LOSS RESERVE TESTING: A REPORT YEAR APPROACH

WAYNE H. FISHER AND JEFFREY T. LANGE

"Two roads diverged in a wood, and I—
I took the one less traveled by,
And that has made all the difference."
– Robert Frost

During the nineteenth century, the functions of the actuary were twofold: the calculation of premium rates and the setting of reserves. While reserves for life insurance may be set by actuarial formulae, casualty reserves are more frequently established by claims adjusters on an individual case basis. One of the tasks of the casualty actuary is to test the adequacy of reserves. Such tests are generally carried out on a statistical basis, reviewing a whole portfolio of case reserves at one time. As a result of the actuary's test, it may be necessary to increase or decrease existing reserves, add special bulk reserves, or issue new instructions redirecting the claims adjusters in the setting of reserves. The significance of these steps for the financial solvency of the company cannot be overestimated. As Balcarek has clearly shown,[1] changes in the degree of reserve adequacy have a very substantial impact on the earnings of casualty insurance companies. Thus, reserve tests should be, and usually are, the concern of senior management.

The most familiar reserve tests are those incorporated in the annual statement. Currently, Schedule P is organized on an accident year basis and provides a means of performing a reserve test. However, it can be argued that Schedule P alone cannot be used to determine the amount of overstatement or understatement of the current reserves for a given line. Skurnick[2] includes more elaborate accident year tests in his survey of reserve calculations and, no doubt, the accident year approach is favored by many actuaries for reserve evaluation.

This paper will discuss a reserve test which is based on an alternative scheme for organization of the data. Fundamental to this approach is the tabulation of claims (both reserves and payments) by report year. The

[1]R. J. Balcarek, "Effect of Loss Reserve Margins in Calendar Year Results", *PCAS* Vol. LIII, p. 5.
[2]D. Skurnick, "A Survey of Loss Reserving Methods", *PCAS* Vol. LX.

latter is defined to be the year in which the claim was reported to the company, regardless of the accident date. In his survey, Skurnick mentions two methods of using report year data. The first is a projection, or loss development, approach dealing with total report year losses at various points of development. The second is a payment development method in which it is assumed that the percentage increase in the ultimate incurred value will be the same as the increase in average paid claim costs for claims of a similar age. In this paper, we will describe a third, more complex method utilizing report year data, and will then proceed to show how this approach is used in practice.

The report year approach is designed to test the adequacy of a portfolio of reserves for known cases. By known cases, we mean cases that have been reported to the company and for which the claim department has established a reserve. Since the method is a test, it is possible to apply it even in situations where the claim department employs an average reserving or similar system in setting some reserves. The test is usually applied to the entire known case reserve for a given line, such as auto bodily injury liability; however, in some cases it has been applied to subdivisions of a line, when the subdivision itself constitutes a unique reserving problem. The test is customarily applied only to third party lines. It is designed to reveal whether the reserve is adequate, and to measure: (1) the extent of any redundancy or inadequacy, (2) the slippage or strengthening of the equity position of the reserve since the last evaluation, and (3) the contribution of various report years to the overall reserve position. The first two results are significant in the financial position of the company in that one deals with solvency while the second deals with possible distortions in the income statement. The third result is of value in the administration of the claim department in that it tells us whether any redundancy is due to old or new cases. In other words, it indicates where corrective action is needed and, in later evaluations, monitors that corrective action. While the first two results are, of course, attainable under an accident year approach, the third result is not available in exactly the same format unless one uses a report year approach. It is believed that, from the point of view of administering a claim department, the report date is more significant than the accident date. Claim department policy would deal with claims reported during some time frame. That policy, or changes in it, can be tested by examining appropriate report (not accident) years.

