## DISCUSSION BY GARY PATRIK

I read Mr. Ferguson's paper with great interest. His topic is critical to the reinsurance business, since so-called burning cost rating is the reinsurance underwriter's favorite pricing technique. An actuarial analysis of it is long overdue.

The paper analyzes burning cost rating and quantifies the degree to which loss results will usually exceed the target loss ratio. This occurs because traditional burning cost rating formulas ignore both loss development and the inflationary growth of losses over time. As actuaries, we are astounded by this formula deficiency.

Burning cost rating is historically a property insurance technique, hence the name. When there is little or no loss development (including IBNR) and rates are essentially constant and there is no rapid change in underlying exposure, burning cost rating can work well enough. However, the very real problem is that this technique is still being used at a time when none of those conditions hold. I have seen burning cost rating formulas used to price excess liability coverage!

The paper concentrates upon the problem of inflationary growth in losses. It ignores the side issue of loss development which is sometimes accounted for by including an extra loading in the loss conversion factor. But we must realize that if a burning cost rating formula does not account for loss development, the resulting situation is even worse than depicted here.

The type of contract which the paper analyzes is one covering loss excess of a fixed retention. The limiting value of the excess loss ratio is given by Ferguson's formula (3) as:

$$\left[\frac{5i(1+i)^5}{(1+i)^5-1}\right] \times \frac{100}{LCF}$$

where $LCF$ = loss conversion factor, traditionally taken as $100 \div$ (target loss ratio).

$1 + i$ = annual inflationary growth factor for losses.

The term in brackets is $5 \div a_{\overline{5|}}^{\,i}$ in annuity notation. This term is greater than 1 whenever $i > 0$. Thus, this loss ratio will usually be greater than the target loss ratio.

The author suggests that we can solve this problem by redefining the loss conversion factor to be:

(1) $$LCF = \left[\frac{5i(1+i)^5}{(1+i)^5 - 1}\right] \times \left(\frac{100}{\text{target loss ratio}}\right)$$

Before discussing some problems regarding mathematical details, I want to emphasize that the author's result holds true in one general case with a suitable interpretation of the notation. And his result can be modified to account for other conditions so as to hold true in another general situation. I will be discussing details, not general direction. Reinsurers are losing money by using traditional burning cost rating formulas. We are all very concerned by this.

*Mathematical Details*

The author's result, his formula (3), is correct if we interpret his notation as follows:

1. the burning cost premium is defined to be the average of the gross excess losses for the preceding 5 years (including loss development) and multiplied by LCF.
2. $a(1+i)^{t+5}$ is the expected value of the gross excess loss in year $t$ (counts times amounts). Drop the symbol $R$.
3. $1+i$ is the inflationary growth rate of the gross *excess* losses.

With this interpretation, the expected value of the burning cost premium for year 0 is given by the formula in his Appendix (dropping the symbol $R$) as:

(2) $$\frac{1}{5}[a + a(1+i) + a(1+i)^2 + a(1+i)^3 + a(1+i)^4] \times LCF$$

In this case, the ratio of the expected values of the excess loss and the burning cost premium for the year 0 is exactly the limiting value in Mr. Ferguson's formula (3)[1]:

(3) $$\frac{a(1+i)^5}{\frac{a}{5}\cdot\left[\frac{(1+i)^5 - 1}{i}\right] \times LCF} = \left[\frac{5i(1+i)^5}{(1+i)^5 - 1}\right] \times \frac{1}{LCF}$$

Note, that you need not take limits. Also, remember that 1 + i is the excess inflationary growth factor; it is 1.25 or more[2], so that the term in brackets is at least 1.85. Thus, the expected loss ratio will be 85% worse than the target loss ratio.

