THE CALIFORNIA TABLE L

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DISCUSSION BY FRANK HARWAYNE

This is both a review of and an alternative to the program described by Mr. Skurnick in *The California Table L* as a generalization of Table M. Table M focuses attention upon risks of a given expected loss size. The aggregate losses of each risk are ordered with the risks producing the least amount of such losses appearing first, the next lowest amount appearing second, etc. From this order, Table M charges or excess pure premium ratios are developed. Simon's[1] methodology generates a family of curves of Table M Values according to expected loss size.

Skurnick's paper carries Simon's program a step further by introducing the accident limitation into the system of excess pure premium ratios. The mathematics are impressive to the point of rivalling some college textbooks. Dropkin's[2] statement on Simon, "It is not to be read casually, commuting to and from work", applies equally here. The theorems and lemmas have been developed and abstracted for general application. Wrestling with them should give the theoretical mathematician or sophisticated actuary some sense of satisfaction. The formulae are sound and useful in developing Table L which sets forth the excess pure premium ratios when claims arising from a single accident are limited for specific amounts.

Application of Skurnick's theorems and lemmas to produce Table L poses a dilemma. If one requires a separate Table L for every accident limit (or excess loss premium factor of which there are 36 in Rhode Island) in each of fifty-two states, one might need as many as 1800 Table L's. Considering that Table M requires 111 printed pages, we could expect to be printing 200,000 pages of Table L, and the more we print, the more difficult is the annual rate approval process required by rate regulation.

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[1] L. J. Simon, "The 1965 Table M," PCAS, LII (1965), p. 1

[2] L. B. Dropkin, "Discussion of 'The 1965 Table M,' L. J. Simon," PCAS, LII (1965) p. 46

In an effort to stem this ruinous tide of paper, I have tried to reevaluate Skurnick's methodology from the practical side. What essentially is Table L? It is Table M on which has been engrafted the charge required for limiting accidents to a specified amount. The difficulty encountered in attempting to combine Table M with excess loss premium factor charges is that Table M is developed from losses on a risk basis and the other is developed from losses on a per accident basis. This means that at certain entry values Table M (which may already contain charges for individual losses in excess of the accident limitations) needs to be coupled with elements that are not normally compiled on a risk basis. It is clear that at entry ratios corresponding to aggregate losses for risks which produce less than the amount equal to the accident limit, Table M contains no overlap problem. It should also be apparent that at the extremely high entry ratios there will be some risks whose losses will consist solely of accidents where claims exceed the accident limitation. In between, there will be some overlap between Table M charges and excess loss premium factors.

If we define the following terms,

r^M_T = attachment point value such that $r^M_i = \dfrac{\text{Accident imit}}{E^M_i}$

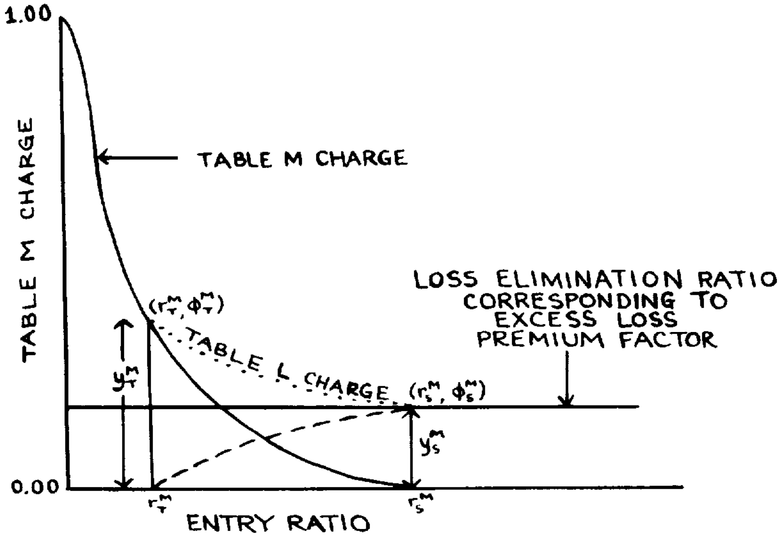
E^M_i = risk expected losses

$Y^M_T = \phi^M_T$ = Table M charge at point r^M_t

r^M_S = asymptotic point at which $\phi^M_i + \Delta\phi(r) = \Delta\phi(r)$

$Y^M_S = \phi^M_S$ Table M Charge at point r^M_S for which the Table L charge is approximately equal to the loss elimination ratio corresponding to the excess loss premium factor.