There are three key features in the report year methodology. As previously mentioned, the first is that the data are organized by report year. This implies that the number of cases for any report year to be tested is fixed at the close of the year. Such a method is superior to an accident year approach in which the number of cases is subject to change at each successive evaluation. Thus, the report year approach substitutes a known quantity for an estimate. The second key characteristic is that virtually all parameters for the reserve test may be estimated from paid loss data. This is an advantage over those techniques employing loss development factors in which reserves are included in the calculation of the parameters of the test. Hence, the report year approach provides a truly independent check on the reserves. A third characteristic of the methodology is that it can be readily adjusted to reflect management's views on the change in the way in which claims will be disposed of by the claim department and on the change in the future rate of inflation. It is possible to show the impact of different rates of inflation on the adequacy of the reserves.

In the next section, we explain the report year methodology using a detailed example. First, the organization of the data base is outlined. Next, the two fundamental calculations are described: the estimation of claim costs and the computation of the disposal rate of claims. Then, these results are combined and the equity position of the reserve is determined. Since one goal of the paper is to instruct actuarial students who are unfamiliar with loss reserving techniques, a complete step-by-step description is provided. While actual data have not been used in this exercise, it has been constructed so that it is representative of third party lines.

This report year test first requires that the various outstanding claims in a given reserve be divided into groups (report year reserves) depending on the year in which the case was initially reported to the company. For example, assume the December 31, 1973 reserve for known cases in a particular line is $55.0 million on 13,761 cases; of these 13,761 cases, 8,372 cases were reported in 1973, 2,764 cases in 1972, 1,416 in 1971, 787 in 1970, and the remainder reported in prior years. Each group of cases, of course, has associated with it an aggregate dollar value—the sum of the individual estimates established for those cases by the claim department.

In order to develop the individual report year reserves, claim data must be available which separate and track the development of claims reported in a given year. The following data, for example, would be maintained on those claims reported in 1970:

TABLE 1

1970 Report Year Statistics on Paid, Outstanding and Incurred Bases

Calendar Year	Cumulative Paid/Closed		Outstanding		Incurred		
	Number	Dollars (000)	Number	Dollars (000)	Number	Dollars (000)	Average
1970	10015	$ 5458	11248	$18304	21263	$23762	$1118
1971	17478	14011	3785	12916	21263	26927	1266
1972	19477	20758	1786	8762	21263	29520	1388
1973	20476	25071	787	5154	21263	30225	1421

The report year data above are assembled in this format to facilitate the comparisons and calculations described later. Several items on the above table should be noted now, however. First, the number of cases incurred for the report year (21,263) is determined by adding the number of 1970 report year cases actually paid in 1970 (10,015) to those 1970 report year cases still outstanding at the end of 1970 (11,248). This figure for the number incurred is then "frozen" for this report year in the subsequent years of development (i.e., a claim closed without payment after the initial year does not decrease the number of claims incurred for the report year). The values in the first column are a combination of claims paid and claims closed—with or without payment. For the initial year, in this case 1970, the figure represents the number of 1970 report year cases actually paid during 1970 (10,015). For the subsequent years, the number in this column is the sum of the cases paid during the initial year and those cases closed, with or without payment, in the following years. The number of cases closed is the difference between the number outstanding at the beginning of the period and those still outstanding at the end of the period.

Arranging the data in the above format allows one to notice several characteristics which are fundamental to the report year methodology. During 1970, roughly half of the total number of claims incurred for the report year were settled, but less than one-fourth of the total dollars incurred were paid on these cases; thereby implying a relatively low average claim cost. In the subsequent years, fewer claims were actually settled, but they were settled at relatively higher averages. In their report year ap-

proaches, Harnek and Sampson[3] did not utilize this difference in the average claim costs by age; however, this difference is an essential feature of the approach presented in this paper.

