GENERALIZED PREMIUM FORMULAE

JAMES P. ROSS

DISCUSSION BY ALAN E. KALISKI

James Ross, in his paper "Generalized Premium Formulae," has mathematically set forth a methodology for determining rate level adjustment factors (i.e., factors to convert actual earned premiums to a present rate level basis) when the earned premiums being put on-level consist of contributions from policies written with different terms. An example of this situation, as posed by Mr. Ross, occurs when: (1) three-year policies are converted to annual policies upon renewal as of a certain date, and (2) the premiums being adjusted to present level consist of earnings from both three-year policies (written prior to date of annualization) and one-year policies (written after date of annualization). In solving this problem, the author has formularized and illustrated many fundamental, yet important relationships among earned exposures, written exposures, rate of exposure writing, and policy term.

With regard to technical ratemaking procedures, this paper is especially relevant at the present time. The Insurance Services Office has recently filed, and received approval of in several states, annualization endorsements for their Special Multi-Peril (SMP) policy form. By attaching these endorsements, SMP policies previously written and rated for three-year terms will now, for the most part, be subject to annual re-rating at first and second anniversaries of policy inception. From a ratemaking standpoint, when faced with the problem of determining rate level adjustment (on-level) factors, the theory developed in the paper under consideration has application to this situation. Some minor modifications are necessary in this case, however, because certain policies—three-year pre-paids and those which develop annual premiums of $500 or less—are *not* annually re-rated for practical reasons. The following paragraphs discuss the nature of the modifications required in order to make Mr. Ross' paper directly applicable to the ISO annualization of SMP policies.

On page 53, an example is given in which the policy term is changed from three years to one year at time X_0, and the exposure prior to time X_0

had been written at a constant rate K_o. The author then states that $f(x)$ is as follows:

$$f(x) = \begin{cases} \tfrac{1}{3} K_o, \text{ for } X_o < X \leq X_o + 1 \\ \tfrac{2}{3} K_o, \text{ for } X_o + 1 < X \leq X_o + 2 \\ K_o, \text{ for } X_o + 2 < X \leq X_o + 3 \\ K_o, \text{ for } X > X_o + 3 \end{cases}$$

In the case of ISO annualization, a modification to the above definition of $f(x)$ is necessary for the following reason: Not all SMP policies are subject to annual re-rating at policy inception anniversary. (More specifically, Deferred Premium Payment (DPP) plan policies where the annual premium is less than \$500 and three-year pre-paid policies are excluded from the effects of annualization.)

Suppose 5% of all SMP policies fall in either of the above two categories and are thus not subject to annualization. Then, under the author's assumptions, $f(x)$ would be defined as follows:

$$f(x) = \begin{cases} .95 \tfrac{1}{3} K_o + .05 K_o, \text{ for } X_o < X \leq X_o + 1 \\ .95 \tfrac{2}{3} K_o + .05 K_o, \text{ for } X_o + 1 < X \leq X_o + 2 \\ .95 \tfrac{3}{3} K_o + .05 K_o, \text{ for } X_o + 2 < X \leq X_o + 3 \\ .95 \tfrac{3}{3} K_o + .05 K_o, \text{ for } X > X_o + 3 \end{cases}$$

Coincident with the annualization of SMP policies, the ISO is changing the term multiple from 2.7 to 3.0 for all policies and is maintaining its 5% installment surcharge *only* for DPP plan policies whose annual premium is less than \$500. Hence, when determining rate level adjustment factors, the following should be considered as normal rate changes effective as of the date of annualization:

<div align="center">Rate Change</div>

DPP Plans—premium at least \$500
$$\frac{3.0}{2.7 \,(1.05)} = 1.058 \text{ (or } + 5.8\%)$$

DPP Plans—premium less than \$500
$$\frac{3.0 \,(1.05)}{2.7 \,(1.05)} = 1.111 \text{ (or } + 11.1\%)$$

3-Year Pre-Paid Policies
$$\frac{3.0}{2.7} = 1.111 \text{ (or } + 11.1\%)$$

(The above three rate level effects would be weighted by the respective premiums within each category to obtain the average rate level change effective with annualization.)

Having made the above described modification to the function $f(x)$, and by including the rate level effect of the change in term multiples and installment surcharges, the methodology set forth in Mr. Ross' paper can be applied directly to the ISO situation.

As a final note with respect to the specifics of the paper, there is one place where I believe the author interchanged words. In particular, at the bottom of page 54, an example is given followed by the statement ". . . with an annual term a rate change at the beginning of the year will result in one-half of the premium earned at the old rate and one-half at the new rate." I believe the author meant to say that one-half of the exposures (*not* premium) are earned at the different rate levels, r_1 and r_2.

Although Mr. Ross' paper addresses itself to the solution of a particular problem—the determination of rate level adjustment factors—it is nevertheless recommended to anyone interested in the mathematical formulation of certain fundamental insurance relationships. Also, it can be shown that the rate level adjustment (on-level) factors calculated via the formulae set forth are equivalent to those determined from the traditional parallelogram approach. Hence, this paper can also serve to clear up the "mystique" of the parallelogram approach that might exist among those first introduced to it.

While of a technical nature, "Generalized Premium Formulae" by James Ross touches on a subject of which more needs to be written. In particular, a gap in the literature seems to exist with respect to Commercial Multi-Peril Package policy ratemaking and pricing. This topic would appear to warrant consideration in that CMP has recently become the predominant commercial line. Also, it would be interesting to learn of pricing and packaging approaches used by companies that market their own independent policy forms in addition to the ISO's SMP policy.