A NOTE ON BASIC LIMITS TREND FACTORS

ROBERT J. FINGER

It is widely accepted that excess layers of insurance suffer an inflationary impact greater than that attributable to the overall growth in claim costs. A necessary corollary of this thesis, and perhaps one not often acknowledged, is that the primary layer (basic limits) suffers a lesser impact than the overall rate. In other words, one may assume that aggregate claim costs are increasing at a certain annual rate. The trend in basic limits costs will be less than this overall rate. The trend in excess layer costs will be more than this rate. This paper will discuss the relationship between basic limit trends and the overall increase in claim costs. A method is presented for estimating the basic limit trend when the overall trend is known.

TERMINOLOGY

The term "claim costs" can have different meanings. Claim costs can change in several ways, for many different reasons. Fundamental changes in costs are due to changes in claim frequency (the number of claims per exposure unit) and claim severity (the average claim size). Claim severity is impacted by these forces, as a minimum: changes in the overall price level in the economy, changes in claim settlement practices and changes in social forces.

This paper is not concerned with changes in claim frequency. If it is assumed that such changes do not affect the claim size distribution, the conclusions of this paper will apply to any level of claim frequency.

This paper does not differentiate between the various sources causing changes in claim severity. It is assumed that these different causes can be suitably combined and that changes in their relative impact over time does not change the claim size distribution. Trend is defined as the change in claim severity.

Liability insurance ratemaking methods usually define certain limits as the basic limits. For example, this could be $25,000 per claim and $75,000 for all claims occurring within the 12-month policy period. In most cases, no insurance policy is sold for limits of less than this amount. In this paper it is assumed that there is a single basic limit per policy (e.g., the $25,000 above). The total amount of insured losses will be referred to as unlimited. The average claim size of the unlimited losses will be referred to as the mean of the claim size distribution.

For a given overall trend in claim costs, the trend in basic limit costs will generally depend upon the relationship between the basic limit value and the mean. The shape of the claim size distribution is also of some importance. If the basic limit is much higher than the mean, relatively few claims are affected by the basic limit; consequently most of the overall trend is felt within the basic limit. If on the other hand, the basic limit is close to the mean, relatively many claims are necessarily above the basic limit. The trend on claims above the basic limit is obviously not reflected in the basic limit cost. The relative trend is defined as the ratio of the basic limit trend to the overall trend. The relative trend varies as the relationship between the basic limit and the mean changes. As the mean gets larger, the ratio of the basic limit to the mean becomes smaller. The average relative trend is the average of the instantaneous relative trends over a period of time.

METHODOLOGY

The basic assumption made in this paper is as follows. When there is a trend in claim costs, the claim size distribution itself does not change, but the value of money does. In effect, this is equivalent to assuming that if overall costs increase 25%, each individual claim increases 25%. Finding the average relative trend is analogous to the following situation. Suppose a Mexican insurance company writes a policy limit of 100,000 pesos on risks located in the United States. When the peso is devalued, what is the increase in claim costs? The ratio of the change in claim costs to the revaluation of the dollar is analogous to the average relative trend, for a basic limit of 100,000 pesos.

Assume that the claim size distribution is known. For a given basic limit A, the unlimited losses, T, can be divided into basic limit losses, B, and excess limit losses, E:

$$T(M) = B(A/M) + E(A/M)$$

where: M is the (unlimited) mean claim size
T(M) is the total amount of losses
B(A/M) is the total amount of losses limited to A per claim
E(A/M) is the total amount of losses in excess of A per claim.

The basic limit losses are defined as:

$$B(A/M) = CMX2(A/M) + CA[1 - X1(A/M)]$$

where: C is the number of claims

X2(A/M) is the percentage of the total amount of losses (moment distribution) on claims which are less than A

X1(A/M) is the percentage of the total number of claims which are less than A.

