### LOSS RESERVE ADEQUACY TESTING: A COMPREHENSIVE, SYSTEMATIC APPROACH

### JAMES R. BERQUIST AND RICHARD E. SHERMAN VOLUME LXIV, PAGE 131 DISCUSSION BY JOSEPH O. THORNE

### INTRODUCTION

A model for estimating loss and loss expense reserves is presented in the paper. This model is extensive, and the authors are to be commended for their clarity and for the enormous effort required in its preparation. However, some of the concepts of the model may be difficult to extract due to the length of the paper. In my discussion, I shall review a few of the concepts I believe to be fundamental. In addition I shall highlight steps in the application of the model that require particular caution and recommend areas of possible improvement.

### USE OF PAID LOSSES

For the most part, the methodology used in the model is designed for the analysis of paid losses rather than incurred losses. This emphasis on paid loss development can partly be attributed to the actuary's search for an objective standard with a minimum of dependence on case estimates. Although paid losses are an objective measure of past losses, the projection of future payment patterns from past ones has several potential sources of distortion.

### Adjustment for Shifts in Claim Settlement Rates

One of the primary causes of distortion in payment patterns, as was pointed out in the paper, is variation in the rate of settlement of claims. Settlement can be influenced by a multitude of factors. Some factors such as the workload of the claims department and directives from management may be within the control of the company, while others such as late reporting of claims may not be within its control. In either case, the effect on payment patterns can be substantial.

One technique is presented to adjust paid losses for shifts in the rate of settlement of claims. The primary assumption is that if a higher percent of ultimate claims is closed, then a higher percent of ultimate losses will be paid. Lack of recognition of the settlement patterns by size of loss can be an important source of error. As mentioned in the paper, it may be necessary to modify the technique to apply to size of loss categories adjusted for "inflation". In Exhibit I-A an example is given to illustrate the need for recognition of settlement patterns by size of loss. In this simplified example it is assumed that the number of small claims (\$3,000) is steadily decreasing and the number of larger claims (\$20,000) is steadily increasing. As shown in Exhibit I-B, the primary assumption is not satisfied; the percent of claims closed decreases from Accident Years 1973 to 1976, and yet the percent of losses paid increases due to the underlying shifts by size of loss. Thus the technique actually adjusts paid losses to be less comparable among accident years and increases the error in the reserve estimate as shown in Exhibit I-A. Although the example is hypothetical, it was selected recognizing the recent trend toward an increasing proportion of severe, late closing claims in many lines of business and demonstrates the hazards of not recognizing settlement patterns by size of loss.<sup>1</sup>

### "Tail of Payments"

In projecting paid losses to ultimate, the payments beyond a selected point of development are often grouped to form a "tail of payments". Although the paper did not specifically address its estimation, the tail can be a key element of the loss reserve. The selected point of development typically can vary from less than five years for property coverages to fifteen or more years for Medical Malpractice. For example, ten years has generally proved satisfactory for Workers' Compensation since losses paid more than ten years after the accident year have represented a relatively small percent of the ultimate payments (approximately 10% or less). Care must be taken in projecting the tail from older accident years to recent accident years. For example, in Workers' Compensation the tail percentage may increase due to trends in cumulative injury, shifts to unlimited medical benefits, and increases in the proportion of pension claims. On the other hand, the percentage may decrease due to trends in settlement practices for lump sum awards or for compromise and release of claims. The effects of certain factors may be quantified by analysis of loss experience (such as claims by size or injury type) or by specific sampling; other factors may require considerable judgment. In either case, it should be recognized that the adoption of a fixed percentage for the tail of payments may not be appropriate.

### Use of Ultimate Severity for Recent Accident Years

The techniques of traditional paid loss development as represented in Methods I, II, and V may be satisfactory in estimating loss reserves for older, more mature accident years. However, such techniques are many times inaccurate and unstable for recent accident years as shown in Exhibit II. The estimates in that

<sup>&#</sup>x27;See Exhibits I-C, I-D and I-E for a complete application of the adjustment technique for this example.

exhibit have been developed from the Medical Malpractice example presented in the paper. The potential inaccuracy of methods based only on paid losses can be seen by comparing Columns (2) to (4) with Column (1), while the instability can be seen by comparison among Columns (2), (3) and (4). These weaknesses of paid loss methods for recent accident years can be improved by separating the estimation of ultimate losses into two components—number of claims reported and average severity. For many lines of business the estimation of ultimate claims reported is stable, as is the estimation of ultimate severity for older accident years. The ultimate severity for recent accident years can then be projected by trending from prior accident years.

Although the use of ultimate severity can improve the stability and accuracy of the reserve estimates for recent accident years, the periodic warnings in the paper regarding procedural changes in the processing of claims should not be overlooked. A change in the meaning of a "claim" can cause substantial errors in the resulting reserve estimates when relying on the projection of ultimate severity for recent accident years. These changes need not even be internal to the company. For example, changes in waiting periods, statutes of limitation, and no-fault coverage can have a significant effect on the meaning of a "claim" and thus on ultimate severity.

