A SURVEY OF LOSS RESERVING METHODS

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INTRODUCTION

Proper loss and loss expense reserves are vital for an insurance company, both for financial security and for the production of accurate income statements. The readings for the loss reserving section of the Casualty Actuarial Society examinations describe dozens of different methods for estimating loss and loss expense reserves. Some of these methods are in common use, others are not used at all. Some are in universal use, since they are required in the Annual Statement. Some are quite complex, others are extremely simple. Some are explicitly described in every detail, others are merely outlined. Underlying most of these methods are a few basic principles which have been combined in different ways, sometimes with different terminology.

The primary purpose of this paper is to describe the various loss and loss expense reserving methods using consistent terminology, and to explain the relationships between them. I will also clarify the assumptions underlying the methods. However, I have not compared the methods for accuracy. Such a judgment should be based upon detailed studies of the various methods, covering how well the assumptions are satisfied, the effect of data errors, inflation, large losses, the amount of premiums or losses required for credibility, the expense and difficulty involved in applying the method, and the accuracy of the method as demonstrated by its use with actual data. Perhaps this paper will encourage such studies. In their absence, I have made a few critical comments, confining them to pointing out the inconsistency of certain assumptions with actual data.

Like any statistical analysis, reserving requires grouping the data into appropriate categories, which should be homogeneous but large enough to be credible. More categories also means more labor. The definitions of the categories must therefore vary from company to company. All the loss reserving methods can be applied to any such category, although some methods are recommended as being particularly applicable to certain types of claims. This paper will not cover the proper categorizing of claims by line, class, or geographic regions, but it does examine the categorizing of claims by time units.

Types of Reserves

The total loss reserve for a line of business as of a given date is a liability which should equal the amount of paid loss that will be required to settle all claims which took place prior to the date, not including payments already made. Insurance companies are required by law to carry adequate total loss reserves on the company books. The reserve on the company books will be referred to as the carried total loss reserve.

Of course, it is extremely unlikely that the carried total loss reserve will ever be the precise amount necessary to settle all the claims for which it is meant to provide. We will refer to the precise amount necessary to settle all claims which have taken place as the required total loss reserve. The required total loss reserve, as of a point in time, cannot be known until many years later. A reserving method will produce an estimated total loss reserve. We can estimate current reserves, past reserves, or future reserves. A good reserving method will produce an estimated total loss reserve which is close to the required total loss reserve. An insurance company's carried total loss reserve will generally be set equal to its currently estimated total loss reserve. By definition, the carried total loss reserve as of a given date and the required total loss reserve as of a given date can never change (although the required total loss reserve is generally unknown) but the estimated total loss reserve as of a given date will change with time. Estimates become more accurate (i.e. closer to the required total loss reserves) with the passage of time as more data becomes available.

The total loss reserve provides for payments subsequent to a given date on claims occurring prior to this date. This date is called the reserve date. The evaluation date for a reserve estimate means the date of the most recent accounting or statistical data entering the calculation. Reserve estimates can be categorized as prospective or retrospective. An estimate is retrospective if the evaluation date is later than the reserve date and is prospective if the evaluation date is equal to or earlier than the reserve date.

A reserve test refers to a comparison of an estimated reserve with a carried reserve. The developed reserve is another name for an estimated reserve when the estimate is retrospective.

The total loss reserve can be divided into the reserve for known claims and the incurred but not reported reserve (IBNR). The reserve for known claims represents the amount of paid loss that will be required to settle all reported claims not including payments already made on these claims. The IBNR reserve represents the amount of paid loss that will be required to settle all incurred but not reported claims. Like the total loss reserve, these two reserves can be discussed in terms of required, carried and estimated. The concepts of reserve date, evaluation date, prospective and retrospective estimate, developed reserve, and reserve test all apply to the IBNR and reserve for known claims.

The reserve for known claims is also referred to as the Unpaid Losses Excluding Incurred But Not Reported (Annual Statement Part 3A), the regular reserve, the reserve for claims adjusted or in the process of adjustment and the case reserve. The IBNR is sometimes referred to as the bulk reserve (bulk reserve more commonly refers to any loss or loss expense reserve which is a bulk amount, rather than the sum of individual case reserves). Incurred but not reported claims are referred to as late-reported claims.

