *continuous* cash flow models for different lines of business.
- The manner of reporting loss reserve confidence intervals along with the attendant probabilities.
- Extension of the proposed concepts to lines of business insuring rare events—low frequency/high severity combinations.

These are but a few of the research possibilities connected with the concepts introduced in this paper.

SUMMARY

In this paper the loss reserving process is directly identified as a twin of the ratemaking process. Just as actuarial assumptions underlie the ratemaking process, it is suggested that actuarial assumptions underlie the loss reserving process. Just as the pure premium represents an estimate surrounded by a confidence interval, it is proposed that the loss reserve be defined as an estimate with its own confidence interval. Just as the actuary is normally accountable for the ratemaking performance, it is proposed that he also be held accountable for the loss reserving performance, along with full disclosure of how prior loss reserve estimates affected current financial results. For each of these concepts, an illustrative methodology is introduced.

It is this writer's belief that employing these ideas can enhance the clarity and prominence of the loss reserving process. Also, if and when a certification requirement should be introduced, these concepts should help in delineating the specific areas with which the actuary should deal. Finally, viewing the loss reserving process in the framework introduced here may sharpen the practitioner's awareness of the value of loss reserving performance standards, and in the process help motivate an even better work product.

NOTES AND REFERENCES

[1] D. Skurnick, "A Survey of Loss Reserving Methods," *PCAS* LX (1973).

[2] R. Salzmann, "How Adequate Are Loss and Loss Expense Liabilities?," *PCAS* LIX (1972).

[3] Lewinson, Brian, Kreisel, Nadler, and Balcarek, "How Do Investment Analysts View Company Loss Reserves?" Casualty Actuarial Society Loss Reserve Symposium, 1976.

[4] Supplemental reserves, as used here, denote the sum of the reserve for incurred but not reported claims and the reserve for future development on previously reported cases which are still open.

[5] C. Hewitt, "Credibility For Severity," *PCAS* LVII (1970).

[6] Mayerson, Jones and Bowers, "On the Credibility of the Pure Premium," *PCAS* LV (1968).

[7] C. Hewitt, "Loss Ratio Distributions: A Model," *PCAS* LIV (1967).

[8] January 1, 1979 is used instead of December 31, 1978 only as a matter of convenience.

[9] Volatility in reserve development may be deduced from the observed historical variance in age-to-age loss development factors.

[10] W. Fisher and E. Lester, "Loss Reserve Testing in a Changing Environment," *PCAS* LXII (1975).

[11] In order to simplify the process of developing present value reserves, it will be helpful to develop P assuming zero interest. This construction necessitates the discounting of all historical payout data to a present value basis.

[12] Using a continuous payment mode is not only possible, but preferable. The discrete case is used here solely to simplify the presentation.

[13] R. Balcarek, "Effect of Loss Reserve Margins in Calendar Year Results," *PCAS* LIII (1966).

[14] Above and beyond the reassembly of various Schedule O and Schedule P pieces.

[15] It is also possible to distribute each year's actuarial gain/loss among each of the five assumptions.

EXHIBIT I

SPLIT OF FINANCIAL RESULTS DURING CALENDAR YEAR $t + 1$
BETWEEN CURRENT OPERATIONS AND LOSS DEVELOPMENT

	Calendar/Accident Period		
	$(t + 1)$	All Other	All Years
1. Earned premiums	$10,000	$ 0	$10,000
2. Investment income on unearned premiums*	500	0	500
3. Investment income on loss reserves	300	1,500	1,800
4. Total income attributable to insurance operations	$10,800	$ 1,500	$12,300
5. Claim payments	$ 2,000	$ 5,000	$ 7,000
6. Loss reserves as of December 31, t	0	25,000	25,000
7. Loss reserves as of December 31, $(t + 1)$	4,000	22,000	26,000
8. Incurred losses [(5) + (7) − (6)]	$ 6,000	$ 2,000	$ 8,000
9. Incurred expenses**	$ 3,500	$ 600	$ 4,100
10. Net income due to insurance operations [(4) − (8) − (9)]	$ 1,300	$ (1,100)	$ 200

* Only with respect to line 1. See narrative.

** Includes all loss adjustment expenses.

EXHIBIT II

COMPOSITION OF A TYPICAL LOSS RESERVING CYCLE

Input	Processed by	Output
1. Day-to-day transactions of an insurance business.	Operating departments	Raw data
2. Environmental factors and nature of raw data.	Actuary	Assumptions
3. Raw data, assumptions, and method.	Actuary	Reserve point estimate
4. Loss ratio distributions and raw data.	Actuary	Confidence intervals
5. Reserve point estimate and confidence interval and ?	President	Final reserve estimate
6. Final reserve estimate.	Actuary	Annual Statement allocations and pricing inputs

EXHIBIT III

Testing of the Actuarial Assumptions Underlying the Rates and Reserves of Accident Year t

Assumptions		Valuation Date				
Category	Basis	$1.1.t$	$12.31.t$	$12.31.t+1$	$12.31.t+2\ldots$	$12.31.t+n-1$
1. Frequency	Ultimate	f	f_1	f_2	f_3	$\hat{f}$
2. Severity	Ultimate	s	s_1	s_2	s_3	$\hat{s}$
3. Interest	t	$i(t)$	$\hat{i}(t)$	$\hat{i}(t)$	$\hat{i}(t)$	$\hat{i}(t)$
	$t+1$	$i(t+1)$	$i_1(t+1)$	$\hat{i}(t+1)$	$\hat{i}(t+1)$	$\hat{i}(t+1)$
	$t+2$	$i(t+2)$	$i_1(t+2)$	$i_2(t+2)$	$\hat{i}(t+2)$	$\hat{i}(t+2)$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$t+n-1$	$i(t+n-1)$	$i_1(t+n-1)$	$i_2(t+n-1)$	$i_3(t+n-1)$	$\hat{i}(t+n-1)$
4. Inflation	t	$j(t)$	$\hat{j}(t)$	$\hat{j}(t)$	$\hat{j}(t)$	$\hat{j}(t)$
	$t+1$	$j(t+1)$	$j_1(t+1)$	$\hat{j}(t+1)$	$\hat{j}(t+1)$	$\hat{j}(t+1)$
	$t+2$	$j(t+2)$	$j_1(t+2)$	$j_2(t+2)$	$\hat{j}(t+2)$	$\hat{j}(t+2)$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$t+n-1$	$j(t+n-1)$	$j_1(t+n-1)$	$j_2(t+n-1)$	$j_3(t+n-1)$	$\hat{j}(t+n-1)$
5. Payout	t	$p(1)$	$\hat{p}(1)$	$\hat{p}(1)$	$\hat{p}(1)$	$\hat{p}(1)$
Increments	$t+1$	$p(2)$	$p_1(2)$	$\hat{p}(2)$	$\hat{p}(2)$	$\hat{p}(2)$
	$t+2$	$p(3)$	$p_1(3)$	$p_2(3)$	$\hat{p}(3)$	$\hat{p}(3)$
	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
	$t+n-1$	$p(n)$	$p_1(n)$	$p_2(n)$	$p_3(n)$	$\hat{p}(n)$

Pricing Assumptions