AN ACTUARIAL NOTE ON EXPERIENCE RATING NUCLEAR PROPERTY INSURANCE

RICHARD D. McCLURE

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DISCUSSION BY ROBERT L. HURLEY*

There may possibly be some danger that in his impatience to push right into the subject and offer a few thoughts of his own thereon, a reviewer might occasionally be unmindful of his responsibility first to identify the author's argument and evaluate it objectively. With this prime duty, however, once discharged, the reviewer should be free to comment as his conscience dictates and the indulgence of the reader may be expected to reasonably allow.

Our Society is singularly indebted to Mr. McClure for his papers on nuclear energy insurance. His previous paper[1] chronicled for the future student the first decade of the insurance industry's truly remarkable response to the economic and social challenges posed by the peace-time use of nuclear energy. The current McClure paper treats of the insurance industry's plan to relate its prices for nuclear energy property insurance to the developing loss and expense experience.

This "Actuarial Note on Experience Rating Nuclear Property Insurance" presents the background leading to development of the rating plan by the Actuarial Subcommittee of the Nuclear Insurance Rating Bureau; various of the particulars considered by the Subcommittee; the details of the resulting plan and a specific example of its application. Mr. McClure has again contributed a comprehensive and easily readable document and a definitive paper to the CAS *Proceedings*.

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* The relevancy of E. J. Gumbel's *Statistics of Extremes* published in 1958 by Columbia University Press to the actuarial problem of "rare events" was suggested by Thaddeus L. Smith, ISO Mathematical Statistician, but the reviewer reserves as his own any faults in the application thereof.

[1] R. D. McClure, "A review of Nuclear Energy Insurance", *PCAS* Volume LV, (1968) pp. 255-294.

Experience rating of atomic energy losses—would not any such proposal be unthinkable to the earlier generation of casualty actuaries, the Rubinows, the Flynns, the Mobrays? But in all fairness, let us not demean the imaginative skills or the ability of our predecessors to come up with solutions to pressing social and economic problems.

The implications of the horseless carriage, insurance-wise, must have been unfathomable to all but the very few. Who else could have guessed the impact of the social significance of the first workmen's compensation laws under whose shadow, or better promise, our Society was founded and has since flourished? Where otherwise could one have turned to satisfy the needs for aircraft liability and property insurance?

And in stride, the industry has successfully coped with problems presented by hurricanes, earthquakes, flood and social unrest. Why should the atomic energy hazard prove the first instance in which we fail?

With remarkable candor, Mr. McClure draws the reader's attention to the considerations that were given to the practices in other property insurance fields in the process of formulating rating standards for nuclear property insurance. He notes that it was decided to treat the first \$5 million of any nuclear property insurance loss as normal experience with the equivalent of 50% weight in the overall rate level, although there was some evidence that losses of such magnitude might reasonably be expected to account for a significantly larger portion of commercial fire losses.

And the selection of the 20 year review period for excess losses was made not without advertence to the then operative critera for extended coverage insurance. The credibility formula, $P/(P+K)$, adopted for excess nuclear property insurance has long been a hallowed tradition for many casualty and property lines stemming from the early researches in our Society.

It has sometimes been observed that no great misfortune would likely stem from giving some modest credibility, say 10%, to an individual account's experience. And in this instance of nuclear property damage insurance, the credibility is not being applied to a single account for a relatively brief time interval, but rather to all accounts, over all states, for a not insignificant period of years.

We need not possess any profound discernment to appreciate the potential vexations of the business assured who has paid considerable premiums, year after year, and witnessed substantial underwriting margins with no resulting reduction in his rates. Such an insurance purchaser recognizes the prime importance of financial stability, but he often expects that continuing favorable underwriting results should ultimately be reflected in rate levels.

It is probably doubtful that the property insurance industry can yet come up with solutions to the credibility problem of rare events that will, at the same time, prove workable for the practical situations confronting us and also satisfy the rigorous standards customarily required by statistical theory. While there has been no flagging in the energy with which scholars have pursued their research, it is not unlikely that Gumbel's 1958 work still serves as benchmark against which all subsequent theoretical contributions are measured on the very difficult questions of the significance to be attached to the occurrence of rate events in the sense of large losses.