The expression "IBNR" is sometimes used to denote a gross IBNR which includes provision for both late reported claims and the deficiency in the reserve for known claims. If the reserve for known claims is redundant, this gross IBNR would denote a reserve for late reported claims minus the redundancy in the reserve for known claims. For example, Bornhuetter and Ferguson recommend this usage.[1] This definition says,

$$\begin{pmatrix}\text{Required gross}\\ \text{IBNR}\end{pmatrix} = \begin{pmatrix}\text{Required total}\\ \text{loss reserve}\end{pmatrix} - \begin{pmatrix}\text{Carried reserve}\\ \text{for known claims}\end{pmatrix}$$

In effect, the error in the total loss reserve becomes fully attributable to the gross IBNR.

In this paper IBNR will always denote the reserve purely for incurred but not reported claims. This usage is consistent with the name of the reserve and, I believe, leads to a clearer exposition.

There is a lag between the date a claim is first reported to an insurance company and the date it is recorded on the company books. It is actually the recorded date which distinguishes whether a claim is provided for in the IBNR or the reserve for known claims. For this reason some people recommend that IBNR be used to abbreviate incurred but not recorded. The incurred but not recorded reserve could be divided into an incurred but not reported reserve and a reported but not recorded reserve. However, under the usual terminology, the word reported is used to mean recorded on the company books. This paper throughout employs the usual terminology.

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[1]R. L. Bornhuetter and R. E. Ferguson, "The Actuary and IBNR", *PCAS*, Vol. LIX, p. 181.

METHODS OF ESTIMATING LOSS RESERVES

RESERVE FOR KNOWN CLAIMS

Individual Case Estimates

An individual case estimate or per case reserve is the value assigned to a specific claim by a field adjuster or home office claims department official based upon an investigation of the claim. This estimated value of the claim, referred to as the gross case reserve, may be revised as more information is discovered. The net case reserve is the gross case reserve less partial payments. It represents the current claims department estimate of the payments remaining to be made on the claim.

The sum of the individual net case reserves for all open claims in a line or other category provides an estimated reserve for known claims for the category. Many companies use this estimate as their carried reserve for known claims; when they do, their carried reserve for known claims is called the case reserve.

The assumption of the individual case estimates method is that the claims department can accurately evaluate a claim. If a large number of claims is included in the given category, over-reserved claims and under-reserved claims will tend to compensate for one another, and the case reserve will be fairly accurate as long as there is no bias in the individual reserves.

Fast Track Reserves

Like the individual case reserve, the fast track reserve is also applied on a claim by claim basis. The fast track reserve is the estimated average value of a claim in the category. If a claim remains open beyond a certain length of time, an individual case reserve will be substituted for this average value.

The use of fast track reserves represents a saving of effort over the use of individual case reserves since many claims will close before they need to have an individual case reserve established for them. Fast track reserves can be accurate if the average value used is accurate. This average value is based upon the average value of similar claims from earlier years. Fast track reserving is appropriate for lines of insurance whose claims, such as auto collision, are similar in size.

Tabular Value

Tabular value reserves are used for certain claims under accident and health or workmen's compensation. Each individual case reserve is taken from a table. For example, a workmen's compensation reserve for benefits to a widow and dependent children arising from a death claim would be based upon tables reflecting the ages of the widow and children, the remarriage probability of the widow, and the benefit level in the state. The tabular reserve is a kind of fast track reserve with an average reserve applied to all claims in a category, where one particular category might be workmen's compensation death claims in New York state with a widow age 46 and dependent children ages 7 and 10. The accuracy of the method depends upon the accuracy of the tables and their applicability to a given company's claims.

Notice-Average Method

This method, as described by Michelbacher and Roos, is an accident year version of the Fast Track Method.[2]

Estimated 12/31/y reserve for known claims, evaluated as of 12/31/y =

$$\sum_{t \le y} \left[\begin{pmatrix} \text{No. of reported} \\ \text{claims for} \\ \text{accident year } t \end{pmatrix} \times \begin{pmatrix} \text{Estimated average} \\ \text{cost per claim for} \\ \text{accident year } t \end{pmatrix} - \begin{pmatrix} \text{Paid loss to} \\ \text{date on} \\ \text{accident year } t \end{pmatrix} \right]$$

The number of claims used in the formula is not subject to development, since unreported claims are provided for in the IBNR. The method does require an accurate value for the average claim. The value chosen should be based upon average claims from past accident years.