The average relative trend, ART, is a function of the (beginning) basic limit value and the unlimited trend. In other words, the unlimited losses will be increased by certain trend, i. At the same time the basic limit losses will be increased by a lesser amount. The average relative trend is the percentage increase in basic limit losses as a fraction of the percentage increase in total limit losses. Thus:

$$ART(A, i) = \frac{\dfrac{B(A/(1+i)M) - B(A/M)}{B(A/M)}}{\dfrac{T((1+i)M) - (T(M)}{T(M)}}$$

To derive usable results, two assumptions are made. It is assumed that unlimited losses are proportional to the unlimited mean. Symbolically:

$$\frac{T(M')}{T(M)} = \frac{M'}{M}$$

It is also assumed that the percentage distributions X1 (claim count) and X2 (moment) are a function of the ratio of the basic limit to the unlimited mean. This assumption holds, for example, for the log-normal and Pareto distributions.

By the second assumption:

$$B(A/M) = CM(X2(R) + R[1 - X1(R)]) = CMX(R)$$

where: $R = \dfrac{A}{M}$ and X(R) is the percentage of the total amount of losses which are below a basic limit value of R times the unlimited mean, per claim

This leads to the redefinition of ART as:

$$\mathrm{ART}(R, i) = \frac{\dfrac{B\left(\dfrac{R}{1+i}\right) - B(R)}{B(R)}}{(1+i) - 1}$$

$$\mathrm{ART}(R, i) = \frac{1}{i}\,\frac{(1+i)X\left(\dfrac{R}{1+i}\right) - X(R)}{X(R)}$$

In plain English, these equations state that there exists a distribution, X(R), which represents the percentage of unlimited losses which are less than R per claim, where R is defined as a ratio to the unlimited mean. Assume there is a trend in overall claim costs of fraction i during a period; only basic limit losses which were previously less than $\frac{R}{1+i}$ will now be included within the new basic limit. The entire distribution, however, will be $(1+i)$ times larger. In other words, assume the initial basic limit is \$25,000 and inflation is 25%. Under the new circumstances only the basic limit losses under the previous \$20,000 basic limit will be below the new basic limit. The entire loss distribution, however, is 25% larger. Algebraically:

$$\mathrm{ART} = \frac{1}{.25}\,\frac{1.25X(20) - X(25)}{X(25)}$$

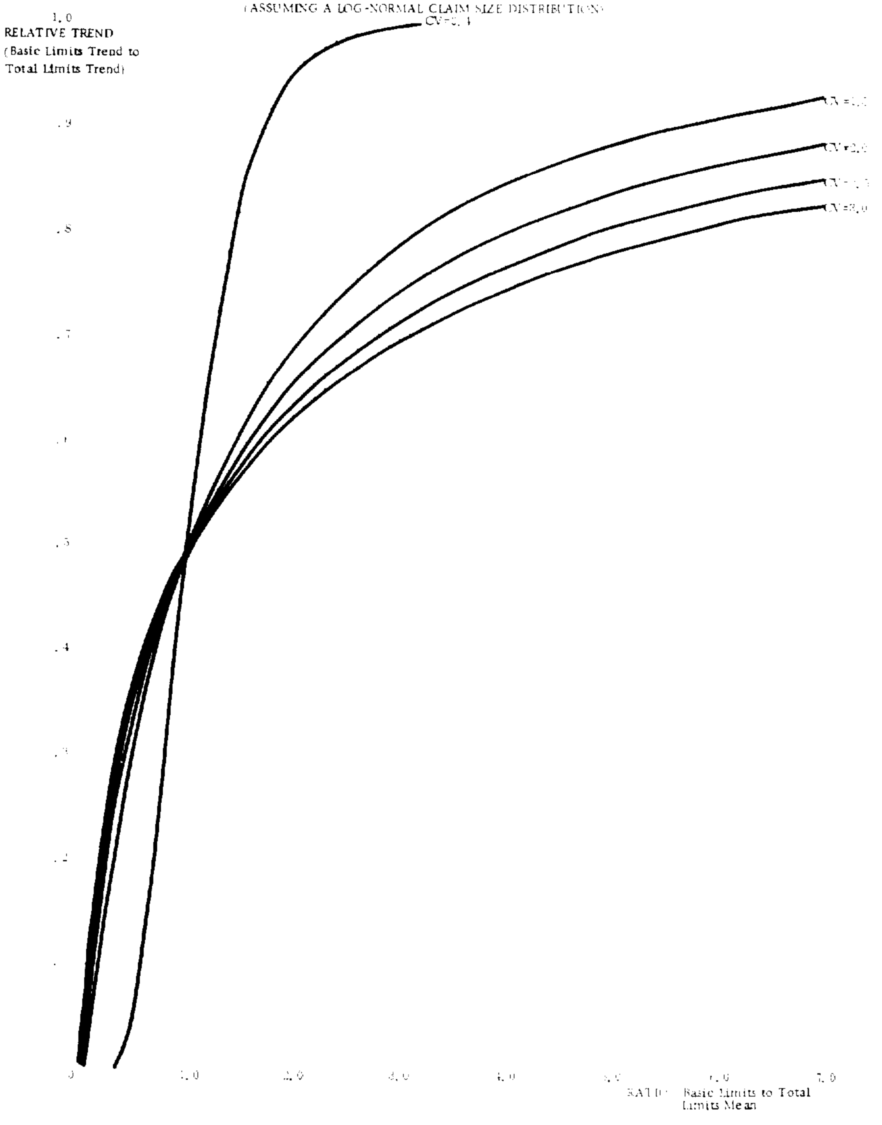
where: X(x) is the percentage of the total amount of losses below x per claim.