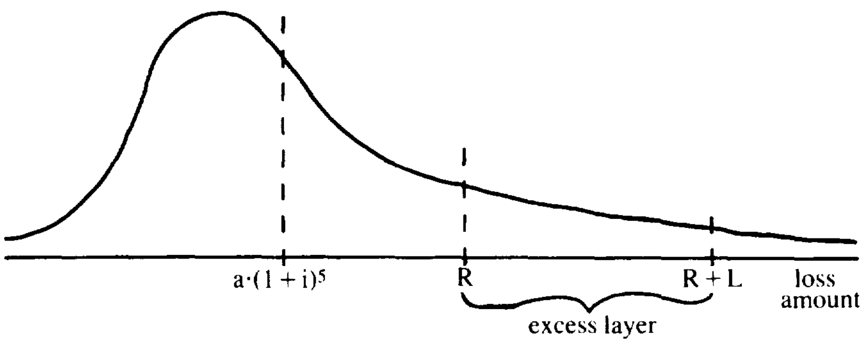
With any other straightforward interpretation of the notation, the formulas do not work. For instance, suppose we take the symbol $R$ to be an aggregate retention. In this case, I believe the author intends that $a(1+i)^{t+5}$ denote the expected value of the gross (excess?) loss subject to the aggregate retention $R$ in the year $t$[3].

According to the Appendix, the expected value of the burning cost premium for the year 0 is:

$$(4) \qquad \frac{1}{5}\left[a + a(1+i) + \ldots + a(1+i)^4 - 5R\right] \times LCF$$

I suppose the intention here is to take the average over the last 5 years of the gross losses excess of the retention $R$ for each year and then look at the expected value of the resulting premium. However, the expected value of the loss excess of $R$ in the year $t$ is usually *not* $a(1+i)^{t+5} - R$.[4] For example, suppose the graph of the probability density function for the gross loss is of the following form:

Figure 1




The term $a(1+i)^5 - R$ is negative if the situation is as in Figure 1. This is, in fact, usually the case for excess coverage.

The true expected value of the loss excess of $R$ in the year $t$ can best be written (assuming an upper limit of $L$ on the excess loss):

$$(5) \qquad \int_{R}^{R+L} (x - R)\ dF_t(x) + L \cdot \int_{R+L}^{\infty} dF_t(x)$$

where $F_t(x) = \text{Probability}\left[\text{total loss} \leq x \mid \text{year } t\right]$

This exact value can be rewritten from (5) as:

$$(6)\quad [a(1+i)^5 - R] + \text{Probability}\,[x \le R] \cdot (R - E[x \mid x \le R])]$$
$$- \left\{ \text{Probability}\,[x \ge R+L] \cdot (E[x \mid x \ge R+L] - [R+L]) \right\}$$

*where* $x$ is the random variable denoting gross loss[5].

The positive term added to $a(1+i)^5 - R$ is the "insurance savings" in the Table M sense and the term subtracted is the "insurance charge".

The other straightforward interpretation takes $R$ as an individual loss retention. In this case, I believe the author intends that $a(1+i)^{t+5}$ denote the expected value of the total loss (ground-up) per loss event (occurrence) in the year $t$. If this were true, one might try to repair the premium formula in the Appendix by including a factor for the number of loss events. One might then suppose that this factor could cancel out of the loss ratio formula (3) and that the rest of the formulas might hold. But once again we are faced with the certainty that $a(1+i)^{t+5} - R$ is *not* the expected value of the individual loss excess of $R$ in year $t$.

*Another Burning Cost Formula*

Most applications of burning cost rating that I have seen do not compute a flat premium as in the preceding discussion. Instead, what is usually computed is a burning cost rate. The burning cost is, as the paper mentions on page 2, "generally defined as the unmodified excess losses divided by the total subject premium"; this total subject premium is usually the total direct premium for the total direct coverage[6]. The burning cost is then multiplied by a loss conversion factor to obtain a final burning cost rate.

Next year's excess premium is the product of the burning cost rate and next year's total subject premium. The total subject premium is estimated in advance and a provisional excess premium is calculated; this may be adjusted later when the actual total subject premium becomes known.