In the introductory sections of his magnum opus, which represents a veritable lifetime of intellectual devotion to the subject, Gumbel starts with the Intensity Function from which he develops the Distribution of Repeated Occurrences. The latter is not unrelated to the Pascal or Geometric Distribution which has underscored certain developments of considerable significance to Casualty Actuaries as may be typified by the Dropkin paper[2] concerning the use of the Negative Binomial and other equally distinguished research by the various outstanding CAS pioneers in actuarial theory.

Gumbel presents the Intensity Function as

$$M(x) = \frac{f(x)\,dx}{[1 - F(x)]} \geq f(x)dx$$

where the probability of a value "equal to" or "larger than" x is $[1 - F(x)]$ and $f(x)\,dx$ is the probability of a value between x and $(x + dx)$ and cites as a reference the work of one of our past Presidents, L. H. Longley-Cook.

Gumbel then proceeds to develop the distribution of repeated occurrence of any Large Value x. He starts with q the probability of any occur-

[2] L. B. Dropkin, "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records", *PCAS* Vol. XLVI, p. 165.

rence smaller than the Large Value x: $q = (1 - p) = F(x)$. Observations are made regularly until the occurrence of a value "equal to" or "larger than" x. The chance that the Large Value x occurs at the *vth* trial equals pq^{v-1} and decreases as v increases and has a moment generating function $G_v(t) = p/(e^{-t} - q)$ which can be shown as follows:

$$G_v(t) = \sum_{1}^{\infty} pq^{v-1} \cdot e^{vt} = \frac{p}{q} \sum_{1}^{\infty} q^v \cdot e^{vt}, \text{ and}$$

$$\sum_{v=1}^{\infty} q^v e^{vt} = (qe^t + q^2e^{2t} + q^3e^{3t} + \cdots)$$

$$= qe^t(1 + qe^t + q^2e^{2t} + \cdots) = qe^t(1 - qe^t)^{-1}$$

$$\text{thus: } G_v(t) = \frac{p}{q}\left(\frac{qe^t}{1 - qe^t}\right)$$

$$= \frac{pe^t}{1 - qe^t} = \frac{p}{(e^{-t} - q)}, \text{ the mean } \bar{v} = \frac{1}{p}$$

which Gumbel identifies as the Return Period $T(x) = \dfrac{1}{[1 - F(x)]} > 1$.

Thus if in any given year a nuclear energy catastrophe equal to or greater than a given magnitude is 2%, the Return Period is 50 years. Thus, Return Period is the reciprocal of the chance of a loss of a given size.

Students of actuarial mathematics, and even occasional dabblers therein like this reviewer, will recognize the similarity of Gumbel's development with Pascal-Fermat "Problem of Points" as detailed in the Probability Chapter in Hall and Knight[3]. This approach also parallels the development of the Geometric Series which, as Feller shows[4], has a variance equal to $\dfrac{q}{p^2}$.

[3] H. S. Hall and S. R. Knight, *Higher Algebra* (MacMillan and Co., London).

[4] W. Feller, *An Introduction to Probability Theory* (John Wiley & Sons, Inc., New York), Vol. I.

And using Gumbel's relationships above: $q = (1 - p) = F(x)$ and $T(x) = \dfrac{1}{[1 - F(x)]}$ we develop a Standard Deviation σ for the Return Period $\sqrt{T^2 - T}$ as follows:

$$\sigma^2 = \frac{q}{p^2} = \frac{F(x)}{[1 - F(x)]^2} = \left[\frac{T-1}{T}\right] \cdot \left[\frac{1}{T^{-2}}\right] = (T)(T-1)$$

$$\therefore \sigma = \sqrt{T^2 - T}$$

Gumbel notes that the cumulative probability of the event "at" or "before" the *vth* trial is $W(v) = (1 - q^v)$, or in terms of the Return Period, T (i.e. equals $[1 - q]^{-1}$):

$$W(v) = 1 - \left(\frac{T-1}{T}\right)^v = 1 - \left(1 - \frac{1}{T}\right)^v$$

If x is large and p is small, T will be large and $W(v)$ may be written $(1 - e^{-\frac{v}{t}})$

$$\underset{T \to \infty}{\text{limit}}\, W(\mu T) = \left[1 - \underset{T \to \infty}{\text{limit}} \left(1 - \frac{1}{T}\right)^{\mu T}\right] = (1 - e^{-\mu})$$