### Ex Ante Analysis

In the evaluation of Methods I to VI, the statistical technique of ex ante analysis<sup>2</sup> was used. In this technique past bias is determined by comparing the past actual average payments with the estimates made at that time. The percentage deviations of the actual from estimated average payments are illustrated in Exhibit V for Method II. To consolidate these deviations two measures are considered-the average percentage deviation and the median percentage deviation. The average percentage is rejected due to its tendency to be overly influenced by large individual percentage deviations. The median is adopted as the measure of bias. While I agree that the median is preferable to this arithmetic average percentage deviation, I feel that a more direct measure is possible. By assigning weights to the percentage deviations by payment year, a weighted average deviation could be determined. That weighted average deviation would relate directly to the calendar year reserve. The weights would be the estimated percent of the calendar year reserve that is contributed by each payment year. One approach to the estimation of the weights and of the weighted percentage deviation is illustrated in Exhibits III-A and III-B for the Automobile Bodily Injury Liability example from the paper.

<sup>2</sup>McLagan, Donald L., "A Non-econometrician's Guide to Econometrics", Business Economics Vol. VIII, No. 3, May 1973, p. 38.

The use of a weighted average deviation should be less susceptible to the large individual percentage deviations that eliminated the arithmetic average deviation from consideration. These individual deviations may be large as percentages, particularly for later development periods. When related to their contributions to the calendar year reserve, though, they should have a smaller effect on the weighted average. I would not expect the weighted average to differ significantly from the median percentage selected in the paper. However, the weighted average has the advantage of relating *directly* to the calendar year loss reserve. It is the calendar year loss reserve that we are estimating—not a set of unweighted percentages.

The technique of ex ante testing can be a useful tool in evaluating past bias in reserve estimates. However, care must be taken that it does not create an unwarranted confidence in the projected loss reserve estimates. The variability inherent in the projection of the future will not be eliminated by the existence of stable indications in the past.

#### CASE RESERVE ADEQUACY

One method of projecting ultimate losses using incurred loss development rather than paid loss development is presented in the paper. The method addresses the problem of changes in case reserve adequacy. For example, incurred loss development factors can be too high if the claim adjusters have been improving the adequacy of their case estimates. In the method presented in the paper, the current calendar year adjusters' estimates for each accident year are adopted and an underlying trend in severity is assumed. Adjusted incurred loss development factors and ultimate loss estimates are then derived.

The estimation of the underlying trend in severity requires much care due to the sensitivity of the reserve estimates to the selected rate, and due to the substantial judgment often necessary. The sensitivity of the reserve estimate is illustrated in Exhibit IV for the Medical Malpractice example presented in the paper. The loss reserve estimate prior to adjustment by the method is approximately \$750 million, based on average incurred loss development and corresponding to a 30% severity trend. The adjusted estimate of the method is approximately \$430 million, corresponding to the 15% severity trend selected in the paper. Thus by reducing the estimated severity trend from 30% to 15%, the effect on the loss reserve estimate will be a decrease of 43%—nearly one-half. The degree of judgment necessary in the estimation of the severity trend makes this substantial effect on the loss reserve estimate particularly critical. For example, estimation of the severity trends for Medical Malpractice is complicated by several factors. The slow payment of losses substantially reduces the experience available by accident year for trending in Exhibit C of the paper. Less than 3% of ultimate losses are paid during the first two payment years of an accident year and less than 30% during the first five payment years. Furthermore, the trends in severity are distorted by irregular settlements and variation in the rate of claims closed without payment. For example, the claim severities from which the calendar year trend of 15.0% is derived in the paper are average paid losses per claim closed *with* payment, while the severities in Exhibit C are average paid losses per claim closed *with or without* payment. Since the rate of claims closed without payment is typically in excess of 60% for Medical Malpractice (over 70% for the example), then variation in the rate can distort the trend in the average reserves per outstanding claim in Exhibit B.

The importance of the type of complicating factors mentioned above is not that 15% or 20% or 25% is the best estimate of the rate. Instead the importance is that *any* selected rate will have a high degree of uncertainty. As shown in Exhibit IV this uncertainty in the rate is directly translated to the reserve estimate.

#### HINDSIGHT OUTSTANDING SEVERITY

The methods presented in the paper concentrate primarily on the projection of ultimate losses, from which the implied loss reserve estimates are determined. An alternate approach is to concentrate directly on the outstanding losses. For example, the average outstanding case estimates (Exhibit B for Medical Malpractice) provide a direct basis for the estimation of loss reserves. However, three disadvantages with these case estimates stand out:

- 1. The estimates are distorted by varying levels of adequacy from year to year.
- 2. IBNR is not included in the estimates.
- 3. Settlement patterns and reporting patterns can make the averages less comparable at corresponding points of development.

The effects of the first two can be reduced if we use our current hindsight knowledge of case development and reportings to adjust these case estimates. The loss reserve estimates of such a method are presented in Exhibit U of the paper. This "hindsight average outstanding losses" technique is not discussed in the paper, but it can be a valuable tool in the evaluation of loss reserve adequacy.