The reason for tying the excess premium to the total subject premium for the year of coverage is that most changes in the underlying exposure will be reflected in the total subject premium, and will then be automatically reflected in the excess premium. However, if the individual loss amounts are growing over time, the excess premium should grow even faster. A moment's reflection on the fact that liability increased limits factors are (necessarily) growing should convince you of this[7].

We can postulate a particularly simple model wherein the total subject premium is growing at a rate of $1 + j$ while the excess loss is growing at a rate of $1 + i$. In this case, the expected value of the burning cost rate for year 0 would be (assuming that IBNR is taken into account and that the total subject premium is deterministic, and generally using the paper's notation).

$$(7) \qquad \left[\frac{a + a(1+i) + \ldots + a(1+i)^4}{b + b(1+j) + \ldots + b(1+j)^4}\right] \times LCF$$

or

$$(8) \qquad \left(\frac{a}{b}\right) \times \left(\frac{j}{i}\right) \times \left[\frac{(1+i)^5 - 1}{(1+j)^5 - 1}\right] \times LCF$$

where $a(1+i)^{t+5} =$ expected value of excess loss in year $t$

$b(1+j)^{t+5} =$ total subject matter premium in year $t$

The expected value of the excess premium for year 0 would be:

$$(9) \qquad \left(\frac{a}{b}\right) \times \left(\frac{j}{i}\right) \times \left[\frac{(1+i)^5 - 1}{(1+j)^5 - 1}\right] \times LCF \times \left[b(1+j)^5\right]$$

$$= a \cdot \left[\frac{j(1+j)^5}{i}\right] \times \left[\frac{(1+i)^5 - 1}{(1+j)^5 - 1}\right] \times LCF$$

And the ratio of the expected values of the excess loss and excess premium would be:

$$(10) \qquad \left[\frac{i(1+i)^5}{j(1+j)^5}\right] \times \left[\frac{(1+j)^5 - 1}{(1+i)^5 - 1}\right] \times \frac{1}{LCF}$$

or simply

$$(11) \qquad \left(\frac{a_{\overline{5}|}^{\,j}}{a_{\overline{5}|}^{\,i}}\right) \times \frac{1}{LCF}$$ in annuity notation

We could then define LCF so that (10) is equal to our target loss ratio.

Of course, if loss development is not taken into account when computing the burning cost rate, the situation is more complex. We must then make some assumptions regarding loss development and we must modify the formulas. Rather than go through this exercise, I would urge you not to use burning cost rating.

*Don't Use Burning Cost*

I immensely distrust burning cost rating. I would go so far as to say that it should only be used when you cannot get more information. If you must use it, loss development and the inflationary growth in excess losses should be accounted for directly in the rating formula year-by-year. And even then, if you cannot get more information, perhaps you should not write the contract.

Why do I so intensely distrust burning cost? The first reason is that burning cost formulas bury all information pertaining to changes in underlying exposure to loss, both counts and amounts. It is better to get more information for each past year and dig into the data to attempt to forecast the next year.

The second reason relates to the variance of the resulting estimate of the proper rate for next year. If the only loss information explicitly considered are the realized losses excess of a fixed retention $R$ for the last 5 years, there may be almost nothing to work with.

For example, suppose that the *overall* growth rate of individual losses is $1 + i$ from year-to-year. That is, assume a simple constant inflation rate which relates the individual loss distribution functions from year-to-year via:

(12) $$F_{t}(x) = F_{(t+n)}[x(1+i)^n] \qquad \text{for all } t, n \text{ and } x$$

*where* $1 + i$ = annual inflationary factor for (ground-up) individual losses

In this case, the retention $R$ in the year 0 (next year) is equivalent to the retention $R \cdot (1 + i)^t$ in the year $t$. In particular, for year $t = -5$, it is $R \cdot (1 + i)^{-5}$. Thus, we see that by considering only losses in excess of $R$, we will have less and less to work with from earlier years. Thus, the earlier the data, the larger the variance relative to the expected value, or the larger the coefficient of variation (standard deviation divided by expected value). Other sources of variance are the loss development factors and inflationary trend factors. Since these are estimates, they are random variables and thus have variance. So, when a burning cost rate estimate is multiplied by loss development factors and trend factors, the resulting estimate of the proper rate for next year will have even more variance.