Hence, the approximation for large T

$$W(T\lambda) - W(T/\lambda) = e^{-1/\lambda} - e^{-\lambda}$$

If we wish to select a $P = 0.9545$ which for the normal curve corresponds to an interval of 2 sigmas about the mean, v would be expected to fall within an interval equal approximately to .05 to 21.5 times the return period. And if the expected annual probability of a \$25 million nuclear property damage loss were 2 percent, the range within which such an event would occur would be 2.5 years to 1,075 years.

While the thoughtful reader will likely concede that more than a modicum of genius and scholarship must have been exercised in devising these forecasting techniques, he may wonder how such findings might be used to establish actuarially based rates for a "rare event" insurance commitment. In fairness to these scientists, it must be recognized that their prime interest gravitated toward pure research into mathematical techniques. They had not been charged with the responsibility of developing actuarial based insurance rates. It would be most unfortunate, however, if the above observations were to be construed that such mathematical research might be dismissed cavalierly as of no concern for us. This would be misreading of our intent.

Possibly a fair appreciation of some of the limitations in our presently developed mathematical forecasting techniques might be obtained from an August, 1954 article in the *Proceedings of the American Society of Civil Engineers*,[5] by H. Alden Foster who has been mentioned as one of the pioneers into mathematical flood forecasting techniques. After admitting that an efficient forecasting service would be helpful in a flood insurance program, Foster stated that he had some doubts as to how much reliance could be placed on present methods. He then proceeded to cite ten different types of floods, some of which would be difficult, or even impossible, to forecast from a probability standpoint.

In the October, 1961 *Journal of the Boston Society of Civil Engineers*, the report of the Committee on Floods cites a number of the statistical formulas which have variously been used for computing the recurrence intervals of floods. While the New England flood records extend back over a respectably long period of years (*i.e.*, over 325 years in some instances), the reader may well get the impression that the predictability of large floods must not be considered amenable to the same statistical precision as in the case of the small flood traceable to less uncommon meteorological conditions.

And it may be somewhat awesome to learn that if we had 10,000 years of records, we could expect 100 large floods, but if these 10,000 years were divided into 100 centuries each, on the average 37 of these centuries would have no such flood, 37 would have one, about 18 of the centuries would experience two floods, 6 would have three, and about 2 of the centuries would have four or more large floods. But it would be impossible to predict into which of these centuries each of these frequencies would occur!

It may seem to the reader that we have reached an impasse. The establishment of any actuarial based system of rates, it has been observed, demands a credible volume of statistics designed for the specific underwriting enterprise with advertence to the possible classification breakdowns for significant differential in loss expectancies.

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[5] Volume 80, Separate No. 483.

It is not always possible to launch a new insurance enterprise with a system of detailed classification rates substantiated by credible statistical experience. But it is a common experience in insurance, as well as in other commercial and industrial enterprises, to set a system of prices (or rates) on a judgement evaluation of all available information, including whatever statistics that may be helpful to the purpose, and then to adjust the rating schedule as the subsequent experience indicates revisions are warranted.

We are indebted to Mr. McClure for keeping us abreast of current developments in nuclear energy insurance and for affording us an insight into the first glimmerings of sound actuarial rating techniques for nuclear energy property losses. Let us hope that the Society will be equally fortunate in that the next generation of actuaries will have an equally competent and experienced chronicler to record the insurance industry's further contributions to sound actuarial rating of nuclear energy property insurance.