The derivation of hindsight average outstanding losses is illustrated in Exhibits V-A, V-B and V-C for the Automobile Bodily Injury Liability example of the paper. In Exhibit V-A the hindsight outstanding losses are derived as in the retrospective test from cumulative paid losses and estimated ultimate losses. The hindsight outstanding claims (including IBNR claims) are similarly determined in Exhibit V-B. Then the hindsight outstanding losses are divided by the hindsight outstanding claims to give the hindsight average outstanding losses in Exhibit V-C. Thus the averages in Exhibit V-C are the loss severities per outstanding-plus-IBNR claim that "should have been" assigned in the past based on our current hindsight knowledge.<sup>3</sup>

The hindsight average outstanding losses developed in Exhibit V-C are the key to the technique. These hindsight outstanding severities have two particular applications in loss reserve analysis. First, they can be used to evaluate the loss reserve estimates of various other methods. For example, the loss reserve estimates of Methods I to VI can separately be translated into hindsight outstanding severities and evaluated at comparable points of development. A loss reserve estimate that seems otherwise appropriate may not be reasonable when viewed from this perspective. Secondly, the hindsight outstanding severities can be used to develop methods for estimating loss reserves, as in the paper. For example, the hindsight outstanding severities for recent accident years can be trended from older accident years and multiplied by the hindsight outstanding claims. Alternately, they can be compared to claim adjusters' case estimates (Exhibit B for Medical Malpractice) to determine past case adequacy. The current claim adjusters' case estimates can then be adjusted for this indicated past case adequacy. The estimates in the paper use the former method.

Since we concentrate on outstanding rather than paid losses in this technique, two adjustments become especially important. First, just as with closed claims, the mix of outstanding claims can be changed by shifts in settlement patterns. An adjustment for these shifts was discussed earlier. I recommend that the method used in the paper be extended one step further to include this adjustment. Exhibit V-A and V-B have been adjusted. Secondly, the treatment of partial payments can alter the meaning of the averages. By adjusting the average outstanding values to include partial payments, we could convert them to average incurred values per outstanding claim. These average incurred values would provide a more consistent trend, particularly in lines such as Workers' Compensation where significant variations in the extent of partial payments can occur between accident years.

<sup>&</sup>lt;sup>3</sup>Salzmann, Ruth, "Estimated Liabilities For Losses and Loss Adjustment Expenses". Chapter 3, Property-Liability Insurance Accounting, ed. Robert W. Strain, The Merritt Company, Santa Monica, California, 1974.

The hindsight outstanding severity technique is vulnerable to inaccuracies in certain key estimates—especially the ultimate claims closed and the adjustment for shifts in settlement patterns. However, in application the technique has proved to be a valuable approach since it provides an additional perspective with a more direct relationship to the loss reserve being estimated. It can be an important tool which, when combined with the many other methods, can provide the actuary with an improved basis for his judgmental selection of the loss reserve.

### TRIANGLE VS PARALLELOGRAM

The accident year experience analyzed by the authors is in "triangular" form, as is illustrated in Exhibit VI. In such a form, the experience of the older accident years is lost (1973 and prior in Exhibit VI). The experience from the early development years of these accident years may be difficult to compile and in many instances is only of marginal value due to its age. However, the experience from the later years of development is often not as difficult to compile and may be well worth the extra effort. The expansion of the triangle to a parallelogram, as shown in Exhibit VI, could result in a gain in the accuracy and stability of the reserve estimates at nearly every phase in the model.<sup>4</sup>

#### CONCLUSION

I have reviewed certain stages of the model to which I believe the reserve estimates are particularly sensitive. However, the recognition of these crucial stages does not imply a rejection of the model. On the contrary, in application to a variety of companies and lines I have found that with recognition of their sensitivity such techniques can be useful tools in the evaluation of loss and loss expense reserves. The model presented has many positive features, particularly its flexibility in the recognition of the effects of the common but crucial considerations reviewed in Appendices B and C of the paper. It is the vulnerability of the various reserve models to such effects and the need for considerable actuarial judgment at key stages that concerns me, especially in view of the tendency of non-technicians to expect a "mechanized" reserving procedure. I believe a reserve model can only be expected to be a tool on which the actuary can impose his judgment.

16

<sup>4</sup> For example, tail of payments, ex ante testing, trend in paid and outstanding claim severity. Methods I to VI, hindsight average outstanding losses.

### **EXHIBIT I-A**

## Adjustment for Shifts in Claim Settlement Rates Changes in Distribution of Claims by Size of Loss

### Pattern of Payment Assumed

|                  | Claim<br>From 0   | s Closed<br>-12 Mos. | Claims Closed<br>From 13-24 Mos. | Claims Closed<br>From 25-36 Mos. | Total No.<br>of Claims<br>Closed |
|------------------|-------------------|----------------------|----------------------------------|----------------------------------|----------------------------------|
| Accident<br>Year | \$3,000<br>Claims | \$20,000<br>Claims   | \$20,000<br>Claims               | \$30,000<br>Claims               |                                  |
| 1973             | 50,000            | 0                    | 30.000                           | 20.000                           | 100.000                          |
| 1974             | 46,000            | 1.000                | 32.000                           | 20.000                           | 99.000                           |
| 1975             | 42,000            | 2.000                | 34,000                           | 20.000                           | 98.000                           |
| 1976             | 38,000            | 3,000                | 36,000                           | 20,000                           | 97,000                           |

## Projected Ultimate Losses as of 12/31/76 Before and After "Adjustment for Claims Disposed"

| A    | Actual      | Projected Ultimate Losses |             |  |
|------|-------------|---------------------------|-------------|--|
| Year | Losses      | Before Adj.               | After Adj.  |  |
| 1973 | \$1,350,000 | \$1,350,000               | \$1,350,000 |  |
| 1974 | 1,398,000   | 1,398,000                 | 1,398,000   |  |
| 1975 | 1,446,000   | 1,502,496                 | 1,527,030   |  |
| 1976 | 1,494,000   | 1,560,258                 | 2,058,350   |  |

Notes: 1. The above example illustrates how the adjustment for shifts in the settlement of claims can potentially increase rather than decrease errors in reserve estimates unless variation in distribution of claims by size of loss is considered.