For this paper, it is assumed that the claim size distribution follows the log-normal probability law[1]. Results for this law can be produced in terms of two parameters: the coefficient of variation (CV), and the ratio to the unlimited mean. The second parameter can be used to represent the basic limit. Results vary somewhat as a function of CV, but this parameter is not as crucial as the basic limit value. Exhibit I illustrates the relative trend for several choices of CV. A method for calculation of the average relative trend is described in the appendix.

[1] For a discussion of this distribution, see Finger, R. J., "Estimating Pure Premiums By Layer—An Approach", PCAS LXII (1976).

EXHIBIT I

RELATIVE TREND FOR VARIOUS COEFFICIENTS OF VARIATION
(ASSUMING A LOG-NORMAL CLAIM SIZE DISTRIBUTION)



EXAMPLE NO. 1—LIABILITY INSURANCE

Basic limits rates are being prepared for liability insurance. For this purpose two policy years are used which are 3.5 and 4.5 years removed from the average effective date for the new rates. The basic limit trend is measured at 10% per annum based on claims occurring an average of 10 years prior to those expected under the new rates. For claims entering the trend calculation, the basic limit is about 3.8 times the observed unlimited mean.

Looking at Exhibit I, and assuming a CV of 3, the relative trend is about .74 at 3.8 times the mean. This implies an unlimited trend of about $\frac{10\%}{.74} = 13.5\%$. If the CV is 2, the relative trend is .79 and the unlimited trend is 12.7%. Assuming the CV is 3, the basic limit is expected to be $3.8 \times 1.135^{(4.5-10)} = 1.89$ and 1.67 times the mean for the above policy years and 1.07 times the mean in the policy year for the new rates. The average relative trend from 1.89 to 1.07 is .56 and from 1.67 to 1.07 is .55. Thus the average basic limit trend should be $(.56)13.5\% = 7.6\%$ and $(.55)13.5\% = 7.4\%$ for the two policy years. The basic limit trend factors should be $(1.076)^{4.5} = 1.39$ rather than $(1.10)^{4.5} = 1.54$ and $(1.074)^{3.5} = 1.28$ instead of $(1.10)^{3.5} = 1.40$.

Assuming the CV is 2, the basic limit would be 1.97 and 1.75 times the mean for the given policy years and 1.15 in the new policy year. The average relative trends would be .59 and .58. The basic limit trends would be 7.5% and 7.4%. The basic limits trend factors would be 1.38 and 1.28.