The data in the above table are presented on a cumulative basis; reassembling these data on a segmented basis, year by year, enables one to observe the pattern in average claim costs more readily. For example, the paid data for the 1970 report year would be displayed as follows:

TABLE 2

1970 Report Year

Calendar Year of Closing	Time Since Beginning of Report Year	Number Paid/Closed	Dollars Paid (000)	Average Claim Cost
1970	(0-12 months)	10015	$5458	$ 545
1971	(13-24 months)	7463	8553	1146
1972	(25-36 months)	1999	6747	3375
1973	(37-48 months)	999	4313	4317

With the report year data in this format, one notices a pattern of increased average claim costs as the time of settlement moves farther away from the year in which the claim was reported. This pattern is typical of that seen for other report years, and implies that the claims settled three years after the year in which they were reported are substantially different from the claims settled, say, in the first year.

In addition, this methodology incorporates the assumption that the claims settled in the same time period (i.e., 13 to 24 months after the beginning of the report year) are essentially similar type claims, and can be compared with claims from other report years which were settled in the same relative time period. In the following table, the data for the 1970 report year are combined with data for other report years, grouping together, for comparison, those averages relating to claims settled in the same time frame.

[3] R. F. Harnek, "Formula Loss Reserves", *Insurance Accounting and Statistical Association Proceedings,* 1966 and R. T. Sampson, "Establishing Adequacy of Reserves on Slow Closing Lines—Use of Paid Formulae", *Insurance Accounting and Statistical Association Proceedings,* 1959.

TABLE 3

Average Claim Cost for Claims Settled in Interval Indicated

Age of Claim Measured in Number of Months from Beginning of Report Year to Settling of Claim	Report Year									
	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
0—12	398	393	413	444	495	577	545	577	612	698
13—24	790	871	837	961	1084	988	1146	1181	1466	
25—36	2348	2128	2288	2471	2438	2865	3375	3598		
37—48	2430	2500	2998	3146	4261	4344	4317			
49—60	3429	2630	3425	3173	4681	5285				
61—72	2572	3629	2944	4034	5211					
73—Ultimate*	1934	3114	5931	4228	4934					

*These averages include the current claim department estimate for any claims still outstanding.

This composite exhibit allows one to observe the effect of inflationary factors on the average claim costs from year to year. It is important to note that the claim costs are increasing at difference rates for the various age-of-claim groups. For example, the claim costs in the 49 to 60 month group are increasing nearly twice as fast as the claims in the 0 to 12 month category. The actual increases are shown later, and at this point only the fact that the percentage increases are different, or at least can be different, is important. This methodology incorporates these different trends by projecting average claim costs independently for each age-of-claim group by utilizing the historical trend for that group. The approaches described by Harnek and Sampson do not reflect this phenomenon.

The requirement for projecting these claim costs can be seen in Table 3. For the 1970 report year, we know that the cases settled in the initial year were settled at an average of $545. Similarly, those settled in the next year were settled at an average of $1,146; in the next year, $3,375; and so forth. We do not know, however, what the average will be for those cases that will be settled in 1974 (49 to 60 months), or in the years after that. Assuming for the moment that we know the number of cases that will be settled in each age-of-claim group for this report year and that we can project the necessary future average claim costs, then we can obtain a weighted average incurred claim cost for the entire report year. This weighted average incurred claim cost can then be compared to the actual average incurred claim cost, based on the claim department estimates, to determine the current reserve adequacy for the particular report year. An average incurred claim cost, rather than an average outstanding claim cost, is utilized for clarity in presentation. This will be explained later.

Projecting the necessary average claim costs can be done in many ways; however, certain techniques work well with this methodology. Table 4 displays, in parentheses, projected averages based on a least squares fit of an exponential curve to the available data for that age group. The exponential curve was utilized as it implies a constant percentage increase in inflation, and this was felt to be most indicative of the situation today. In addition, projections based on a weighted exponential least squares fit are also valuable, as they give added weight to the more recent experience. A linear projection could also be utilized; however, this is unrealistic due to its implied decreasing percentage trend.

