DISCUSSION BY FRANCIS J. LATTANZIO AND FRANK C. TAYLOR

Mr. Harwayne's paper describes the new procedure for developing Excess Loss Premium Factors (ELPF's). In addition, he mentions two portions of the old method not adopted until after Dunbar Uhthoff[1] wrote his paper. These are: (a) the use of the 1.6 development factor; and (b) the spreading of the average ELPF over the Hazard Groups through the use of Hazard Group differentials. Mr. Harwayne briefly mentions that these differentials were under study. Since this paper was presented, the National Council on Compensation Insurance (NCCI) has filed a revision in the current differentials. The new relativities for all loss limitation sizes are those found in Appendix B, Exhibit B-4, Column (8) of Mr. Harwayne's paper.

The new procedure differs from the most recent procedure in the following ways:

(1) There are now two tables instead of four, with the Fatal Limited tables also being used for Fatal Unlimited and Permanent Total,

(2) The ratios to average have been extended from 3.00 to 3.50 for the Limited Fatal table and from 3.00 to 6.00 for the Major Permanent Partial table, and

(3) The 1.6 development factor will be eliminated when fourth reports of losses by type of injury become available.

The reviewers believe the most important change was the decision to use development by injury type. The remainer of this review will be devoted to a discussion of the necessity of using this type of development and to the presentation of a method for obtaining development factors beyond fifth report.

The ELPF is an important factor not only for ensuring the correctness of the retrospective rating formula, but also as an integral factor in the calculation of excess premiums as used by reinsurers and by the national accounts departments of large insurers. The development of the correct ELPF is essential in all these situations. The question then becomes, how are the proper ELPF's derived?

[1] D. R. Uthoff, "Excess of Loss Ratios via Loss Distributions," *PCAS*, Vol. XXXVII, (1950).

The formula for computing the Excess Loss Ratio (using Snader's notation[2]) is $e^* = w_d e^*_d + w_p e^*_p + w_m e^*_m$. The w's represent the ratios of losses by injury type (death, permanent total, and major permanent partial, respectively) to total losses, while the e*'s represent the excess loss ratios applicable to cases by injury type.

The correctness of the excess ratios by injury type will be accepted for the present. (However, the next revision could be based on data from more states.) The ELPF, then, depends next on the ratios of losses by injury type to total losses, as these are the weights used to combine the individual excess ratios. One assumption of this review is that the development of serious losses is greater than that of total losses and that this difference increases as a direct function of unit report number. Mr. Harwayne indicates that development by injury type will be used as soon as fourth reports of losses become available. This review also assumes that development to fifth report will be used when available. This latter development is currently unavailable and loss development by injury type beyond fifth report is not obtainable under the current unit statistical plan. The absence of data could be especially important in states with large total development beyond fifth report, but it will not be possible to empirically prove this until proper data is available.

If it is assumed that loss development by serious injury type is greater than that for total losses, the problem can be stated as follows: how to measure loss development by injury type beyond fifth report without changing the statistical plan. Even if the statistical plan were changed, the relevance of such factors beyond fifth report is questionable.

There are currently too few development factors by injury type through fifth report for an analysis to be performed. However, loss development factors for losses in excess of certain loss limitation sizes are available. This data is from the New York Compensation Insurance Rating Board and is used in their calculation of ELPF's (a calculation which differs from that of the NCCI). The N.Y.C.I.R.B. displays loss development by unit report from first to fifth for losses in excess of the following loss limitation sizes: \$10,000, \$15,000, \$20,000, \$25,000, and \$50,000.

[2] R. H. Snader, "Fundamentals of Individual Risk Rating and Related Topics," *CAS Study Note*, Part II, Page 16.

This data, together with certain curve fitting techniques, has been used to test not only the New York ELPF charge adequacy in the aggregate, but also the ELPF charge equity by loss limitation size. This technique, while not testing the impact of loss development by injury type beyond fifth report, can be modified to estimate such later development once raw injury type development data is available through fifth report.

