JOINT UNDERWRITING AS A REINSURANCE PROBLEM

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DISCUSSION BY WALDO A. STEVENS

Mr. Strug is to be commended for his fine effort in handling a very difficult reinsurance problem. The usual proportional reinsurance arrangement (on a coinsurance basis) is not applicable here due to the criterion that the Dental Plan maintain a contingency reserve, R_E, no less than 10% (W) of its portion of the annual incurred losses on premium accounts, $X \bullet L_p$, in order to preserve a reasonably acceptable (to potential clients and state insurance examiners) financial statement position.

In view of the fact that Mr. Strug's formula for the value of X, the percentage retention of the Dental Plan, is overburdened with numerous variables, the following stubstitutions will prove helpful in delineating the primary variables from those which may be considered, more or less, as parameters:

$$
\begin{array}{ll}
D_p = f^D \bullet P_p & 0 < f^D < 1 \\
E_p = f^E \bullet P_p & 0 < f^E < 1 \\
A_p = k \bullet D_p = (k \bullet f^D) \bullet P_p & k > 0 \\
L_p = Q \bullet P_p & \\
I = f^R \bullet R_B &
\end{array}
$$

Since the cost of the servicing agreement between the Dental Plan and the Hospital Plan was set as a percent of premium, we may assume the value of f^E to be predetermined at the beginning of each calendar year (this may not be exactly true, since a contract year between insurer and reinsurer is not likely to coincide with a calendar year, but for illustrative purposes our assumption is not unreasonable). Hence, any favorable or unfavorable expense experience associated with E does not affect the value of X.

Because D is a conserative estimate of the direct expenses borne entirely by the Dental Plan, f^D may be considered as a constant for any given calendar year. A, the actual direct expenses incurred by the Dental Plan, is expressed as a percentage, 100k%, of D. Generally k lies somewhere between 0 and 1.00. By its nature, k must be determined at year-end.

Investment income, I, may be represented as a percentage, $100f^R\%$, of the contingency reserve at the beginning of the calendar year, R_B. In essence, f^R represents an interest rate which must be determined at year-end. Normally, $0 < f^R < 1$, but it may be negative or greater than 1. Q, of course, represents the loss ratio.

The revised formula for X is:

$$X = \left[\frac{1}{Q \bullet (1+W) + f^D + f^E - 1}\right] \bullet \left[\frac{(1+f^R) \bullet R_B + U_C + P_P \bullet f^D \bullet (1-k)}{P_P}\right]$$

The beauty of this formula lies in the fact that it is much easier to isolate the key variables affecting the value of X. For any given calendar year we know in advance the values of W, f^D, f^E and R_B. Hence, the value of X may be expressed as a function of f^R, U_C, P_P, k and Q.

Since we are dealing with one year term insurance, and since f^R and k are reasonably stable from year to year, we may treat these variables as parameters.

An analysis of how fluctuations in the remaining variables affect X reveals that X is most sensitive to Q, moderately sensitive to P_P and generally insensitive to U_C (one wouldn't expect U_C to be very large in relation to P_P since all of U_C must come from favorable expense experience on the cost-plus business). What we find most interesting, though, is that X varies inversely to Q *and* P_P. Hence, if the loss ratio was unexpectedly large in a given calendar year, the retention of the Dental Plan would be less than if the loss ratio had been smaller. assuming all other factors were equal. Although from the viewpoint of the Hospital Service Plan this appears to be a "Heads you win, tails I lose" proposition, one must keep in mind that the loss ratio need not be unexpectedly large. A redundant premium per unit of exposure, though less competitive (and can competition be much of a factor in this market?) is of extreme importance, especially to the Hospital Service Plan.

In contrast, due to the fact that X varies inversely to P_P, the Hospital Plan actually gains if earned premium income increases (either via a rate per unit increase—assuming no significant loss in volume of business—or via a volume increase) and all other factors remain within reasonable range of expected values. This retention pattern helps to compensate, may even overcompensate, for the fact that the Hospital Service Plan stands to gain little and lose much under the first retention pattern (X decreases as Q increases). Thus, the price the Dental Plan pays for their protection is in terms of an exceedingly slow rate of growth.

In due time, as the volume of business increases and as the business on the books matures (loss ratios stabilize), the point may be reached where the Dental Service Plan has a year-end surplus which equals or exceeds 10% (W) of the incurred losses on all of the premium accounts business, in which case the Dental Plan's retention will reach 100%. However, the moment at which total recapture is achieved may be so far in the future that the reinsurance agreement becomes a losing proposition as far as the Dental Service Plan is concerned. As a possible solution, a time limit on the agreement could be resorted to; but, understandably, it would be extremely difficult, if not impossible, to determine an equitable limit based upon actuarial methods. A "gentleman's agreement" may be the best recourse.

AUTHOR'S REVIEW OF DISCUSSION

I concur with Mr. Stevens' comment that the formula as it appears in the paper is somewhat overburdened with numerous variables. Most of these variables were introduced to represent those monetary elements which could affect the operating results of the Dental Plan.

In developing the initial approach to this reinsurance problem, the basic formula contained only those elements which were critical in establishing whether the sharing between the two corporations was equitable and feasible.

The first pass at the formula presumed no income from investment (I) or a gain from any cost plus (U_c) operation. In addition, actual direct expense (A_p) was set equal to expected direct expense (D_p). A further simplification is possible as the Direct Expense (D_p) and Indirect Expense (E_p) allowances in the rates are known. In our example, these were set at $.18\ P_p$. W is also known and was set at .10.

Introducing these simplifications into the formula for X produces the following:

$$X = \frac{R_B}{L_p(1.10) + .18P_p - P_p} = \frac{R_B}{1.10\ L_p - .82\ P_p}$$