Table 4 also displays the percentage increases underlying the individual exponential projections. These percentages can then be weighted

TABLE 4

Average Claim Cost for Claims Settled in Interval Indicated

	Report Year										Average % increase in Claim Cost (exponential)
Age of Claim	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973	
0—12	398	393	413	444	495	577	545	577	612	698	6.6
13—24	790	871	837	961	1084	988	1146	1181	1466	(1426)	7.0
25—36	2348	2128	2288	2471	2438	2865	3375	3598	(3639)	(3906)	7.4
37—48	2430	2500	2998	3146	4261	4344	4317	(5251)	(5883)	(6591)	12.0
49—60	3429	2630	3425	3173	4681	5285	(5368)	(5986)	(6676)	(7445)	11.5
61—72	2572	3629	2944	4034	5211	(5624)	(6546)	(7620)	(8869)	(10322)	16.4
73—Ult.*	1934	3114	5931	4228	4934	(7216)	(8973)	(11158)	(13874)	(17252)	24.3
											9.1%**

NOTE: Numbers in parentheses are projected values.

*These averages include the current claim department estimate for any claims still outstanding.

**Weighted average of percentage increases by age of claim, with weights proportional to the product of the appropriate claim costs (above) and disposal rates (from Table 6) for the latest report year (1973).

to obtain the overall percentage increase inherent in the estimate, in this case 9.1%. If this increase is felt to be either excessive or inadequate (presumably because of information from some external source), the projected averages can be modified to reflect the anticipated rate of increase.

After obtaining projected average claim costs, the second step is to determine the number of claims which will be settled for each age group. From Table 1 we observe that, for the 1970 report year, 47.1% of the total cases incurred were settled in the first year, 35.1% in the second year, 9.4% in the third, and so forth. Combining these percentages, which we will refer to as disposal rates, with similar data for the remaining report years, we notice definite patterns from report year to report year. Table 5 displays these disposal rates in the same format as the claim cost data in Table 3.

Examining the disposal rates in this format allows one the opportunity to observe any trends in the pattern of claim settlement. For example, these data show a lessening in recent years in the time required to settle claims. Settling 50.2% of the 1973 report year claims in the first year (as compared to 47.7% in this time frame for each of the two prior report years), 36.7% of the 1972 report year claims in the 13-24 month group (as compared to 35.0% and 35.1%), and 10.1% for the 1971 report year (as compared to 9.4% and 7.9%) bear out this observation. It should be noted that these percentages—50.2%, 36.7%, and 10.1%—along with the remaining values on that diagonal, pertain to those claims settled in the latest calendar year (1973).

The speed-up in claim settlement noted above may be the result of a deliberate plan by the claim department; in this example we assumed that it was planned and would extend into the future. Accordingly, the anticipated disposal rates shown in parentheses in Table 6 were selected with this in mind. The selected values may appear to be low for certain intervals; however, this is the result of settling more claims in the earlier periods, thereby leaving fewer claims to be settled later.

Selecting disposal rates can be done in a variety of ways. If one feels the recent pattern is representative of current claim settlement practices, the disposal rates from the most recent calendar year can be utilized exclusively; or a weighted average of the last few years can be employed if such is felt to be more in line with anticipated trends. Disposal rates from

TABLE 5

Percentage of Report Year Total Claims Incurred Settled in Interval Indicated

Age of Claim Measured in Number of Months from Beginning of Report Year to Settling of Claim	Report Year									
	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
0—12	.510	.503	.496	.505	.500	.497	.471	.477	.477	.502
13—24	.333	.333	.340	.334	.345	.344	.351	.350	.367	
25—36	.073	.081	.084	.087	.083	.079	.094	.101		
37—48	.037	.036	.038	.035	.033	.040	.047			
49—60	.021	.022	.020	.019	.021	.024				
61—72	.012	.012	.012	.010	.011					
73—Ultimate	.016	.013	.010	.010	.007					