Average Value Method[3]

First estimate the average net case reserve of an open claim, then

$$\begin{pmatrix} \text{Estimated reserve} \\ \text{for known claims} \end{pmatrix} = \begin{pmatrix} \text{No. of open} \\ \text{claims} \end{pmatrix} \times \begin{pmatrix} \text{Estimated average} \\ \text{net case reserve} \end{pmatrix}$$

[2] G. F. Michelbacher and N. R. Roos, *Multiple Line Insurers, Their Nature and Operation* (McGraw-Hill Book Company, Missouri, 1970)

[3] Michelbacher and Roos, *op. cit.*

An estimate of an average value for a particular time grouping of claims should be based on data specifically for that type of time grouping. For example, to estimate the average net case reserve over all claims open at a point in time one should use the average net case reserve over all claims open at an earlier point in time. The average cost of a claim over an accident year as used in the notice-average method is often less than the average net case reserve as used in the average value method. The reason is that for some lines of business, smaller claims close faster than larger ones. The set of open claims at a point in time includes a large share of slow closing larger claims, so the average gross case reserve at a point in time is much larger than the average accident year claim, and even the average net case reserve may be larger than the average accident year claim.

A numerical example will clarify this point. Assume that there are five claims each year, all occurring on June 1. The distribution of sizes and closing times is:

No. of Claims	Closing Time	Average Amount
3	1 month	$100
1	1 year	400
1	2 years	700

Average accident year claim = $1,400 ÷ 5 = $280.

As of any December 31, a $400 claim and a $700 claim from the current accident year will still be open, along with a $700 claim still open from the previous accident year. The average gross case reserve as of December 31 is $600 [($400 + $700 + $700) ÷ 3].

Schedule P, Part 1—Total Suit Liability is an application of the average value method. The number of suits remaining is shown for each accident year and a net reserve amount per suit is specified by the Annual Statement depending upon the age of the accident year. (For 1968 and prior, policy year is used rather than accident year because Schedule P was formerly on a policy year basis.)

Runoff

A runoff is an estimate of a past reserve. The reserve for known claims can be described as the anticipated future payments on known claims. The runoff estimate is

$$\begin{pmatrix}\text{Actual future payments}\\ \text{on known claims}\\ \text{up to a given date}\end{pmatrix} + \begin{pmatrix}\text{Anticipated remaining future payments}\\ \text{on known claims}\\ \text{as of the given date}\end{pmatrix}$$

More precisely, the runoff estimate of the reserve for known claims as of 12/30/70 evaluated as of 6/30/72 is

$$\begin{pmatrix}\text{Paid loss}\\ \text{during the period 1/1/71 through 6/30/72}\\ \text{on all claims}\\ \text{reported prior to 12/31/70}\end{pmatrix} + \begin{pmatrix}\text{Remaining reserve}\\ \text{as of 6/30/72}\\ \text{on all claims}\\ \text{reported prior to 12/31/70}\end{pmatrix}$$

After a sufficient length of time, this runoff becomes fully accurate, either because all claims reported prior to 12/31/70 are settled or because the remaining open claims are accurately reserved. It is not unusual for a partially developed runoff to be an inaccurate indicator. The two common patterns following show actual company data.

RUNOFF OF RESERVE FOR KNOWN CLAIMS 12/31/66

(000,000 omitted)

	Carried Reserve for Known Claims 12/31/66	Number of Months Development							
		3	6	9	12	24	36	48	60
Workmen's Compensation	67.2	66.9	66.3	65.9	66.3	69.2	70.6	70.7	70.7
General Liability BI	58.7	58.9	59.0	59.8	59.9	62.1	65.4	65.6	65.7

The workmen's compensation runoff first moved down and then up. Over-reserved claims tended to settle early, under-reserved claims did not have their reserves increased until somewhat later. The nine-month development made the tested reserve appear redundant, whereas it was actually deficient. The general liability BI runoff consistently moved up. A runoff of 24 months or less would not have shown the full extent of the reserve deficiency.

Schedule G of the Annual Statement is a runoff of the reserve for known claims for fidelity and surety, with the current year end as evaluation date and each of the prior seven year ends as reserve dates.