Now, what more information should we obtain and what should we do with it? I would like to suggest two possibilities:

1. Ideally, it is best to obtain individual reports of all losses which exceed some suitably low but yet manageable retention. Use this information, together with general exposure information, to estimate suitable parameters for a stochastic risk model such as described by Hans Bühlmann and others in the actuarial literature[8]. We have computers and there is plenty of mathematics lying around for us to use. The problem is that building such a model takes time. However, the major advantages of a stochastic risk model are that (1) the important conditions which affect the losses are explicitly taken into account and (2) the model can reflect changing conditions through explicit parameter changes.

2. The next best, and more easily implemented, suggestion is hinted at by formula (12). That is, obtain individual reports on all losses which exceed $R(1 + i)^t$ in the year $t$, i.e., $t = -5$, $t = -4$, etc. Also gather general exposure information which allows you to predict either the total number of loss events or the number of excess losses for year 0. Put these two pieces together to estimate the gross excess loss, or the gross excess loss with respect to total subject premium, for year 0.

For example, suppose that $L$ is the excess limit per loss in year 0 along with the retention $R$. Let $XL(t)$ be the realized excess loss for past year $t$, in the layer $R(1 + i)^t$ up to $(R + L) \times (1 + i)^t$. Suppose that $N(t)$ is the total number of losses for year $t$. Then an estimate of the expected value of the excess loss per loss event in year 0 may be written:

(13)

$$\frac{(1 + i)^{-t} \cdot XL(t)}{N(t)} \qquad \text{(e.g., } t = -5\text{)}$$

We get an estimate like this from each past year and we can multiply some suitable average by an estimate of $E[N(0)]$ to obtain an estimate of the expected excess loss for year 0. An analogous procedure holds if $N(t)$ above is the number of losses excess of $R(1 + i)^t$. This estimate has a lower relative variance than does the typical estimate using only past losses excess of $R$ multiplied by a highly variant excess-of-loss trend factor.

In summary, I agree with Mr. Ferguson that burning cost rating leads to inadequate pricing. And I understand that he is addressing the problem: if underwriters insist upon using burning cost, let us actuaries at least supply them with better factors. However, I would go further and say that the situation is even worse than he depicts, e.g., it is excess inflation—not ground-up inflation—in formula (3), and we should avoid the use of burning cost rating altogether.

## REFERENCES


[1] The expected value of the ratio is usually not the ratio of the expected values. Thus, the following is not the expected value of the loss ratio. The terminology "expected loss ratio" would be statistically confusing in this case.

[2] See Lange, Jeffrey T., "The Interpretation of Liability Increased Limits Statistics", *PCAS* LVI, 1969, pp. 168ff and Miccolis, Robert S., "On the Theory of Increased Limits and Excess of Loss Pricing", *PCAS* LXV, 1977.

[3] The actual loss is a random variable. What Mr. Ferguson means by a $(1 + i)^{t+.5}$ is apparently the expected value.

[4] The expected excess loss is equal to the expected total (ground-up) loss minus the retention if and only if the loss cannot be below the retention or above the reinsurer's limit. As an example, take the degenerate distribution wherein the loss is a constant value.

[5] This reformulation of formula (5) is due to Charles Hachemeister.

[6] See also the Munich Re monograph, *Reinsurance and Reassurance*, volume 2, page 43 (CAS Exam 10 syllabus 1978).

[7] Same as Reference [2].

[8] Bühlmann, Hans, *Mathematical Methods in Risk Theory*, Springer-Verlag, 1970; Beard, R. E., Pentikainen, T. and Pesonen, E., *Risk Theory, The Stochastic Basis of Insurance*, 2nd Edition, Chapman and Hall, 1977; Seal, Hilary, *Stochastic Theory of a Risk Business*, John Wiley & Sons, 1969.