TABLE 6

Percentage of Report Year Total Claims Incurred Settled in Interval Indicated

Age of Claim Measured in Number of Months from Beginning of Report Year to Settling of Claim	Report Year									
	1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
0 - 12	.508	.503	.496	.505	.500	.497	.471	.477	.477	.502
13 - 24	.333	.333	.340	.334	.345	.344	.351	.350	.367	(.349)
25 - 36	.073	.081	.084	.087	.083	.079	.094	.101	(.091)	(.087)
37 - 48	.037	.036	.038	.035	.033	.040	.047	(.040)	(.036)	(.035)
49 - 60	.021	.022	.020	.019	.021	.024	(.022)	(.019)	(.017)	(.016)
61 - 72	.012	.012	.012	.010	.011	(.010)	(.009)	(.008)	(.007)	(.007)
73 - Ultimate	.016	.013	.010	.010	.007	(.006)	(.006)	(.005)	(.005)	(.004)

the most recent year were used in this example. Specifically, the 1973 report year value for the 13-24 month age group was determined as follows:

$$\frac{\text{(1972 report year claims settled 13-24 mos.)}}{\text{(1972 report year claims outstanding at 13 mos.)}} = \frac{.367}{1.000 - .477} = .702$$

$$.702 \times \text{(1973 report year claims outstanding at 13 mos.)} = .702 \times (1.000 - .502) = .349$$

Values for the remaining disposal rates were determined in a similar manner, utilizing the latest data for percentage of claims settled in each age group. A more mathematical projection technique, similar to that used in estimating claim costs, could have been used in determining disposal rates instead of that described above.

After determining the projected average claim costs and corresponding disposal rates, the estimated average incurred claim cost for a report year can be calculated. For example, the 1970 report year average incurred claim cost is determined by weighting the 1970 report year average claim costs in their respective age groups from Table 4 by the corresponding 1970 disposal rates from Table 6. Note that the first four products in the weighted average are actual observed values whereas the latter three are projections. The resulting estimated average incurred claim cost of $1,410 may be compared to the actual average claim cost incorporating the current claim department reserves ($1,421, as shown in Table 1). This difference ($1,421 − $1,410 = $11) is then multiplied by the total number of incurred claims (21,263 from Table 1) to determine the dollar redundancy for this reserve ($11 × 21,263 = $234,000). In this case, the actual average incurred claim cost exceeds the estimated value and the reserve is redundant by $234,000. If the estimated value had exceeded the actual average, the reserve would have been deficient. This redundancy, or deficiency, is the reserve equity. In the following table, the report year equity positions for the remaining years are calculated.

TABLE 7

Calculation of Report Year Reserve Positions

	Report Year				
	1969	1970	1971	1972	1973
Estimated Average Incurred	$ 1,253	$ 1,410	$ 1,493	$ 1,618	$ 1,679
Actual Average Incurred (at 12/73)	$ 1,259	$ 1,421	$ 1,508	$ 1,613	$ 1,637
Margin per Claim Incurred	+$6	+$11	+$15	−$5	−$42
Number of Claims Incurred	20,462	21,263	22,613	23,124	23,716
Report Year Resaerve Position	+$123,000	+$234,000	+$339,000	−$116,000	−$996,000

The current, overall equity position for this reserve can now be determined by adding the individual results (i.e., the last line in the table) obtained for the various report years. Accordingly, this reserve appears to be deficient by $416,000. It is interesting to note, however, that the equity positions vary greatly between the various report years, with the older claims being over-reserved and the more current claims being under-reserved. Information of this type may be useful in giving guidance to the claim department.