Correct Case Reserve for Bias

This method, as described by J. A. Scheibl, estimates the reserve for known claims by applying a correction factor to the case reserve. The bias in the prior case reserve is estimated by comparing it with the current estimate of the prior reserve for known claims. The assumption is then made that the same percentage bias exists in the current case reserve.[4] For example,

$$\left(\begin{array}{l}\text{Estimated 12/70}\\ \text{reserve for}\\ \text{known claims}\end{array}\right) = \left(\frac{\text{12/70 estimate of 12/69 reserve for known claims}}{\text{12/69 case reserve}}\right) \times \left(\begin{array}{c}\text{12/70 case}\\ \text{reserve}\end{array}\right)$$

This type of adjustment could alternately be applied to the gross case reserve.

$$\left(\begin{array}{l}\text{Estimated 12/70}\\ \text{reserve for}\\ \text{known claims}\end{array}\right) = \left(\begin{array}{c}\text{Correction}\\ \text{factor}\end{array}\right) \times \left(\begin{array}{c}\text{12/70 gross}\\ \text{case reserve}\end{array}\right) - \left(\begin{array}{c}\text{Paid to date}\\ \text{on open claims}\\ \text{as of 12/70}\end{array}\right)$$

In this case the correction factor is the ratio of the developed 12/69 gross case reserve to the carried 12/69 gross case reserve.

Report Year Loss Development

A report year consists of all claims reported in a given year regardless of accident year or policy year. We can use the report year incurred loss to calculate the reserve for known claims. For example, let us assume that all losses are settled within ten years of being reported. Then

12/70 required reserve for known claims =

Report year 1970 contribution to the reserve for known claims
+ report year 1969 contribution to the reserve for known claims
+ . . . . .
+ report year 1961 contribution to the reserve for known claims.

[4] J. A. Scheibl, "Developments in Formula Reserving Methodology", *Insurance Accounting and Statistical Association Proceedings,* 1970.

$$\begin{pmatrix}\text{Report year y}\\ \text{contribution to the}\\ \text{reserve for known claims}\end{pmatrix} = \begin{pmatrix}\text{Report year y}\\ \text{ultimate}\\ \text{incurred loss}\end{pmatrix} - \begin{pmatrix}\text{Report year y}\\ \text{paid loss to date}\\ \text{as of 12/70}\end{pmatrix}$$

This formula is a mathematical indentity. The key is the estimate of the ultimate incurred loss for past report years. Various approaches are recommended by Harnek[5] and Sampson[6] as well as in the Examination of Insurance Companies put out by the New York Insurance Department.[7] These methods will be discussed in the next two sections.

Projection Method

This method, as described by R. F. Harnek, suggests obtaining the estimated report year incurred loss from the paid loss to date, by applying a factor based on the past.[8] The New York Insurance Department Examination of Insurance Companies uses the term "Projection Method" to refer to a paid loss development by "Loss or Report Month, Quarter, or Year, or by Loss Year within policy year or by any other convenient grouping of the 'Time Elements'."[9] Here is a simple example to show how the projection method might be applied.

REPORT YEAR PAID LOSS DEVELOPMENT
(000 omitted)
Based upon Loss through December 31, 1971

Report Year	Age in Years			
	1	2	3	4
1968	1,000	1,100	1,210	1,210
1969	1,200	1,320	1,452	
1970	1,400	1,540		
1971	1,600			

For the sake of simplicity, assume that the reserve for known claims at age 3 years is always zero; that is, all claims are closed by the end of the third year of development.

[5] R. F. Harnek, "Formula Loss Reserves", *Insurance Accounting and Statistical Association Proceedings*, 1966.

[6] R. T. Sampson, "Establishing Adequacy of Reserves on Slow Closing Lines—Use of Paid Formulae", *Insurance Accounting and Statistical Association Proceedings*, 1959.

[7] New York (State) Insurance Department, *Examination of Insurance Companies*, Volume 3.

