ON THE THEORY OF INCREASED LIMITS AND EXCESS OF LOSS PRICING

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DISCUSSION BY SHELDON ROSENBERG

Bob Miccolis has presented a paper which discusses the mathematical theory underlying many aspects of increased limits ratemaking. Committees and staff of Insurance Services Office have put much of this theory into practice in reviewing increased limits loss experience. In so doing, practical problems have arisen and interim solutions developed pending further study. Some of these problems and solutions comprise this discussion.

CONSISTENCY IN INCREASED LIMITS TABLES

Extension to 2 Dimensional Tables

The Miccolis test for consistency is that "the marginal premium per $1000 of coverage should decrease as the limit of coverage increases." As discussed later in the paper, this consistency test can be extended to two-dimensional tables as well. That is, consider an increased limits table that appears as follows:

Aggregate Limit (in thousands)	Occurrence Limit (in thousands)			
	25	50	100	250
25	1.00			
50	1.50	1.70		
100	1.80	2.03	2.50	
250	2.00	2.25	2.80	3.20

This table must now "pass" the consistency test for each occurrence limit (down each column), as well as for each aggregate limit (across each row). The table passes this test for the $25,000 occurrence limit because

$\frac{.50}{25} > \frac{.30}{50} > \frac{.20}{150}$. That is, marginal premiums per $1000 of aggregate coverage are decreasing, as the occurrence limit is held fixed. The table fails to pass this test for the $250,000 aggregate limit because

$\frac{.25}{25} < \frac{.55}{50}$.

There is yet another consistency test that can be performed on a two-dimensional table. This test has in fact been used in conjunction with the previously discussed test in developing occurrence/aggregate and claim/accident tables at Insurance Services Office.

The test itself can best be described by examining the hypothetical table drawn up above. Consider an insured who is considering switching from a \$25,000 occurrence limit to a \$50,000 occurrence limit. The question becomes the following: if one assumes that his aggregate limit remains constant, then for which aggregate limit will his decision to change occurrence limits give him the greatest vs. the smallest increases in coverage?

His increase in coverage will be most significant when the aggregate is highest. To see this, consider the extreme situation of having no aggregate at all (i.e. infinite aggregate coverage). In this case, the insured's switching from a \$25,000 occurrence limit to a \$50,000 occurrence limit will give him a potential increase of \$25,000 for each and every occurrence, since no aggregate can ever be applied to stop payments at a certain amount. The other extreme is for an insured who has a \$25,000 aggregate and is contemplating a switch from \$25,000 to \$50,000 in his occurrence coverage. Of course, he gets nothing in additional coverage because the \$25,000 aggregate acts as a cap on his occurrence coverage as well.

The pattern thus emerges. For any two occurrence limits in an Increased Limits Table, the differences in the factors must not decrease, as the aggregates grow. This follows from the above discussion because increasing aggregates imply increasing differences in coverage between the given pair of occurrence limits. The way to reflect this in increased limits tables is to make sure that differences between the occurrence limit factors increase (or at least do not decrease) as aggregates increase. Note that our hypothetical table passes this test for the \$25,000 and \$50,000 occurrence limits because $1.70-1.50 \leq 2.03-1.80 \leq 2.25-2.00$.

This argument can of course be extended to test the differences in any pair of aggregate limits, for all occurrence limits. In comparing the "100" and "250" aggregate limits for the "25", "50", and "100" occurrence limits, the table again passes the test since $2.00-1.80 \leq 2.25-2.03 \leq 2.80-2.50$.

Anti-selection and Consistency

Miccolis notes that while "there can be anti-selection . . . this should not restrict the general applicability of the consistency test".

If one assumes that there is no anti-selection or that its effects are minimal, then one can assume that the inherent underlying severity distribution is identical for all insureds. Thus increased limits factors can be developed using one smooth curve to represent all policy limits. The factors resulting from this curve will then automatically pass the consistency test as described by Miccolis because the curve will produce decreasing severity values by layer. However, if we wish to reflect anti-selection, then each policy limit must have a curve fit to its own loss data.

Increased limits factors for the various policy limits will now result from the various loss curves, each of which is unrelated to any other one. In such a situation, marginal premium per \$1000 of additional coverage can increase from one policy limit to the next.

Anti-selection can take two forms. One often encountered is adverse selection, in which purchasing higher limits is associated with adverse loss experience. This can occur for two different reasons. Firstly, insureds who can expect higher loss potential could be more inclined to purchase higher limits. Secondly, liability law suits or settlements may be influenced by the policy limit. Thus the same accident may result in higher losses for the insured that purchased higher policy limits.