This same picture of slippage in the recent claims can be seen in a comparison of the estimated and actual average incurred claim costs set forth in Table 7. The estimated average (i.e., the first line of data in the table) for the 1973 report year is up 3.8% over the 1972 report year average. A greater increase would have been expected based on the 9.1% rate of inflation inherent in the estimate; however, the substantial acceleration in disposal rates serves to lower the estimated averages and the percentage increases from year to year. This 3.8% increase in the estimated average exceeds the 1.5% increase in the actual average (the second data line in the table), thereby indicating that the claim department may be establishing too low an average on the recent cases. Similarly, the 1972 report year estimated average shows an 8.4% increase over 1971, while the actual average increased only 7.0%. Graphing these estimated and actual average incurred claim costs, or the percentage increases from report year to report year, can be helpful in presenting the results of the report year test.

We have now determined the equity position of the December 31, 1973 reserve for this particular line. In order to obtain the strengthening, or slippage, in this reserve during 1973 we must first calculate the equity position of this reserve at December 31, 1972. An increase in reserve equity at December 31, 1973 indicates a strengthening during 1973, while an in-

crease in the reserve deficiency (or decrease in savings) indicates a slippage during the year.

Table 8 sets forth equity positions for the 1971, 1972, and 1973 year-end reserves. These equity estimates are obtained by adding the savings already emerged for the report year to the current estimate of the savings to emerge in the future. The savings already emerged would be determined from report year information similar to that set forth in Table 1. Specifically, subtracting the 1970 report year dollars incurred as of December 31, 1973 ($30.2 million) from the dollars incurred as of December 31, 1971 ($26.9 million) yields the emerged savings on the December 31, 1971 reserve (−$3.3 million).[4]

Examining Table 8, we notice a substantial strengthening ($3.6 million) for this reserve during 1973 (going from a deficit of $4.0 million at December 31, 1972 to a current deficit of $0.4 million) following a slippage ($0.7 million) during 1972 (from a deficit of $3.3 million to a deficit of $4.0 million). These movements in the level of reserve equity have a direct effect on underwriting results, and accordingly could be used to restate such results on a more accurate basis.

It must be noted that this methodology, as described in the example, contains two potential sources of distortion: reopened claims and partial claim payments.

Reopened claims, if included in the report year data, will distort both the disposal rates and the average claim costs. For those lines with a substantial volume of reopened cases, such as workmen's compensation, the approach described in the example can be appropriately modified. However, since reserves are frequently maintained and tested separately for reopened cases, excluding these cases from the report year data developed for this test is probably the best solution.

[4] Equity positions are not calculated for report years prior to 1969. This methodology is not sensitive to report years which are already at 72, or more, months of development. Equity positions for these report years could be determined either by increasing the number of age groups—73 to 84 months, 85 to 96 months, etc.—or by projecting an estimated average incurred loss by applying a development factor to the average paid loss to date. In practice, the method is extended for certain slow closing lines. For this example, we will assume these early report years to be correctly reserved (no inadequacy or redundancy).

TABLE 8

Reserve Equity Position

($000)

Report Year	Outstanding Reserve (at 12/71)	Emerged Savings (as of 12/73)	Current Position	Reserve Position
	December 31, 1971 Reserve			
1968 and Prior	7,576	+ 110	—	+ 110
1969	8,724	+ 400	+123	+ 523
1970	12,916	−3,299	+234	−3,065
1971	18,432	−1,213	+339	− 874
TOTAL	47,648	−4,002	+696	−3,306
	December 31, 1972 Reserve			
1968 and Prior	3,566	− 360	—	− 360
1969	5,201	− 300	+123	− 177
1970	8,762	−1.522	+234	−1,288
1971	14,671	−1,843	+339	−1,504
1972	20,472	− 567	−116	− 683
TOTAL	52,672	−4,592	+580	−4,012
	December 31, 1973 Reserve			
1968 and Prior	904	—	—	—
1969	1,832	—	+123	+123
1970	5,154	—	+234	+234
1971	8,246	—	+339	+339
1972	15,125	—	−116	−116
1973	23,761	—	−996	−996
TOTAL	55,022	—	−416	−416

Partial claim payments also can be handled by modifications in the reserve test. In practice, this is done only for workmen's compensation, where partial payments are prevalent, although similar modifications may become necessary in auto bodily injury as No-Fault begins to exert greater influence. Partial payments involving allocated loss adjustment expense must also be considered in lines where allocated expenses are a major factor. Performing the test on pure indemnity data would, of course, remove this problem.