This example points out some general conclusions:

- The choice of CV has relatively little impact on the results.

- The use of a basic limit trend factor based solely on previous experience may overstate the projected basic limit losses; in the given example it was by about 10%.

EXAMPLE NO. 2—WORKERS' COMPENSATION PAYROLL OFFSET

The same general approach can be taken to evaluate the effect of increasing wages on collectible premiums in workers' compensation insurance. A few states have a payroll limitation which acts much like a basic limit to curb the growth of subject payroll. The main practical difference between a payroll limitation and basic limits is that the subject distribution is much less skewed for workers' compensation payroll. Table I compares the Standard Wage Distribution Table with a log-normal distribution with CV of 0.4. These tables are based on claimant data and may not represent the same distribution as that for exposed workers.

TABLE I
COMPARISON OF STANDARD WAGE DISTRIBUTION TABLE AND LOG-NORMAL DISTRIBUTION

Ratio to	Standard Wage Table[2]		Log-Normal (CV = 0.4)	
Mean	% Workers	% Wages	% Workers	% Wages
.1	.1	—	—	—
.2	.5	.1	—	—
.3	1.3	.3	.2	—
.4	2.9	.8	1.4	.5
.5	6.3	2.4	5.4	2.3
.6	12.7	8.9	12.9	6.4
.7	22.1	12.0	23.2	13.2
.8	33.2	20.4	35.0	22.0
.9	44.9	30.2	46.8	32.1
1.0	56.5	41.2	57.6	42.4
1.1	66.4	51.6	67.0	52.2
1.2	74.4	60.8	74.7	61.1
1.3	80.5	68.4	80.9	68.7
1.4	85.4	75.0	85.7	75.2
1.5	89.0	80.2	89.3	80.5
1.6	91.6	84.3	92.1	84.8
1.7	94.1	88.4	94.2	88.2
1.8	95.7	91.1	95.7	90.9
1.9	97.0	93.5	96.8	93.0
2.0	98.0	95.5	97.7	94.6

[2] Source: Fratello, Barney, "The 'Workmens Compensation Injury Table' and 'Standard Wage Distribution Table,' " PCAS XLII (1955).

Assume the statewide average wage is \$200 and the payroll limitation is \$300. If total wages can be expected to grow by 7%, subject premium will only grow by 5.6%. That is, the payroll limitation changes from 1.5 to 1.4 times the mean and the relative trend is about .8. Currently used ratemaking methods consider many other factors and may indirectly adjust for this shortfall in collectible premium.

SUMMARY

This paper has explored the problem of estimating the basic limits trend once the overall trend has been determined. Although the log-normal, has been used for numerical examples, it can be expected that the general conclusions hold for most actual claim size distributions.

Generally speaking, the relative trend (that is, the basic limit trend relative to the unlimited trend) is less than 1.0 and decreasing as the ratio of the basic limit to the unlimited mean is decreasing.

Practical applications of the relative trend concept are not limited to basic limits ratemaking. An example is presented to show what the increase in subject wages will be for workers' compensation insurance, given a fixed dollar payroll limitation.

APPENDIX

FINDING THE AVERAGE RELATIVE TREND

The relative trend varies as the relationship between the basic limits value and the mean changes. To measure the average relative trend over a period of time, one must take into account the changes in that relationship.

The relative trend, f(x), is defined at the particular instant when the ratio of the basic limit to the unlimited mean is x. This function can be defined as a limiting distribution of ART, or:

$$f(x) = \lim_{i \to o} \frac{1}{i} \frac{(1+i)X\left(\frac{x}{1+i}\right) - X(x)}{X(x)}$$

The relationship between a fixed basic limit value, A, and the mean of the unlimited distribution is not changing as a linear function of time. For example, after one time period of inflation i, the new unlimited mean is $\frac{A}{1+i}$ where A was the original mean. After two time periods the mean is $\frac{A}{(1+i)^2}$. For fractional time periods, t, we can use the function $e^{-\delta t} = (1 + i)^{-t}$ to represent the changing value of the mean. Thus $\frac{A}{(1+i)^t} = Ae^{-\delta t}$.