THE MODEL

The hypothesis set forth here is that loss development factors used in the calculation of ELPF's are a function of time and loss limitation. Hence, the objective is to define a loss development factor function f(T,L) where:

T: T^{th} report to $(T + 1)^{th}$ report, $T \geq 1$

L: Losses in excess of $(L) \times \$5{,}000$ per claim, $L \geq 2$.

For example, f(7,4) is the loss development factor from report 7 to report 8 for the losses in excess of $20,000 per claim.

The starting point in attempting to define f(T,L) is actual loss development data from the N.Y.C.I.R.B., compiled September 19, 1975. The development factors contained therein can be arranged in the form of the following matrix:

Development	Loss Development Factors by Loss Limitation (L):				
From (T):	$10,000	$15,000	$20,000	$25,000	$50,000
1st to 2nd Report	1.610	1.651	1.660	1.665	1.711
2nd to 3rd Report	1.337	1.409	1.474	1.546	1.551
3rd to 4th Report	1.174	1.212	1.247	1.277	1.316
4th to 5th Report	1.109	1.132	1.170	1.166	1.227

Thus, the actual data is defined for: L = 2, 3, 4, 5, 10 and T = 1, 2, 3, 4. The function derived below will calculate loss development during any two adjacent reports for any excess over a given amount per claim ($10,000 or greater), i.e. for all intregal $T \geq 1$, and $L \geq 2$.

In order to arrive at f, the columns and rows of the matrix are analyzed separately. That is, one variable is held fixed, and loss development patterns are examined as the other varies.

Column Analysis

It is assumed that as one proceeds out in time, the loss development from one report to the next report approaches unity as a lower limit. However, each of the equations displayed on Exhibit I, Section A fit data using least squares with an upper limit property. Therefore, the reciprocals of the loss development factors are used as input for the equations. Curve No. II was chosen as most appropriate for the loss development data varying by time after reviewing the indices of determination for the various amounts of excess.

Row Analysis

Exhibit I, Section B lists the curve types to which the loss development data was fit by time T to the independent variable L. Upon review of the indices of determination of the curves, curve type $Y = C + D \ln (L)$ was best overall.

Form of f(T,L)

While it is of interest to extend the original matrix in both directions, i.e., for L greater than 10 as well as for T greater than 4, the primary concern is in the latter direction. Thus, the form of the function f(T,L) will be $f(T,L) = (1 + A_L e^{B_L T})$ where A_L and B_L are functions of L. The constants A and B from curve type No. II (Exhibit I, Section A) are as follows:

Losses in Excess of:	A_L	B_L
\$10,000 (L = 2)	1.07258	−0.58284
15,000 (L = 3)	1.14694	−0.54483
20,000 (L = 4)	1.10248	−0.47205
25,000 (L = 5)	1.20771	−0.48480
50,000 (L = 10)	1.10787	−0.39826

One would hope that there would be some functional relationship between either A_L and L or B_L and L. It has been seen that the loss development factors themselves, for each fixed report, fit very well with the function $Y = C + D \ln (L)$ as L varies. The B_L's do as well, with an index of determination of 0.9501. However, as one can see, the A_L's do not exhibit such a relationship. It becomes necessary to choose a constant A. The function now has the form:

$$f(T,L) = 1 + Ae^{(C + (\ln L)D)T}$$

where the function $C + (\ln L)D$ is substituted for B_L with $C = -0.58785$, $D = 0.08492$. After testing constants in the range of A_L, A is chosen to be 1.10.

Various loss development factors calculated from the function $f(T,L) = 1 + 1.10e^{(0.58785 + (\ln L)(0.08492))T}$ are displayed on Exhibit II. A comparison of Exhibit II with the matrix of actual data reveals an extremely close fit. The ultimate loss development factors as calculated can now be compared with those which would be used by the N.Y.C.I.R.B. in their calculation of ELPF's.

	$10,000	$15,000	$20,000	$25,000	$50,000
1. First Report to Ultimate as Calculated by f(T,L)	3.2232	3.8016	4.2740	4.6869	6.4612
2. First Report to Fifth Report (N.Y.C.I.R.B.)	2.8018	3.1917	3.5718	3.8331	4.2844
3. Increase in LDF Obtained ((1) ÷ (2))	1.1504	1.1911	1.1966	1.2227	1.5081

In summary, the reviewers feel that current ELPF charges are inadequate to the extent that they fail to recognize the ultimate development of losses, both in total and by injury type. A method has been presented which estimates this inadequacy. Some improvements in the quality of the derived factors could be achieved if raw loss development factors were available for: (a) no loss limitation, and (b) additional loss limitation sizes.