2. Ultimate losses are projected using average paid loss development. See Exhibits I-C, I-D and I-E for their derivation.

3. Amounts are in thousands of dollars.

#### LOSS RESERVE ADEQUACY TESTING

### EXHIBIT I-B

## Underlying Effect of Assumed Shift in Size of

### Loss Distribution

### Ultimate Claims Disposed Ratio

| Accident |       | Month of | Development |        |
|----------|-------|----------|-------------|--------|
| Year     | 12    | 24       | 36          | Ult.   |
| 1973     | .5000 | .8000    | 1.0000      | 1.0000 |
| 1974     | .4747 | .7980    | 1.0000      |        |
| 1975     | .4490 | .7959    |             |        |
| 1976     | .4227 |          |             |        |

### Percent of Ultimate Losses Paid

| Accident | Month of Development |       |        |        |
|----------|----------------------|-------|--------|--------|
| Year     | 12                   | 24    | .36    | Ult.   |
| 1973     | 11.11                | 55.56 | 100.00 | 100.00 |
| 1974     | 11.30                | 57.08 | 100.00 |        |
| 1975     | 11.48                | 58.51 |        |        |
| 1976     | 11.65                |       |        |        |

- Notes: 1. The adjustment would reduce the losses paid in older accident years since the percent of claims closed has decreased. However, the percent of losses paid is already too low for older accident years. Thus in this example the adjustment would make the losses *less* comparable among accident years, not *more* comparable.
  - 2. The ultimate claims disposed ratio is the cumulative closed claims divided by the ultimate claims.

## **EXHIBIT I-C**

# Projected Ultimate Losses Before Adjustment

## Unadjusted Paid Losses

| Accident | Month of Development |           |             |             |
|----------|----------------------|-----------|-------------|-------------|
| Year     | 12                   | 24        | 36          | Ult.        |
| 1973     | \$150,000            | \$750,000 | \$1,350,000 | \$1,350,000 |
| 1974     | 158,000              | 798,000   | 1,398,000   |             |
| 1975     | 166,000              | 846,000   |             |             |
| 1976     | 174,000              |           |             |             |

## **Development Factors**

| Accident     | M              | onth of Develop | ment       |
|--------------|----------------|-----------------|------------|
| Year         | 12 to 24       | 24 to 36        | 36 to Ult. |
| 1973         | 5.0000         | 1.8000          | 1.0000     |
| 1974         | 5.0506         | 1.7519          |            |
| 1975         | 5.0964         |                 |            |
| Arith. Avg.  | 5.0490         | 1.7760          | 1.0000     |
| Cum. Product | <b>8.967</b> 0 | 1.7760          | 1.0000     |

## Projection of Ultimate Losses

| Accident<br>Year | Cum.<br>Paid Losses<br>@ 12/31/76 | Cum. Paid<br>Loss Dev.<br>Factor | Proj. Ult.<br>Losses<br>(1)x(2) |
|------------------|-----------------------------------|----------------------------------|---------------------------------|
|                  | (1)                               | (2)                              | (3)                             |
| 1973             | \$1,350,000                       | 1.0000                           | \$1,350,000                     |
| 1974             | 1,398,000                         | 1.0000                           | 1,398,000                       |
| 1975             | 846,000                           | 1.7760                           | 1,502,496                       |
| 1976             | 174,000                           | 8.9670                           | 1,560,258                       |

Note: Amounts are in thousands of dollars.

#### LOSS RESERVE ADEQUACY TESTING

### **EXHIBIT I-D**

### Projected Ultimate Losses After Adjustment

## **Adjusted Paid Losses**

| Accident | Month of Development |           |             |             |  |
|----------|----------------------|-----------|-------------|-------------|--|
| Year     | 12                   | 24        | 36          | Ult.        |  |
| 1973     | <b>\$</b> 99,070     | \$733,762 | \$1,350,000 | \$1,350,000 |  |
| 1974     | 121,717              | 789,804   | 1,398,000   |             |  |
| 1975     | 146,724              | 846,000   |             |             |  |
| 1976     | 174,000              |           |             |             |  |

## **Development Factors**

| Accident     | Month of Development |          |            |  |
|--------------|----------------------|----------|------------|--|
| Year         | 12 to 24             | 24 to 36 | 36 to Ult. |  |
| 1973         | 7.4065               | 1.8398   | 1.0000     |  |
| 1974         | 6.4889               | 1.7701   |            |  |
| 1975         | 5.7659               |          |            |  |
| Arith. Avg.  | 6.5538               | 1.8050   | 1.0000     |  |
| Cum. Product | 11.8296              | 1.8050   | 1.0000     |  |

## Projection of Ultimate Losses

| Accident<br>Year | Cum.<br>Paid Losses<br>@ 12/31/76 | Cum. Paid<br>Loss Dev.<br>Factor | Proj. Ult.<br>Losses<br>(1)x(2) |  |
|------------------|-----------------------------------|----------------------------------|---------------------------------|--|
|                  | (1)                               | (2)                              | (3)                             |  |
| 1973             | \$1,350,000                       | 1.0000                           | \$1,350,000                     |  |
| 1974             | 1,398,000                         | 1.0000                           | 1,398,000                       |  |
| 1975             | 846,000                           | 1.8050                           | 1,527,030                       |  |
| 1976             | 174,000                           | 11.8296                          | 2,058,350                       |  |

- Notes: 1. The adjustment of paid losses for shifts in the rate of settlement of claims is calculated in Exhibit I-E.
  - 2. Amounts are in thousands of dollars.