By summarizing algebraically the report year methodology described above, an alternative method of estimating the average claim costs can be derived. The procedure described thus far makes independent estimates of the average claim costs for each age of claim. The alternative method, on the other hand, estimates these average claim costs simultaneously for all ages of cases.

In making this summary, we will refer to cases settled within the first twelve months of the beginning of the report year as being in age group 1, cases settled within the thirteenth to twenty-fourth month after the beginning of the report year as being age group 2, and so on.[5] In the following notation, the age group will be denoted by the first subscript $_i$, while the particular report year (e.g., report year 1972) will be denoted by the second subscript $_t$.

n_{it} = number of cases closed in *i-th* age group in report year t

g_{it} = observed disposal rate for *i-th* group in report year t

$$= \frac{n_{it}}{\sum_{j=1}^{h} n_{jt}}$$

where h simply represents the number of age groups into which the report year data has been divided.

X_{it} = paid claim cost for *i-th* age group in report year t

$\overline{X}_{.t}$ = ultimate average paid claim cost for report year t

$$= \sum_{i=1}^{h} g_{it} X_{it}$$

[5]This section will assume that only annual subdivisions of the data are available, although quarterly subdivisions are used in practice.

Of course, X_{it} is unknown[6] for $i > 1974\text{-}t$ in our problem where our latest complete report year is 1973. Thus, estimates of X_{it} must be used in computing $\overline{X}_{\cdot t}$. This latter quantity can be compared to the actual incurred average claim cost, $\overline{W}_t$, carried on the company books, where

$$\overline{W}_t = \frac{\text{Total incurred losses for report year } t}{m_t}$$

m_t = number of cases reported in year t, and

$m_t\,(\overline{W}_t - \overline{X}_{\cdot t})$ = future runoff savings (or deficit if minus) for the report year t reserved at the end of the current year.

Note that, for the current report year, the future savings equal the reserve position for the current reserve. In reevaluating reserves for prior calendar years, the emerged savings to date for that reserve must be considered:

e_c = emerged runoff savings (deficit if minus) on reserve for calendar year c at the end of the current year.

d_c = reserve position for reserve for calendar year c

$$= e_c + \sum_{t=T}^{c} m_t\,(\overline{W}_t - \overline{X}_{\cdot t}),$$ where T is the first year of data included in the analysis.

s_c = strengthening (slippage if minus) of the reserves during calendar year c

$$= d_c - d_{c-1}$$

The goal of our test was to compute d_c and s_c. The first quantity tells us about the equity position of the reserves and is related to the company's financial solvency. The second quantity gives us the dollar impact of reserve movements (change in reserving policy) on the income account for the year. Specifically, by dividing s_c by the earned premium for the line, the impact of a change in reserve policy may be expressed in terms of points of loss ratio.

We now have summarized the methodology as explained in the pre-

[6] The disposal rates, g_{it}, are also unknown. However, in this section, it is assumed that the g_{it} are relatively stable and, when necessary, may be estimated using the method described in the prior section.

vious section and can return to the problem of estimating the unknown X_{it}. We shall use a different weighting and reorganize the data so that the known (as opposed to the estimated) X_{it} can be combined into a calendar year average claim cost, $\overline{U}_{\cdot c}$ where

$$\overline{U}_{\cdot c} = \frac{\sum_{i=1}^{h} n_{i,\, c-i+1} X_{i,\, c-i+1}}{\sum_{i=1}^{h} n_{i,\, c-i+1}}$$

As defined above, the $\overline{U}_{\cdot c}$ are known quantities since the specific X_{it} is selected so that $i \leq 1974\text{-}t$. We now may consider the relative cost, r_{ic} where:

$$r_{ic} = \frac{X_{i,\, c-i+1}}{\overline{U}_{\cdot c}}$$

It may be noted that these r's reveal the relationship of age of claim to the relative cost of settling the claim. This is in accordance with the assumption mentioned earlier that claims settled in a particular age group (e.g., 13 to 24 months) are similar-type claims and can be compared from report year to report year.