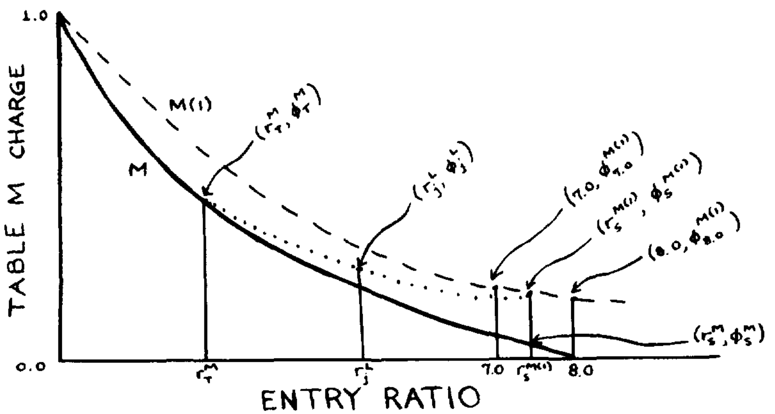
The elements can be graphed as follows:



The broken line represents the loss elimination ratio corresponding to the modified excess loss premium factor (discounted for the overlap with Table M charges). The dotted line connecting the y-values ϕ_T^M and ϕ_S^M represents the net sum of Table M charges and the modified excess loss premium factor. It is the curve of Table L developed by Skurnick. In the form stated here, the degree of overlap of ϕ_i^M and the excess loss premium factor range $r_T^M \leq r_i^M \leq r_S^M$ is not readily expressible as a simple function, or else would entail extensive computation.

The problem may also be looked upon as one of assigning a probability value to the overlap implicit in the Table M values between r_T^M and r_S^M. From this viewpoint, one then asks the question how much should the excess loss premium factor be discounted? Noting that the discount is 100% at point r_T^M and 0% at point r_S^M, and since we are dealing with a continuous function (or nearly so) it is logical that the charge (complement of the discount) be graded in proportion to $r_S^M - r_T^M$. Moreover, in the matter of curve fitting, if we can find another expected loss size M(1) in the same family of curves such that the rate of change at $\phi_T^{M(1)}$ is close to the rate of change at ϕ_T^M and also has the value ϕ_S^M at $r_S^{M(1)}$ with zero or a very small rate of change (i.e. is almost asymptotic), then we will have found two Table M's such that $\phi_j^M \le \phi_j^L \le \phi_j^{M(1)}$ for $r_T^M \le r_j^M \le r_S^M$.

Graphically, the interpolation and curve fitting can be shown to be as follows:



M is a curve of Table M excess pure premium ratios at the given expected loss size.

M (1) is a curve of Table M excess pure premium ratios that $\phi_{7.0}^{M(1)} \leqq$ excess loss premium factor $\leqq \phi_{8.0}^{M(1)}$.

ϕ_j^L can be estimated to be approximately $\phi_j^M + \Delta\phi(rj)$

$$\text{where } \triangle\phi(rj) = \left[\phi_j^{M(1)} - \phi_j^M\right] \times \frac{r_j^M - r_T^M}{r_S^M - r_T^M}$$

$\triangle\phi(rj)$ is the modified excess loss premium factor because $M(1)$ was selected so that $\phi_S^{M(1)} - \phi_S^M$ equals the excess loss premium factor.

It can also be shown that:

$\triangle\phi(rj) + \phi_j^M \geq \phi_S^M$. This means that the adjusted excess loss premium factor plus the table M charge is never less than the undiscounted excess loss premium factor.

The determination of r_S^M for various expected loss size gorups (Table numbers of Table M) forms a new Table of Attenuation Points (see Exhibit I). It is noted that the differences in charges for entry ratios of 7.00 and 8.00 are sufficiently small to meet our requirements regarding asymptotic values. At Table 90 the difference is .065 (less than .00065 for a change of .01 in entry ratio) and at Table 40 the difference is .003 (less than .00003 for a change of .01 in entry ratio). Values for entry ratios less than 7.00 and 8.00 could be selected if less stringent criteria were used. The difference in charge between entry ratio 4.0 and 5.0 is .070 for Table 90 and .025 for Table 40.

Procedurally, the computation of the modified excess loss premium factor is very simple. Entry ratios for the minimum (only if larger than r_T^M), maximum and r_T^M are required. A self-explanatory worksheet (Exhibit II) sets forth the procedure for derivation of the modified loss elimination ratio corresponding to modified excess loss premium factor.

It is possible to construct an equivalent to Table L by adding the modified excess loss premium factor described above to the Table M charge. This was done for Mr. Skurnick's Exhibit 5 for a 25,000 limit and for entry ratios that were above the attachment point ϕ_T^M. Exhibit I was developed from Countrywide Table M charges and these charges for appropriate expected values were used (California Table M would have given somewhat different results). The results are as follows:

		Table M		Table L $25,000 Acc. Limit		
Standard Premium	**Entry Ratio**	**Country-wide**	**Calif.**	**Country-wide***	**Calif.**	**Adjusted#**
$35,000	1.37	.235	.297	.248	.292	.310
50,000	1.32	.215	.268	.243	.268	.296
67,500	1.27	.207	.234	.244	.243	.271
80,000	1.25	.201	.211	.242	.233	.252
254,948	1.08	.157	.187	.214	.219	.244

It should be noted that the differences between countrywide and California Table L values arise from and are smaller than the differences in Table M. It will be seen from the column of Table L, $25,000 Accident Limit, Countrywide Adjusted that the charges for accident limitation are higher than by Skurnick's method and are therefore more conservative.