### **EXHIBIT I-E**

# Estimation of Adjusted Paid Losses at Equal Percentiles of Ultimate Claims

## Unadjusted Closed Claims

| Accident<br>Year | Month of Development |        |         |         |
|------------------|----------------------|--------|---------|---------|
|                  | 12                   | 24     | 36      | Ult.    |
| 1973             | 50,000               | 80,000 | 100,000 | 100,000 |
| 1974             | 47,000               | 79,000 | 99,000  | 99,000  |
| 1975             | 44,000               | 78,000 |         | 98,000  |
| 1976             | 41,000               |        |         | 97,000  |

# Adjusted Closed Claims at Equal Percentiles of Ultimate Claims Closed <sup>1</sup>

| Accident<br>Year | Month of Development |        |         |         |
|------------------|----------------------|--------|---------|---------|
|                  | 12                   | 24     | 36      | Ult.    |
| 1973             | 42,268               | 79,592 | 100,000 | 100,000 |
| 1974             | 41,845               | 78,796 | 99,000  |         |
| 1975             | 41,423               | 78,000 |         |         |
| 1976             | 41,000               |        |         |         |

### EXHIBIT I-E (Continued)

# Adjusted Paid Losses at

## Equal Percentiles of Ultimate Claims Closed

|                  | Ма         | Adj.             | Unadj.<br>Closed Claims           |                             | U<br>Paid                   | Adj.            |                             |
|------------------|------------|------------------|-----------------------------------|-----------------------------|-----------------------------|-----------------|-----------------------------|
| Accident<br>Year | of<br>Dev. | Closed<br>Claims | @ 12 Mo.<br><i>x</i> <sub>1</sub> | @ 24 Mos.<br>x <sub>2</sub> | @ 12 Mos.<br>y <sub>t</sub> | @ 24 Mos.<br>y2 | Paid<br>Losses <sup>2</sup> |
| 1973             | 12         | 42,268           | 50,000                            | 80,000                      | \$150,000                   | \$750,000       | \$ 99,070                   |
| 1973             | 24         | 79,592           | 50,000                            | 80,000                      | 150,000                     | 750,000         | 733,762                     |
| 1974             | 12         | 41,845           | 47,000                            | 79,000                      | 158,000                     | 798,000         | 121,717                     |
| 1974             | 24         | 78,796           | 47,000                            | 79,000                      | 158,000                     | 798,000         | 789,804                     |
| 1975             | 12         | 41,423           | 44,000                            | 78,000                      | 166,000                     | 846,000         | 146,724                     |

- Notes: 1. For 12 months of development the adjusted closed claims are 41,000/97,000 times the ultimate claims closed and for 24 months of development 78,000/98,000 times the ultimate claims closed.
  - 2. The adjusted paid losses y are estimated from an exponential curve of the form  $y = a \cdot b^{y}$ (=  $c \cdot e^{at}$ ) with x representing the claims closed and y the paid losses. The equation for y is then

$$y = y_i \cdot \left(\frac{y_2}{y_i}\right) \left[\frac{x - x_i}{x_2 - x_i}\right]$$

While the magnitude of the effect on the estimates in Exhibit I-A is dependent on the form of the equation, the primary assumption without recognition of size of loss (not the form) is the basic cause for the incorrect direction of the adjustment.

3. Amounts are in thousands of dollars.

### EXHIBIT II

# Medical Malpractice Comparison of Loss Reserve Estimates Methods I, II, V and Selected

| Accident | Selected   | Paid Loss Development Method |           |           |  |  |  |
|----------|------------|------------------------------|-----------|-----------|--|--|--|
| Year     | Paid Proj. | I                            | 11        | v         |  |  |  |
|          | (1)        | (2)                          | (3)       | (4)       |  |  |  |
| 1975     | \$123,432  | \$171,805                    | \$141,817 | \$399,928 |  |  |  |
| 1976     | 111,833    | 212,483                      | 154,901   | 731,930   |  |  |  |

- Notes: 1. Methods I and V trend cumulative paid loss development factors while Method II uses a weighted average. The sensitivity of the indications of the methods and comparison with the selected reserve estimate illustrates the need for measures other than paid losses for the more recent accident years (for example, trended ultimate severity).
  - 2. The above estimates for Methods I, II and V have assumed that the payments beyond 96 months of development for Accident Years 1975 and 1976 will be comparable with those for Accident Years 1969 to 1973; that is, payment beyond 96 months will be approximately 32.5% of the total payments for the accident year.
  - 3. Amounts are in thousands of dollars.