Having reorganized the data, we shall now proceed to use the restructed data to estimate the future X_{it}'s. We will do this by decomposing the known average claim costs, X_{it}, into three components: the impact of inflation, the effect of age of claim, and the general level of costs for the line.

Specifically, to measure inflation let us assume that there exists for each year some underlying rate of increase in claim costs, y_c, which is expressed as an index with the latest calendar year indexed as unity. Furthermore, assume that over any span of a few years, the effect of age of claim, the r_{ic}'s are dependent only on age group and not on the calendar year of observation; hence we will replace r_{ic} by r_i in subsequent equations.

Finally, let us define a scalar B representing the average paid claim cost for the latest calendar year so that the r's and y's appear as indices. This allows us to obtain an estimate $\tilde{X}_{it}$ of the actual average claim cost for a particular age group and report year (X_{it}, where $i \leq 1974 - t$ and $t \leq 1973$):

$$\tilde{X}_{it} = B r_i y_{t+i-1} \text{ and } X_{it} = \tilde{X}_{it} + \xi$$

where ξ is an error term for the difference between the observed value X_{it} and the computed value $\tilde{X}_{it}$. Given five years of data, an h by 5

matrix is formed in which the X's are the entry values. Using the iterative techniques described by Bailey and Simon,[7] we may now solve for the best set of r's and y's (denoted $\hat{r}$'s and $\hat{y}$'s) to minimize the sum of the squares of the error term ξ.

An average annual increase in claim costs, q, can be determined from the $\hat{y}$'s. In practice, we use least squares to fit the $\hat{y}$'s to an exponential curve, although other functional relationships could be utilized.[8]

Using this value for q, and the $\hat{r}$'s determined above, projected average claim costs $\hat{X}_{it}$ can be determined for those claims which will be settled in the future:

$$\hat{X}_{it} = B\hat{r}_i\, q^{t+i-1974}$$

These projected values can then be used along with the actual claim costs, X_{it} where $i \leq 1974 - t$ and $t < 1973$, to determine the ultimate average paid claim cost $\bar{X}_{\cdot t}$ as described earlier. Using the projected values $\hat{X}_{it}$ from this technique, one develops a second estimate of the reserve position.

There are two advantages to this approach. First, all of the data are used simultaneously in computing the projection of the average costs instead of subdividing our data by age and making h separate projections. Second, a number of different values of q can arbitrarily be used to compute the projected $\hat{X}_{it}$ so that the sensitivity of d and s to changes in q can be observed. This is relatively simple to do in practice since, after computing B and r, all values other than q are known; hence, d and s may be expressed as polynomial functions of q of degree h, then numerically evaluated.

It must be emphasized that this particular report year approach is but one way of testing reserve adequacy and no single test is completely reliable. Actuaries would wish to use more than one testing procedure to assess the position of the reserves. In our opinion, the report year method has much to recommend itself as one such procedure.

---

[7] The difference between the observed claim cost and the product of the age relativity and the calendar year index and the constant B was minimized using the "minimum Chi-Square" technique described in Bailey, R. A. and Simon, L. J., "Two Studies in Automobile Insurance Ratemaking" *PCAS* Vol. XLVII, pp. 11 and 12.

[8] In practice, for certain lines the problem must be divided into two parts (e.g., cases less than 3 years old and cases more than 3 years old) to compute two distinct q's in order to obtain a reasonably good fit to the actual data.