ESTIMATING PURE PREMIUMS BY LAYER—AN APPROACH

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DISCUSSION BY LEE R. STEENECK

Mr. Robert J. Finger in his paper Estimating Pure Premiums By Layer —An Approach suggests that the log-normal probability distribution function can be used as a model for the distribution of a single claim in many instances. Use of the log-normal is based on sound statistical theory and has already been applied to numerous actuarial problems. The "long-tail" evident in liability lines of insurance seems to lead us toward an asymmetrical distribution function like the log-normal.

As a model for claim sizes, in order to be practical, a distribution should have the following desirable characteristics: the estimate of the mean should be efficient and reasonably easy to use; a confidence interval about the mean should be calculable; all moments of the distribution function should exist. The log-normal distribution function has these desirable characteristics. Unfortunately, the log-normal has two annoying qualities, too. One, demonstrated by Mr. Finger, is that there may be fitting problems when there are many small values of the variable under consideration. Making adjustments oftentimes requires a great deal of work. Secondly, from a statistical point of view, the integral in the characteristic function cannot be solved and the convolution cannot be expressed explicitly.[1]

An approximation of the real world severity loss distribution is essential from a reinsurance point of view. On an excess of loss basis the reinsurer is directly involved with the tail of the liability loss distribution. Inflation places the excess of loss reinsurer in a leveraged position where reinsurance claim costs are multiplied significantly with even minor errors in severity or frequency estimation. The cost of error in evaluating these long-tails can produce spectacular underwriting loss as claims develop to ultimate. Reinsurance actuaries have previously realized that distribution functions for claim size would be helpful. Unfortunately, although tools like the log-normal, Pareto, Gamma, and Weibull (to mention a few) have been available for some time now, the estimation of the parameters has been difficult. Few losses exist in these upper layers upon which to make accurate estimates.

[1] Lars-Gunnar Benckert, "The Log-Normal Model For The Distribution Of One Claim," Astin Bulletin, Vol. II (January, 1962) Part I, Pages 2-23.

After detailing the calculation of pure premiums by layer using a log-normal distribution, Mr. Finger applies his approach, for example purposes, to data reported in a Special Malpractice Review.[2] Perhaps some of the problems encountered in fitting a log-normal distribution function to this claim data can be traced directly to the use of survey closed claim data. All the criticisms and caveats implied in using closed claim data will not be repeated here, but suffice it to say that claims included within the survey have accident years dating back into the early 1960's (and claim amounts were not trended). Smaller claims belong to the most recent accident years and are higher in volume relative to the older less frequent severe cases. The poor fit over the entire range of loss values can be attributed to the frequency with which losses close by incurral year. As previously mentioned, the need for an accurate barometer of claim frequency by size is essential. If only we could agree on one.

Several other points deserve comment. To emphasize Mr. Finger's definition of an excess loss distribution—it is defined as "the sum of all claims values larger than the attachment point less the number of claims above the attachment point times the value of the attachment point." Using this definition, Table I represents layers of loss between any two attachment points. This then paves the way for the determination of increased limits factors in Table III. The heading of Table III is a bit misleading. A $100,000 policy increased limit factor is being determined (Basic Limits = $25,000). Coverage is not being rated to $125,000. Perhaps a better title to Table III might be: Indications of $100,000 Policy Increased Limits Factor.

Table VI illustrates an estimation process for determining the CV when claims below a given amount are excluded from the analysis. The problem in dealing with the truncated distribution has also been dealt with in the Benckert article.[3] If the censoring point, c, is such that the excess distribution is greater than 80% $(1.0 - L(c))$ estimates of the mean and

[2] "Report To The All Industry Committee Special Malpractice Review: 1974 Closed Claim Survey Preliminary Analysis of Survey Results," Prepared by the Insurance Services Office (December, 1975).

[3] Lars-Gunnar Benckert, "The Log-Normal Model For The Distribution Of One Claim," Astin Bulletin, Vol. II (January, 1962) Part I, Pages 2-23.

variance (hence the coefficient of variation is easily calculated) are approximately given by:

$$\sigma^{2*} = \frac{Y_2}{r} - \frac{(Y_1)(\overline{Y_1})}{r} + 0.4\sqrt{\frac{Y_2}{r} - \frac{(Y_1)(\overline{Y_1})}{r} \cdot \frac{m}{r}} \bullet (\overline{Y_1} - \log c)$$

$$u^* = \overline{Y_1} - \frac{(m)(\sigma^*)(0.4)}{n}$$

Where $Y_1 = \Sigma \log x_i + m \log c$ for $x_i > c$

$Y_2 = \Sigma \log^2 x_i + m \log^2 c$ for $x_i > c$

$$\overline{Y_1} = \frac{Y_1}{n}$$

m is the number of claims $\leq c$

r is the number of claims $> c$

$m + r = n$

One final comment regarding the "Other Applications" section of the paper. Although this reviewer has not researched the problem in depth some European actuaries (Benckert [4] and Beard[5]) have suggested the use of the log-normal in connection with fire losses.

I hope this article will spark additional interest in the use of theoretical loss distributions to characterize claim activity. Certainly other functions exist which may provide even better indications for the tail. Insurance data needs to be collected, fitted, analyzed, and published in the testing process of various model distributions. With the sparsity of large claim data, continuous claim size distributions are needed in the rating of high layers of insurance coverage. We are indebted to Mr. Finger for his enlightening exposition on this most flexible rating tool.

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[4] Lars-Gunnar Benckert, "The Premium in Insurance Against Loss of Profit Due to Fire As A Function of the Period of Indemnity," *Transactions of the XVth International Congress of Actuaries*, Vol. II, (1957), Pages 297-305.

[5] R. E. Beard, "Analytic Expressions of the Risks Involved in General Insurance," *Transactions of the XVth International Congress of Actuaries*, Vol. II, (1957), Pages 230-242.