## **EXHIBIT III-A**

Automobile Bodily Injury Liability

Estimated Weights for Calendar Year 1976

# Reserve Ex Ante Errors

# Distribution of Calendar Year Loss Reserve to Year of Payment

|                  | Year of Payment |           |          |          |         |         |         |        | Est.     |                    |
|------------------|-----------------|-----------|----------|----------|---------|---------|---------|--------|----------|--------------------|
| Accident<br>Year | AY              | AY + 1    | AY + 2   | AY + 3   | AY + 4  | AY + 5  | AY + 6  | AY + 7 | AY + 8 + | Ultimate<br>Pymts. |
|                  | 12.73%          | 30.22%    | 23.47%   | 16.75%   | 9.14%   | 4.15%   | 2.01%   | 0.69%  | 0.84%    | 100.00%            |
| 1969             |                 |           |          |          |         |         |         |        | \$87     | \$10.343           |
| 1970             |                 |           |          |          |         |         |         | \$84   | 103      | 12,218             |
| 1971             |                 |           |          |          |         |         | \$297   | 102    | 124      | 14,757             |
| 1972             |                 |           |          |          |         | \$ 692  | 335     | 115    | 140      | 16,665             |
| 1973             |                 |           |          |          | \$1.679 | 762     | 369     | 127    | 154      | 18,370             |
| 1974             |                 |           |          | \$2.968  | 1.620   | 735     | 356     | 122    | 149      | 17,721             |
| 1975             |                 |           | \$5,018  | 3,581    | 1,954   | 887     | 430     | 148    | 180      | 21,380             |
| 1976             |                 | \$6,647 * | 5,163    | 3.684    | 2.011   | 913     | 442     | 152    | 185      | 21,997             |
| CY1976 Reserve   |                 | \$6,647   | \$10,181 | \$10,233 | \$7,264 | \$3,989 | \$2,229 | \$850  | \$1,122  |                    |

## EXHIBIT III-A

(Continued)

## Estimation of Payment Year Weights for Calendar Year 1976 Loss Reserve

| Pymt.<br>Period | Contribution<br>to CY1976<br>Loss Reserve | Pct. of Total<br>Contr. to<br>CY1976<br>Loss Reserve |
|-----------------|---|--|
| AY + 1          | \$ 6,647                                  | 17.3%  |
| AY + 2          | 10,181                                    | 26.6   |
| AY + 3          | 10,233                                    | 26.7   |
| AY + 4          | 7,264                                     | 19.0   |
| AY + 5          | 3,989                                     | 10.4   |
| Total           | \$38,314                                  | 100.0%   |

Notes: 1. (\*)  $6,647 = 30.22\% \times 21,997$ .

- 2. The estimated ultimate payments correspond to the mean of the Methods I-VI estimates; an alternate basis could have been selected judgmentally.
- 3. The pattern for payment of losses (12.73%, 30.22%, 23.47%...) is derived from the estimated ultimate payments and the cumulative payments as of 12/31/76.
- 4. The contributions to the Calendar Year 1976 loss reserve are restricted to payment periods prior to AY + 6, since lack of loss experience prevented the calculation of ex ante errors beyond AY + 5 (see Exhibit V). The expansion from triangular to parallelogram form would allow estimation of errors beyond AY + 5 and avoid this restriction (see Exhibit VI).
- 5. Amounts are in thousands of dollars.

### EXHIBIT III-B

# Automobile Bodily Injury Liability Estimated Ex Ante Error in Calendar Year 1976 Reserve Projection for Method II

| Pymt.<br>Period | Wgts. for<br>Errors | Arithmetic<br>Avg. Error<br>for Est. of<br>Pymts. During<br>Period (1) | Weighted<br>Avg. Ex Ante<br>Error in Est.<br>of CY1976 Res.<br>(2) × (3) |
|-----------------|---------------------|--|--|
| (1)             | (2)                 | (3)  | (4)  |
| AY + 1          | 17.3%               | - 7.45%  | - 1.29%  |
| AY + 2          | 26.6                | -8.93  | - 2.38   |
| AY + 3          | 26.7                | -0.85  | -0.23  |
| AY + 4          | 19.0                | + 3.25   | + 0.62   |
| AY + 5          | 10.4                | + 2.41   | + 0.25   |
| Total           | 100.0%              | - 4.44%  | - 3.03%  |

Weighted average ex ante error for Method II in estimation of CY1976 loss reserve = -3.03%

- Notes: 1. The method assumes that the ex ante error and the period of payment are correlated; this correlation has been frequently observed, particularly when the trend for later payments has been accelerating faster than for early payments.
  - The arithmetic average errors in Column (3) are "column averages" of the percentage deviations presented in Exhibit V. Note that the accuracy of these "column averages" potentially could be improved by expansion to a parallelogram (see Exhibit V1).
  - 3. The weights in Column (2) are derived in Exhibit III-A.