The mirror image of "adverse" selection might be labeled "favorable" selection, in which the insureds with the highest policy limits show the best loss experience. There are two reasons why this might occur. Firstly, financially secure insureds may be better risks. Yet since they have more assets to protect, they will be inclined to purchase higher limits. Secondly, insurance companies, knowing that these are the better risks, would be more willing to insure them at higher limits.

While anti-selection can affect increased limits factors, it does not always do so. If anti-selection produces differences by policy limit in the relationships among indemnity severity values, then increased limits factors are affected. However, it may well be that anti-selection produces increasing or decreasing basic limits severities by policy limit, but maintains the same proportionate relationship in severities for all other policy limit cut-offs as well. In this case, increased limits factors would not be affected by anti-selection. Consider the following two examples:

Example A: Anti-selection does affect increased limits factors

Average Indemnity severity resulting from purchasers of:

at "cut-offs" of	POLICY LIMIT $50,000	POLICY LIMIT $100,000
$ 25,000	$ 5,000	$ 7,000
50,000	7,500	8,500
100,000	10,000	11,000

Based on these results, increased limits factors (I.L.F.) could be calculated as follows:

Policy Limit	I.L.F. not reflecting anti-selection	I.L.F. reflecting anti-selection
$ 25,000	1.00	1.00
50,000	1.33 (i.e. $ 8,000/6,000)	1.50 ($ 7,500/5,000)
100,000	1.75 (i.e. $10,500/6,000)	1.57 ($11,000/7,000)

Example B: Anti-selection does not affect increased limits factors

Indemnity severity resulting from purchasers of:

at "cut-offs" of	POLICY LIMIT $50,000	POLICY LIMIT $100,000
$ 25,000	$ 5,000	$ 6,000
50,000	7,500	9,000
100,000	10,000	12,000

Based on these results, increased limits factors could be calculated as follows:

Policy Limit	I.L.F. not reflecting anti-selection	I.L.F. reflecting anti-selection
$ 25,000	1.00	1.00
50,000	1.50 (i.e. $ 8,250/5,500)	1.50 ($ 7,500/5,000)
100,000	2.00 (i.e. $11,000/5,500)	2.00 ($12,000/6,000)

In each example, the basic limit is assumed to be $25,000. Indemnity severities are calculated based on smooth loss curves for each policy limit separately, with the severity values "cut off" (i.e. limited) at various points representing key policy limits in an increased limits table. Note that because loss curves extend for infinite loss values, severity values could be calculated for "cut-offs" higher than the policy limit purchased. Increased limits factors not reflecting anti-selection are calculated by averaging the severity values for both policy limits (this assumes equal weights for both policy limits), and dividing these resulting increased limits average severities by the basic limit average severity. To reflect anti-selection, the severity value at a "cut-off" is equal to the value for that policy limit. This is true for the increased limit in question, as well as the basic limit. Note that in Example A, the increased limits factors differ depending on whether anti-selection was reflected, while in Example B, the factors are identical with or without reflecting anti-selection.

LOSS DEVELOPMENT

As Miccolis states in his paper, "it is very likely that [a] distribution of immature claim values will change considerably as these claims develop". In general, this varying development will exhibit an upward pattern over intervals of claim size. Displayed in Table 1 are the mean and standard deviation for one year of data fitted to a log normal distribution over three different evaluation periods for Physicians, Surgeons, and Hospitals. The increase over time in the mean and standard deviation is evidence of the upward pattern of development. An important reason for this is that IBNR claims, when they eventually get reported, tend to wind up in the higher claim size intervals. The I.S.O. closed claim surveys, for both Malpractice and Product Liability, demonstrate that the longer the time interval between the occurrence of a claim and the reporting of that claim, the higher the average size of the claim.

At Insurance Services Office, loss development factors have been calculated by claim size interval by comparing "theoretical" (the result of smooth loss curves) claim counts per interval as of various evaluation levels. These factors have exhibited the generally upward pattern by claim size interval referred to above.

TABLE 1

POLICY YEAR 1972

Evaluated as of	Physicians		Surgeons		Hospitals	
27 months	Mean	12,421	Mean	13,177	Mean	8,772
	S.D.	58,126	S.D.	53,467	S.D.	42,526
39 months	Mean	16,972	Mean	19,384	Mean	14,364
	S.D.	75,410	S.D.	77,923	S.D.	85,968
51 months	Mean	22,974	Mean	24,784	Mean	19,072
	S.D.	126,314	S.D.	115,847	S.D.	148,045

THE USE OF A THEORETICAL CURVE AS A MODEL FOR LOSS DISTRIBUTION

As Miccolis notes, there are many problems in dealing with empirical distributions.