The alternative suggested here is in no way intended to diminish the logic and insights of Mr. Skurnick's paper. Indeed, his valuable contribution in this area has been the spur for solving a thorny problem in a practical way. Undoubtedly more work in developing refined solutions is to be welcomed.

* Table M plus increment of .1244 developed from application of principles using Exhibits I and II.

\# Modified excess loss premium factor using countrywide Table M for discounting ELPF's and adding California Table M. charge.

EXHIBIT I

NATIONAL COUNCIL ON COMPENSATION INSURANCE

TABLE OF ATTENUATION POINTS FOR COMPUTATION OF ADJUSTED ELPF'S FROM TABLE M CHARGES

Table No.	Charge at Entry Ratio 7.00	Charge at Entry Ratio 8.00	Difference	Table No.	Charge at Entry Ratio 7.00	Charge at Entry Ratio 8.00	Difference	Table No.	Charge at Entry Ratio 7.00	Charge at Entry Ratio 8.00	Difference
				74	.255	.211	.044	54	.047	.030	.017
93	.602	.542	.060	73	.246	.202	.044	53	.043	.027	.016
92	.563	.501	.062	72	.237	.194	.043	52	.038	.024	.014
91	.525	.461	.064	71	.229	.187	.042	51	.034	.021	.013
90	.488	.423	.065	70	.220	.178	.042	50	.030	.018	.012
89	.452	.391	.061	69	.211	.170	.041	49	.027	.016	.011
88	.421	.364	.057	68	.202	.162	.040	48	.023	.013	.010
87	.396	.341	.055	67	.193	.154	.039	47	.020	.011	.009
86	.375	.322	.053	66	.184	.145	.039	46	.017	.010	.007
85	.358	.306	.052	65	.175	.137	.038	45	.015	.008	.007
84	.349	.298	.051	64	.165	.118	.047	44	.013	.006	.007
83	.339	.288	.051	63	.142	.100	.042	43	.011	.005	.006
82	.329	.279	.050	62	.121	.085	.036	42	.009	.004	.005
81	.320	.271	.049	61	.103	.072	.031	41	.007	.004	.003
80	.311	.262	.049	60	.088	.061	.027	40	.006	.003	.003
79	.301	.253	.048	59	.074	.050	.024				
78	.292	.245	.047	58	.068	.046	.022				
77	.283	.236	.047	57	.063	.042	.021				
76	.274	.228	.046	56	.057	.038	.019				
75	.264	.219	.045	55	.052	.034	.018				

EXHIBIT II

NATIONAL COUNCIL ON COMPENSATION INSURANCE

Worksheet for Computing Modified
Excess Loss Premium Factor Charges

(1) Loss Elimination Ratio Corresponding to Excess Loss Premium Factor: ______

(2) Highest Table # where charge at entry ratio of 8.0 is less than (1). Table # ______

(3) For (2),. Entry Ratio 7.00 Charge ______(a)
8.00 ______(b)
Difference ______(c)

(d) $(r\frac{M}{S}, \phi\frac{M}{S})$ Attenuation Point $= 7.00 + \frac{(3a)-(1)}{(3c)} = 7.00$
+ ______ − ______ = ______

(4) Expected Losses (a) ______ and (b) Table # ______

(5) Accident Limit ______

(6) $(r\frac{M}{T}, \phi\frac{M}{T})$ Attachment Point = (5) ÷ (4a)
= ______ ÷ ______ = ______

(7) Minimum Ratio ______ (use only if larger than (6)).

(8) Maximum Ratio ______

(9) For (7), values of savings for Table #'s (2)–(4),
______ − ______ = ______*

(10) For (8), values of charges for Table #'s (2)–(4),
______ − ______ = ______

(11) [(7)–(6)] ÷ [(3d)–(6)], [______ − ______] ÷ [______ − ______]
= ______

(12) [(8)–(6)] ÷ [(3d)–(6)], [______ − ______] ÷ [______ − ______]
= ______

(13) Incremental Charge for Max. = (10) × (12)
= ______ × ______ = ______

(14) Incremental Saving for Min. = (9) × (11)
= ______ × ______ = ______*

(15) Modified Loss Elimination Ratio Corresponding to Excess Loss Premium Factor = (13)–(14) = ______ − ______ = ______

* Enter zero if (7) is smaller than (6).