### EXHIBIT IV





|             |             | medited Loss Development l'actors |       |       |       |       |       |        |  |  |  |  |
|-------------|-------------|-----------------------------------|-------|-------|-------|-------|-------|--------|--|--|--|--|
|             | Policy Year |                                   |       |       |       |       |       |        |  |  |  |  |
|             | 1969        | 1970                              | 1971  | 1972  | 1973  | 1974  | 1975  | 1976   |  |  |  |  |
| Before Adj. | 1.000       | 1.027                             | 1.080 | 1.302 | 1.525 | 2.291 | 4.402 | 11.145 |  |  |  |  |
| After Adj.  | 1.000       | .979                              | .944  | 1.003 | .932  | 1.244 | 1.900 | 7.425  |  |  |  |  |

## Incurred Loss Development Factors

Note: The sensitivity of the loss reserve estimate to the selected rate of growth is demonstrated in the graph. The impact of adopting the 15% rate selected in the paper (after adj.) rather than the 30% underlying the claims adjuster estimates (before adj.) is shown both in the loss reserve estimate of the graph and in the implied incurred loss development factors.

## EXHIBIT V-A

# Automobile Bodily Injury Liability

## Calculation of Hindsight Outstanding Losses

# Cumulative Paid Losses

| Accident | Month Of Development |          |          |          |          |          |                  |          |          |  |  |
|----------|----------------------|----------|----------|----------|----------|----------|------------------|----------|----------|--|--|
| Year     | 12                   | 24       | 36       | 48       | 60       | 72       | 84               | 96       | Ultimate |  |  |
| 1969     | \$ 1,398             | \$ 4,222 | \$ 6,452 | \$ 8,522 | \$ 9,585 | \$10,066 | <b>\$1</b> 0,187 | \$10,256 | \$10,343 |  |  |
| 1970     | 1,705                | 5,116    | 7,845    | 10,160   | 11,309   | 11,739   | 12,031           |          | 12,223   |  |  |
| 1971     | 1,938                | 6,168    | 9,580    | 12,261   | 13,571   | 14,235   |                  |          | 14,752   |  |  |
| 1972     | 2,191                | 7,127    | 11,034   | 13,843   | 15,383   |          |                  |          | 16,616   |  |  |
| 1973     | 2,523                | 7,892    | 11,943   | 15,278   |          |          |                  |          | 18,231   |  |  |
| 1974     | 2,240                | 7,189    | 11,771   |          |          |          |                  |          | 17,347   |  |  |
| 1975     | 2,670                | 9,182    |          |          |          |          |                  |          | 20,588   |  |  |
| 1976     | 2,801                |          |          |          |          |          |                  |          | 21,419   |  |  |

## **EXHIBIT V-A**

## (Continued)

## Hindsight Outstanding Losses

| Accident<br>Year | Month Of Development |          |          |          |               |               |        |              |  |  |  |  |
|------------------|----------------------|----------|----------|----------|---------------|---------------|--------|--------------|--|--|--|--|
|                  | 12                   | 24       | 36       | 48       | 60            | 72            | 84     | 96           |  |  |  |  |
| 1969             | \$ 8,945             | \$ 6,121 | \$ 3,891 | \$ 1.821 | <b>\$</b> 758 | <b>\$</b> 277 | \$ 156 | <b>\$</b> 87 |  |  |  |  |
| 1970             | 10,518               | 7,107    | 4,378    | 2,063    | 914           | 484           | 192    |              |  |  |  |  |
| 1971             | 12,814               | 8,584    | 5,172    | 2,491    | 1,181         | 517           |        |              |  |  |  |  |
| 1972             | 14,425               | 9,489    | 5,582    | 2,773    | 1.233         |               |        |              |  |  |  |  |
| 1973             | 15,708               | 10,339   | 6,288    | 2,953    |               |               |        |              |  |  |  |  |
| 1974             | 15,107               | 10,158   | 5,576    |          |               |               |        |              |  |  |  |  |
| 1975             | 17,918               | 11,406   |          |          |               |               |        |              |  |  |  |  |
| 1976             | 18,618               |          |          |          |               |               |        |              |  |  |  |  |

Notes: 1. Cumulative paid losses are adjusted for shifts in the rate of settlement of claims (Exhibit N). The selected ultimate losses correspond to the selected loss reserve estimates in Exhibit U. For example, for Accident Year 1976 \$21,419 = \$18,618 + \$2,801.

2. The hindsight outstanding losses are the selected ultimate minus the cumulative paid losses. For example, for Accident Year 1969 \$8,945 ≈ \$10,343-\$1,398.

3. Amounts are in thousands of dollars.

## EXHIBIT V-B

# Automobile Bodily Injury Liability

# Calculation of Hindsight Outstanding Claims

## Cumulative Closed Claims

| Accident | Month Of Development |       |       |       |       |       |       |       |          |  |  |  |
|----------|----------------------|-------|-------|-------|-------|-------|-------|-------|----------|--|--|--|
| Year     | 12                   | 24    | 36    | 48    | 60    | 72    | 84    | 96    | Ultimate |  |  |  |
| 1969     | 3.327                | 6.018 | 6,926 | 7.417 | 7,644 | 7,748 | 7.788 | 7,806 | 7.822    |  |  |  |
| 1970     | 3,693                | 6,681 | 7,690 | 8,234 | 8,486 | 8,602 | 8,647 |       | 8,674    |  |  |  |
| 1971     | 4,232                | 7.656 | 8,811 | 9.435 | 9,723 | 9,855 |       |       | 9,950    |  |  |  |
| 1972     | 4,121                | 7,455 | 8,580 | 9,188 | 9,469 |       |       |       | 9,690    |  |  |  |
| 1973     | 4,079                | 7,379 | 8,492 | 9,093 |       |       |       |       | 9,590    |  |  |  |
| 1974     | 3,322                | 6,009 | 6,916 |       |       |       |       |       | 7,810    |  |  |  |
| 1975     | 3,442                | 6,226 |       |       |       |       |       |       | 8,092    |  |  |  |
| 1976     | 3,230                |       |       |       |       |       |       |       | 7.594    |  |  |  |