In addition to those mentioned in his paper two other properties of the claim size distribution tend to bias the results. Firstly, the existence of "cluster points" (intervals where the number of claims drastically rises and immediately drops) magnify the discontinuity between intervals. Usually these cluster points appear at intervals which contain a round number such as $25,000, $50,000 or $100,000. As an example see Table 2 which displays Surgeons data of Companies reporting to ISO for Policy Year 1972 evaluated as of March 31, 1975. This clustering phenomenon may result in a poor fit when any continuous curve is applied to the data. The existence of cluster points should not pre-empt the use of a theoretical distribution, however, since their presence may be artificial for the following reasons:

(1) Policy limits will truncate losses to the limit of an insured's policy. This is not the sole reason for cluster points however, since such points were present for amounts such as $25,000 and $50,000 even when the underlying data corresponded to a policy limit of $100,000. To illustrate this point, consider Table 3 which is Physicians data of companies reporting to ISO for policy year 1974 evaluated as of March 31, 1976. This data contains only those losses which were incurred on policies whose limit was $100,000, yet clusters appear at $5,000, $10,000, $15,000, $20,000, $25,000 and $50,000.

(2) At early evaluation points when the raw data is relatively immature, most losses are still outstanding. These claims are rarely reserved for amounts other than round figures.

A second property of the raw data included in the claim size distribution is that gaps are present for certain intervals where no claims appear. Fitting a curve alleviates this problem because of its smooth nature.

ISO has recognized these weaknesses in the empirical distributions and consequently has chosen to fit a theoretical distribution to the data and use this curve in computing increased limits factor. Initially a log-normal distribution was considered to be an appropriate representation of the data; the fitting problem at cluster points was not considered crucial, as mentioned above. The curve was fit by solving the Maximum Likelihood equations for the parameters μ and σ^2.

One other advantage of using a theoretical distribution to represent the data is that it facilitated the computation of variance at each policy limit. This variance was then used as a basis for risk adjustments.

TREND

When discussing the issue of trending loss distributions, Miccolis first considers the case where trend affects all claims in the same way, i.e. each loss is increased by the same multiplicative factor. This assumption leads to the equation:

$$(1) \qquad \mathcal{F}(x) = F(x/a)$$

where $\mathcal{F}(x)$ is the trended cumulative distribution, "a" is the annual trend factor, and $F(x)$ is the untrended cumulative distribution. If one differentiates both sides of this equation the result is:

$$f(x) = \mathcal{F}'(x) = F'(x/a) = 1/a\, f(x/a)$$

The trended probability distribution function $f(x)$ can thus be defined in terms of $f(x)$.

It is interesting to note that if x is lognormally distributed with parameters μ and σ^2 before trend, then x will also be lognormally distributed with

TABLE 2

INSURANCE SERVICES OFFICE

SURGEONS PROFESSIONAL LIABILITY
CLAIMS SIZE DISTRIBUTION

POLICY YEAR 1972*

Claim Size Intervals	Claims	Loss	Claim Size Intervals	Claims	Loss
0- 250	192	16,321	20,001- 21,000	10	203,022
251- 500	122	50,918	21,001- 22,000	1	22,000
501- 1,000	226	205,580	22,001- 23,000	13	293,007
1,001- 2,000	272	444,820	23,001- 24,000	3	71,290
2,001- 3,000	323	838,154	24,001- 25,000	104	2,548,951
3,001- 4,000	150	545,173	25,001- 30,000	45	1,256,077
4,001- 5,000	425	2,076,934	30,001- 35,000	41	1,388,629
5,001- 6,000	53	303,156	35,001- 40,000	27	1,037,669
6,001- 7,000	60	399,909	40,001- 45,000	15	659,608
7,001- 8,000	116	862,936	45,001- 50,000	67	3,345,275
8,001- 9,000	29	254,676	50,001- 55,000	14	731,821
9,001- 10,000	190	1,875,453	55,001- 60,000	10	600,000
10,001- 11,000	33	345,854	60,001- 65,000	13	822,507
11,001- 12,000	31	368,930	65,001- 70,000	7	486,000
12,001- 13,000	34	427,736	70,001- 75,000	26	1,940,495
13,001- 14,000	9	123,698	75,001- 80,000	9	687,631
14,001- 15,000	130	1,916,898	80,001- 85,000	5	425,000
15,001- 16,000	19	294,270	85,001- 90,000	2	180,000
16,001- 17,000	11	181,967	90,001- 95,000	1	90,540
17,001- 18,000	13	230,802	95,001-100,000	61	6,091,246
18,001- 19,000	7	130,650	100,001-110,000	8	835,605
19,001- 20,000	102	2,039,998	110,001-120,000	2	239,000

INSURANCE SERVICES OFFICE
SURGEONS PROFESSIONAL LIABILITY
CLAIMS SIZE DISTRIBUTION
POLICY YEAR 1972*

TABLE 2 (Cont.)