It is admitted that the data is not the most current and that different data may not fit these curves. However, it is believed that the concept is valid and that curves can be found which will produce a proper fit to more recent and more extensive data.

Finally, the reviewers would like to thank Glenn W. Fresch for his guidance and direction in the preparation of the latter part of this review.

EXHIBIT I

A. LOSS DEVELOPMENT FOR LOSSES IN EXCESS OF ($L \times \$5{,}000$)
INDEX OF DETERMINATION

CURVE TYPE	L=2	L=3	L=4	L=5	L=10
I. $Y = (\text{UPPER LIMIT})\ (A^{BT})$	.9983	.9921	.9768	.9324	.9727
II. $Y = (\text{UPPER LIMIT}) \div (1 + Ae^{BT})$	.9976	.9951	.9814	.9465	.9776

B. LOSS DEVELOPMENT FOR TIME T
INDEX OF DETERMINATION

CURVE TYPE	T=1	T=2	T=3	T=4
1. $Y = C + DL$	.8798	.6449	.8417	.9202
2. $Y = Ce^{DL}$	.8764	.6274	.8313	.9150
3. $Y = CL^{D}$	.9722	.8732	.9811	.9608
4. $Y = C + (D \div L)$	.8734	.9040	.8845	.7830
5. $Y = 1 \div (C + DL)$	.8728	.6071	.8200	.9094
6. $Y = L \div (CL + D)$	.8803	.9186	.9024	.8001
7. $Y = C + D \ln (L)$	.9722	.8861	.9831	.9574
8. $Y = 1 \div (C + De^{(-L)})$	.7832	.8983	.8267	.6979
9. $Y = Ce^{(D \div L)}$	.8769	.9119	.8938	.7918

EXHIBIT II

CALCULATED VALUES OF THE FUNCTION f(T,L)

L: LOSSES IN EXCESS OF $ PER CLAIM:

T: Development from:	$10,000	$15,000	$20,000	$25,000	$50,000	$75,000	$100,000
Report 1 to 2	1.611	1.648	1.671	1.687	1.736	1.765	1.785
Report 2 to 3	1.339	1.382	1.409	1.430	1.493	1.531	1.560
Report 3 to 4	1.189	1.225	1.249	1.268	1.330	1.369	1.399
Report 4 to 5	1.105	1.133	1.152	1.168	1.221	1.257	1.285
Report 5 to 6	1.058	1.078	1.093	1.105	1.148	1.178	1.203
Report 6 to 7	1.032	1.046	1.057	1.066	1.099	1.124	1.145
Report 7 to 8	1.018	1.027	1.035	1.041	1.066	1.086	1.103
Report 8 to 9	1.010	1.016	1.021	1.026	1.044	1.060	1.074
Report 9 to 10	1.006	1.009	1.013	1.016	1.030	1.042	1.053
Report 10 to 11	1.003	1.006	1.008	1.010	1.020	1.029	1.038
Report 11 to 12	1.002	1.003	1.005	1.006	1.013	1.020	1.027
Report 12 to 13	1.001	1.002	1.003	1.004	1.009	1.014	1.019
Report 13 to 14	1.001	1.001	1.002	1.002	1.006	1.010	1.014
Report 14 to 15	1.000	1.001	1.001	1.002	1.004	1.007	1.010
Report 15 to 16	1.000	1.000	1.001	1.001	1.003	1.005	1.007
Report 16 to 17	1.000	1.000	1.000	1.001	1.002	1.003	1.005
Report 17 to 18	1.000	1.000	1.000	1.000	1.001	1.002	1.004
Report 18 to 19	1.000	1.000	1.000	1.000	1.001	1.002	1.003
Report 19 to 20	1.000	1.000	1.000	1.000	1.001	1.001	1.002
Report 20 to 21	1.000	1.000	1.000	1.000	1.000	1.001	1.001