## EXHIBIT V-B (Continued)

### Hindsight Outstanding Claims

| Accident _<br>Year |       | Month Of Development |       |     |     |    |     |    |  |  |  |  |
|--------------------|-------|----------------------|-------|-----|-----|----|-----|----|--|--|--|--|
|                    | 12    | 24                   | 36    | 48  | 60  | 72 | 84  | 96 |  |  |  |  |
| 1969               | 4,495 | 1,804                | 896   | 405 | 178 | 74 | .34 | 16 |  |  |  |  |
| 1970               | 4,981 | 1,993                | 984   | 440 | 188 | 72 | 27  |    |  |  |  |  |
| 1971               | 5,718 | 2,294                | 1,139 | 515 | 227 | 95 |     |    |  |  |  |  |
| 1972               | 5,569 | 2,235                | 1,110 | 502 | 221 |    |     |    |  |  |  |  |
| 1973               | 5,511 | 2,211                | 1,098 | 497 |     |    |     |    |  |  |  |  |
| 1974               | 4,488 | 1,801                | ·894  |     |     |    |     |    |  |  |  |  |
| 1975               | 4.650 | 1,866                |       |     |     |    |     |    |  |  |  |  |
| 1976               | 4,364 |                      |       |     |     |    |     |    |  |  |  |  |

Notes: 1. Cumulative closed claims are adjusted for shifts in the rate of settlement of claims (Exhibit M). The ultimate claims are those selected in Exhibit J of the paper.

2. The hindsight outstanding claims are the selected ultimate minus the cumulative closed claims and thus include both reported claims that are still open and IBNR claims. For example, for Accident Year 1969 4.495 = 7,822 - 3,327.

### EXHIBIT V-C

### Automobile Bodily Injury Liability

### Hindsight Average Outstanding Losses

| Accident<br>Year | Month Of Development |         |         |         |         |         |         |         |  |  |  |
|------------------|----------------------|---------|---------|---------|---------|---------|---------|---------|--|--|--|
|                  | 12                   | 24      | 36      |         |         | 72      | 84      | 96      |  |  |  |
| 1969             | \$1,990              | \$3,393 | \$4,343 | \$4,496 | \$4,258 | \$3,743 | \$4.588 | \$5,438 |  |  |  |
| 1970             | 2,112                | 3,566   | 4,449   | 4,689   | 4,862   | 6,722   | 7.111   |         |  |  |  |
| 1971             | 2,241                | 3,742   | 4,541   | 4,837   | 5,203   | 5,442   |         |         |  |  |  |
| 1972             | 2,590                | 4,246   | 5,029   | 5,524   | 5,579   |         |         |         |  |  |  |
| 1973             | 2,850                | 4,676   | 5,727   | 5,942   |         |         |         |         |  |  |  |
| 1974             | 3,366                | 5,640   | 6,237   |         |         |         |         |         |  |  |  |
| 1975             | 3,853                | 6,113   |         |         |         |         |         |         |  |  |  |
| 1976             | 4,266                |         |         |         |         |         |         |         |  |  |  |

Notes: 1. The hindsight average outstanding losses are the hindsight outstanding losses in Exhibit V-A divided by the hindsight outstanding claims in Exhibit V-B.

2. The hindsight average outstanding losses above can be used to test the reasonableness of the selected ultimate loss estimates in Exhibit V-A. Alternately, the loss reserves can be estimated directly.

### **EXHIBIT VI**

# Expansion from Triangular to Parallelogram Form for Loss Experience

### Triangular Form

| Accident<br>Year | Month of Development |     |     |     |  |  |  |  |
|------------------|----------------------|-----|-----|-----|--|--|--|--|
|                  | 12                   | 24  | 36  | 48  |  |  |  |  |
|                  |                      |     |     |     |  |  |  |  |
| 1974             | XXX                  | XXX | XXX | XXX |  |  |  |  |
| 1975             | XXX                  | XXX | XXX |     |  |  |  |  |
| 1976             | XXX                  | XXX |     |     |  |  |  |  |
| 1977             | XXX                  |     |     |     |  |  |  |  |

### Parallelogram Form

| Accident<br>Year | Month of Development |     |     |     |
|------------------|----------------------|-----|-----|-----|
|                  | 12                   | 24  | 36  | 48  |
| 1971             |                      |     |     | xxx |
| 1972             |                      |     | XXX | XXX |
| 1973             |                      | XXX | XXX | XXX |
| 1974             | XXX                  | XXX | XXX | XXX |
| 1975             | XXX                  | XXX | XXX |     |
| 1976             | XXX                  | XXX |     |     |
| 1977             | XXX                  |     |     |     |

- Notes: 1. The expansion from the triangular to the parallelogram form for loss experience could result in possible gains in accuracy and stability of the reserve estimates of the model since indications for later development on older accident years are obtained.
  - 2. The only loss experience prior to Calendar Year 1974 used in the expansion from the triangular to the parallelogram form for cumulative losses and claims is the cumulative paid losses and closed claims as of 12/31/73.