Claim Size Intervals	Claims	Loss	Claim Size Intervals	Claims	Loss
120,001- 130,000			340,001- 350,000	1	350,000
130,001- 140,000	1	140,000	350,001- 360,000		
140,001- 150,000	5	741,467	360,001- 370,000		
150,001- 160,000			370,001- 380,000		
160,001- 170,000			380,001- 390,000	1	390,000
170,001- 180,000			390,001- 400,000		
180,001- 190,000			400,001- 410,000		
190,001- 200,000	2	400,000	410,001- 420,000		
200,001- 210,000	2	413,750	420,001- 430,000		
210,001- 220,000	1	211,000	430,001- 440,000		
220,001- 230,000			440,001- 450,000		
230,001- 240,000			450,001- 460,000		
240,001- 250,000	2	500,000	460,001- 470,000		
250,001- 260,000			470,001- 480,000		
260,001- 270,000			480,001- 490,000		
270,001- 280,000			490,001- 500,000	1	500,000
280,001- 290,000			500,001- 600,000		
290,001- 300,000	1	300,000	600,001- 700,000		
300,001- 310,000			700,001- 800,000		
310,001- 320,000			800,001- 900,000		
320,001- 330,000			900,001-1,000,000		
330,001- 340,000			1,000,001+		
			TOTAL	3,048	41,836,423

*Evaluated as of 3/31/75

TABLE 3

INSURANCE SERVICES OFFICE

PHYSICIANS PROFESSIONAL LIABILITY
CLAIMS SIZE DISTRIBUTION

POLICY YEAR 1974*
POLICY LIMIT 100/300

Claim Size Intervals	Claims	Loss
0- 250	54	7,324
251- 500	57	24,980
501- 1,000	123	117,448
1,001- 2,000	90	147,741
2,001- 3,000	108	281,101
3,001- 4,000	41	149,235
4,001- 5,000	177	876,986
5,001- 6,000	12	70,289
6,001- 7,000	10	67,462
7,001- 8,000	39	295,795
8,001- 9,000	5	43,500
9,001- 10,000	68	680,000
10,001- 11,000	3	31,780
11,001- 12,000	6	72,000
12,001- 13,000	3	37,500
13,001- 14,000	2	27,333
14,001- 15,000	29	434,620
15,001- 16,000	1	15,804
16,001- 17,000	1	17,000
17,001- 18,000	1	17,500
18,001- 19,000		
19,001- 20,000	27	540,000
20,001- 21,000	1	20,500
21,001- 22,000	2	43,565
22,001- 23,000		
23,001- 24,000	1	24,000
24,001- 25,000	24	600,000
25,001- 30,000	7	207,500
30,001- 35,000	8	280,000
35,001- 40,000	3	118,736
40,001- 45,000	1	45,000
45,001- 50,000	17	850,000
50,001- 55,000		
55,001- 60,000	3	180,000
60,001- 65,000	1	65,000
65,001- 70,000	2	140,000
70,001- 75,000	8	600,000
75,001- 80,000	1	79,020
80,001- 85,000	1	85,000
85,001- 90,000		
90,001- 95,000		
95,001-100,000	17	1,700,000
TOTAL	954	8,993,719

*Evaluated as of 3/31/76

parameters $\mu + \ln a$ and σ^2 after trend. To see that this follows, consider equation (1) and the fact that:

$$f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2}$$

$$\text{Then } f(x) = 1/a\, f(x/a) = \frac{1}{a} \frac{1}{(x/a)\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln(x/a) - \mu}{\sigma}\right)^2}$$

$$= \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{\ln x - (\mu + \ln a)}{\sigma}\right)^2}$$

i.e. $f(x)$ is a lognormal distribution with parameters $\mu + \ln a$ and σ^x.

Once again it is important to keep in mind that this argument is valid only if one assumes that the effects of inflation are so that each loss is multiplied by the same multiplicative factor. It is clear though, that if this assumption is made and if the lognormal distribution can be assumed to represent the underlying loss severity distribution then trend should not affect the parameter σ^2. Thus if a good fit is achieved via a lognormal distribution and yet the parameter is observed to be changing over time, then this would indicate that trend does not affect all claims equally.

With this point in mind, consider the following values for μ and σ^2 which were computed by fitting a lognormal distribution to the indicated data.

