A MATHEMATICAL MODEL FOR LOSS RESERVE ANALYSIS

CHARLES L. McCLENAHAN

DISCUSSION BY DAVID SKURNICK

Actuaries generally predict the ultimate cost of a partially paid accident year from the pattern of earlier accident years' payments. But this procedure ignores the development pattern of the current year itself. McClenahan's paper utilizes each year's development pattern by means of certain assumptions concerning the rates of payment, growth, and inflation; the result is a well-defined mathematical model which can serve several useful functions.

The fundamental assumption is that for a given accident month there is a delay of d months before the loss payments begin. Under constant severity these payments then would decrease at a geometric rate. Severity depends upon payment month, and it changes at a uniform geometric rate. Frequency depends upon accident month, and it also changes at a uniform geometric rate. For a given accident month, the combined effects of the decreasing payment rate and the change in severity produce a geometric decline in which each month's payments are r times the prior month's payments.

The assumptions lead to the development of a variety of formulas relating to paid and unpaid losses by accident month and by accident year. The formulas can be used for both cash flow and reserve analyses. The model allows one to measure the effect of a change in frequency, severity, or payment rate. The author also uses the model to evaluate the amount by which loss reserves can be reduced if the payments are discounted. Although the formulas are complicated, the presentation is clear and easy to follow.

Properly estimating a model's parameters is as important as constructing the model. A sensitivity test of the model will show how much accuracy is required for each parameter. For a paid loss development on a casualty line, the indicated reserve is highly sensitive to the rate of payment, particularly at the later ages. For example, under stable conditions, a change of .01 in the 120 month to 132 month age to age factor will produce 1% more loss development for each of the ten most recent years. Thus, it will change the indicated loss reserve by 10% of a year's incurred loss. The age to age factors for the last portion of the development influence the most accident years. Unfortunately, these factors are based on the oldest data; thus they are the least reliable.

The geometric distribution is a special case of the negative binomial. Three methods of estimating the negative binomial's parameters are described by Johnson and Kotz.[1] In the case of a geometric where each term is r times the preceding term all three methods estimate $1/(1 - r)$ as the sample mean, which in this case is the average length of time to pay a dollar of loss. The reviewer applied this method to fit geometric distributions separately to each of three accident years using paid workers' compensation claims. For accident year 0 with actual paid loss amounts $A_0, A_1, \ldots, A_N$ during years 0, 1, . . ., N respectively the geometric rate of decline was estimated from the formula

$$1/(1 - \hat{r}) = \sum_{n=1}^{N} nA_n \Big/ \sum_{n=1}^{N} A_n \tag{1}$$

Note that year 0 was omitted because the initial reporting delay prevents the geometric pattern from beginning until year 1.

As shown on Exhibit 1, the fit is only fair. The fitted curve substantially underestimates actual paid loss at later years. By comparison, in the automobile bodily injury example in the paper the model overestimated paid loss at later years. Probably these results reflect the different characteristics of the two lines of business.

There is a bias in this estimation procedure. It underestimates $1/(1 - r)$ since it represents the mean of a truncated series of payments. Some adjustment should be made because the observations stop at year N, the latest year for which data is available, if substantial amounts of claims remain unpaid at that time.

Probably the best application of McClenahan's results lies in sensitivity analysis. His formulas directly show the effect of changing the discount rate, the growth rate, or the payment rate. Many readers of this paper will want to experiment to see whether his formulas provide more accurate reserve estimates than the usual methods. This thoroughly developed model is a significant addition to the actuarial literature.

[1] Norman L. Johnson and Samuel Kotz, *Discrete Distributions*, Houghton, Mifflin Company 1969, distributed by John Wiley & Sons, Inc., Salt Lake City, Utah, p. 131-137.

Exhibit 1

GEOMETRIC DISTRIBUTION OF WORKERS' COMPENSATION ACCIDENT YEAR PAID LOSSES*

n	Paid During the Year	(1) Actual Paid	(2) Theoretical Paid Kr^{n}**	(3) Difference (2) − (1)	(4) Percent of Actual (3) ÷ (1)
		Accident Year 1968			
0	1968	11,790,455	11,790,455	—	—
1	1969	10,402,479	10,985,242	582,763	5.6%
2	1970	6,370,883	5,976,239	−394,644	−6.2
3	1971	2,847,065	3,251,220	404,155	14.2
4	1972	1,896,985	1,768,743	−128,242	−6.8
5	1973	1,082,910	962,239	−120,671	−11.1
6	1974	658,942	523,482	−135,460	−20.6
7	1975	492,688	284,787	−207,901	−42.2
		35,542,407	35,542,407		
		Accident Year 1969			
0	1969	13,378,723	13,378,723	—	—
1	1970	14,277,955	15,171,631	893,676	6.3%
2	1971	8,027,259	7,927,389	−99,870	−1.2
3	1972	4,029,497	4,142,172	112,675	2.8
4	1973	2,282,755	2,164,343	−118,412	−5.2
5	1974	1,421,190	1,130,899	−290,291	−20.4
6	1975	1,088,689	590,911	−497,778	−45.7
		44,506,068	44,506,068		
		Accident Year 1970			
0	1970	16,816,141	16,816,141	—	—
1	1971	17,593,975	18,995,304	1,401,329	8.0%
2	1972	9,238,517	9,373,211	134,694	1.5
3	1973	4,571,356	4,625,201	53,845	1.2
4	1974	2,914,044	2,282,300	−631,744	−21.7
5	1975	2,084,322	1,126,198	−958,124	−46.0
		53,218,355	53,218,355		

*For each accident year, year 0 was excluded from the distribution

**Accident year values for r and K

	r	K
1968	.544024371	20,192,555
1969	.522513997	29,035,836
1970	.493448876	38,494,978