The arguments of ART will be revised to represent the beginning and ending ratios of the basic limit to the unlimited mean. If A is the beginning ratio and there is an annual trend of i for T years, the ending ratio will be $Ae^{-\delta T}$.

Assume:

1. The total limits annual trend is i; or $1 + i = e^{\delta}$
2. The beginning value of the basic limit is A times the mean
3. f(x) is the relative trend as a function of x, the ratio of the basic limit to the mean
4. The time period under study is T years.

The average relative trend, ART, can be written as

$$ART(A, Ae^{-\delta T}) = \frac{1}{T}\int_{0}^{T} f(Ae^{-\delta t})dt$$

Substituting $y = Ae^{-\delta t}$

$$ART = \frac{1}{\delta T}\int_{Ae^{-\delta t}}^{A} \frac{1}{y} f(y)dy$$

Substituting $z = \ln y$

$$ART = \frac{1}{\delta T}\int_{\ln (A - \delta t)}^{\ln A} f(z)dz$$

Table II shows the tabulation of $\int_{o}^{\ln A} f(z)dz$ for various values of A and several choices of CV. From this table $\int_{\ln(A-\delta T)}^{\ln A} f(z)dz$ can be obtained by one subtraction. The quantity δT is the difference between the natural logarithms of the initial and ending ratios of the basic limits to the mean. This quantity can also be obtained by one subtraction.

Example. Given: 1. i is 15% per annum.

2. A is 5.0 times the mean.

3. T is 5 years.

4. The CV is 3.0.

Solution: $Ae^{-\delta T}$ is about 2.5 times the mean.

From Table II we have:

$$ART(5.0, 2.5) = \frac{1.811 - 1.309}{1.609 - .916} = .72$$

TABLE II

CALCULATION VALUES FOR AVERAGE RELATIVE TREND

RATIO A: Basic Limits to Total Limits Mean	Ln A	$\int_0^{\text{Ln A}} f(z)dz$ where z is Ln[ratio A]			
		CV = 0.4	CV = 2.0	CV = 3.0	CV = 4.0
.1	−2.303	0	.044	.100	.148
.2	−1.609	0	.121	.215	.285
.3	−1.204	0	.199	.315	.396
.4	− .916	.002	.274	.402	.489
.5	− .693	.008	.343	.479	.571
.6	− .511	.021	.408	.549	.644
.7	− .357	.044	.469	.613	.709
.8	− .223	.077	.526	.673	.770
.9	− .105	.118	.580	.728	.825
1.0	0	.165	.632	.779	.877
1.1	.095	.217	.680	.827	.924
1.2	.182	.270	.725	.871	.969
1.3	.262	.325	.768	.914	1.011
1.4	.336	.381	.810	.955	1.051
1.5	.405	.437	.850	.993	1.089
1.6	.470	.492	.888	1.030	1.125
1.7	.531	.545	.925	1.066	1.160
1.8	.588	.597	.961	1.100	1.194
1.9	.642	.647	.996	1.133	1.226
2.0	.693	.696	1.029	1.165	1.257
2.5	.916	.913	1.181	1.309	1.396
3.0	1.099	1.094	1.313	1.433	1.517
3.5	1.253	1.248	1.430	1.543	1.622
4.0	1.386	1.381	1.535	1.641	1.717
4.5	1.504	1.499	1.630	1.730	1.802
5.0	1.609	1.604	1.718	1.811	1.880
6.0	1.792	1.786	1.873	1.956	2.019
7.0	1.946	1.940	2.008	2.082	2.140
8.0	2.079	2.074	2.127	2.194	2.247
9.0	2.197	2.192	2.234	2.294	2.344
10.0	2.303	2.297	2.331	2.385	2.431