While the σ^2 parameter seems to be changing over time, a hasty conclusion should not be drawn since:

(1) The underlying data is relatively immature (as of 27 months) and thus is largely affected by reserving procedures. To get a proper picture of the effects of trend one should analyze fully developed claims.

(2) As mentioned above, the argument depends on whether or not the data is adequately represented by the lognormal distribution. If the quality of fit changes from year to year, then one cannot analyze the effects of trend by tracing the movement of an artificial parameter.

TABLE 4

Policy* Year		Physicians	Surgeons	Hospitals
1972	μ	7.8616	8.0562	7.4799
	σ^2	3.1311	2.8601	3.1988
1973	μ	7.9609	8.3099	7.7801
	σ^2	2.5031	2.6183	3.1040
1974	μ	8.1901	8.4578	7.8295
	σ^2	2.3395	2.4240	3.6419

*As of 27 months of maturity.

ALLOCATED LOSS ADJUSTMENT EXPENSES

Miccolis mentions in the final paragraph of his introduction that there are many practical problems concerning loss adjustment expenses which cannot be resolved solely by the mathematical model presented in his paper.

Perhaps the overriding reason for this is that an insurer's legal costs in defending an insured are not bounded by the limit of the insured's policy. The sum of the indemnity portion and the cost of lost adjustment expenses may and in many cases does exceed the limit of the insured's policy. This is where Miccolis' model becomes inoperative since the equation

$$g(x;K) = \begin{cases} x, 0 < x < K, K > 0 \\ K, x \geq K \end{cases}$$

is no longer applicable.

As an alternative to using the model in pricing allocated loss adjustment expenses for layers of coverage above basic limits, ISO has investigated raw data to actually compute the average amount of allocated per claim for each policy limit for which data was available. An increasing pattern in these numbers may be matched by a similar pattern for the average basic limits severity by policy limit. In such a case, a constant percentage charge of the basic limits indemnity will produce an increasing dollar amount for allocated loss adjustment expense by policy limit. Therefore, another test was to compute the ratio of allocated loss adjustment expenses to basic limit losses by policy limit. If this ratio forms an increasing

TABLE 5

INSURANCE SERVICES OFFICE
ALLOCATED LOSS ADJUSTMENT EXPENSE
BY POLICY LIMIT*

PHYSICIANS

(1) Policy Limit	(2) # of Claims	(3) Avg. B/L Ind.	(4) Avg. ALAE	(5) (4) ÷ (3)
100,000	956	$6,713	$3,482	.519
200,000	83	6,120	2,048	.335
250,000	96	7,833	2,635	.336
500,000	57	5,474	2,088	.381
1,000,000	230	8,591	3,130	.364
All LimitsØ Combined	1,601	6,988	3,066	.439

SURGEONS

(1) Policy Limit	(2) # of Claims	(3) Avg. B/L Ind.	(4) Avg. ALAE	(5) (4) ÷ (3)
100,000	1,693	$8,156	$4,115	.505
200,000	260	7,885	2,085	.264
250,000	322	9,161	2,112	.231
500,000	140	9,286	3,157	.340
1,000,000	457	8,565	3,365	.393
All LimitsØ Combined	3,118	8,270	3,496	.423

HOSPITALS

(1) Policy Limit	(2) # of Claims	(3) Avg. B/L Ind.	(4) Avg. ALAE	(5) (4) ÷ (3)
100,000	565	$4,745	$2,508	.529
200,000	178	5,124	2,011	.392
250,000	368	6,307	3,022	.479
300,000	412	6,391	2,697	.422
500,000	818	6,620	2,502	.378
1,000,000	1,242	8,820	3,738	.424
All Policy Limits CombinedØ	4,411	6,397	2,706	.423

*Policy Year 1974 data evaluated as of March 31, 1976.
ØIncludes limits not listed.

progression as policy limits increase then an increasing percentage of the basic limit rate should be charged for allocated loss adjustment expenses.

As the tables for Physicians, Surgeons, and Hospitals below show, however. a progression did not materialize. Therefore, in initial pricing considerations, a constant percentage of the Average Basic Limits Severity was loaded into each policy limit in order to construct the increased limit factors. Of course this decision, based as it was on empirical data, is subject to change as more data become available for analysis.

SUMMARY

The conclusion of Miccolis' paper is that the complete solution to the problem of pricing increased limits coverage requires "actual data, judgment, and 'some' further study." This review, prepared with the assistance of George Burger and Aaron Halpert, has presented data (of companies reporting to ISO) and has indicated the judgment needed to develop such data to a form in which the mathematical models can be applied.

The CAS is indebted to Mr. Miccolis for his contribution to the *Proceedings*.