[8] Harnek, *op. cit.*

[9] New York (State) Insurance Department, *op. cit.*

This example was constructed so that the age-to-age development factors are:

One Year Factors		Ultimate Factors	
1 to 2	1.10	1 to 4	1.21
2 to 3	1.10	2 to 4	1.10
3 to 4	1.00	3 to 4	1.00

The report year incurred loss estimates are:

1968	$1210 \times 1.00 = 1210$
1969	$1452 \times 1.00 = 1452$
1970	$1540 \times 1.10 = 1694$
1971	$1600 \times 1.21 = 1936$

and the estimated reserve for known claims as of December 31, 1971 is 490. [(1936 − 1600) + (1694 − 1540) + (1452 − 1452) + (1210 − 1210)].

Also, if we assume that all claims in report years prior to 1968 were closed after three years, then we can obtain an estimated December 31, 1970 reserve for known claims which should be more accurate than the one we obtained using data only through 1970. The estimated reserve for known claims as of December 31, 1970 would then be 426. [(1694 − 1400) + (1452 − 1320) + (1210 − 1210)].

For most lines of insurance a great many years of development would be required for all claims in a report year to be paid. Therefore, it is common practice to carry the paid development to a certain age and use an incurred to paid factor at that age. For an illustration, the previous example can be modified by assuming that the report year incurred loss at age 4 is 1.1 times the report year paid loss at age 4. Then the one-year factors and ultimate factors would become:

One Year Factors		Ultimate Factors	
1 to 2	1.10	1 to ultimate	1.33
2 to 3	1.10	2 to ultimate	1.21
3 to 4	1.00	3 to ultimate	1.10
4 paid to 4 incurred	1.10	4 to ultimate	1.10

Under the revised assumption, there would have to be a contribution to the reserve for known claims as of December 31, 1971 from report years 1967 and prior.

The accuracy of the age-to-age factors chosen will determine the accuracy of the report year paid loss development. This principle also applies to policy year and accident year paid and incurred loss developments. For a given pair of ages, the average age-to-age factor over all report years is often used. In discussing accident year incurred loss development, Bornhuetter and Ferguson recommend a type of average: the sum of three years' losses developed to age n years divided by the sum of the same three years' losses developed to age n-1 years.[10] Trend may be reflected using either a judgment approach or a mathematical approach. When using a trend it is possible to project different one year age-to-age factors for different years. For example, in performing a policy year paid loss development, Balcarek fitted a least squares trend line to the one year age-to-age factors and generated a set of estimated one year age-to-age factors that vary by policy year.[11] The proper choice of age-to-age factors is important for ratemaking as well as reserving.

If all claims closed within a year of being reported, then the Projection Method formula for estimated reserve for known claims as of 12/71 would reduce to a factor multiplied by the report year 1971 paid loss as of 12/71. This is the form in which some authors present the Projection Method.

Payment Development Method

This method, devised by Sampson, is a report year loss development that utilizes the number of claims and their average values in order to estimate the report year incurred loss.[12] An average value method is simpler to apply to a report year than to an accident year or policy year because at the end of the report year all the claims are reported. There is no development in the number of reported claims, so the problem of estimating the reserve for known claims reduces to estimating the average size of claim within a report year.

Sampson uses an inductive method to estimate average size of claim. The inductive process begins with an average claim for an initial report year, which is old enough to be fully developed. The average claim for the report year following the initial year is calculated using the assumption that the percentage increase in ultimate average will be equal to the percentage

[10] Bornhuetter and Ferguson, *op. cit.*
[11] R.J. Balcarek, "Loss Reserving in the Sixties," PCAS Vol. LIX, 1972.
[12] Sampson, *op. cit.*

increase in average paid claim as of the same age. This assumption is successively used to obtain the estimated average claim for all report years up to the current one.

For example, we can estimate the ultimate average claim for report year 1958 with the following data (these figures are from Sampson's paper):

DATA THROUGH DECEMBER 31, 1958

Report year	1952	1953
Average paid claim through 6 years settlements	\$647	\$756
Estimated average claim	655	?

The percentage increase in average paid claim based upon claims closed within 6 years of the beginning of the report year is 17%. [(756 ÷ 647) − 1]. The estimated average claim for 1953 is 117% of \$655 or \$766. The same method is used to estimate the average 1954 claim from the average 1953 claim and so on to the current year.

Another method of estimating the average report year claim would be to assume that the individual case reserves are correct on all open claims. This produces the following formula that can be applied to each report year.

$$\begin{pmatrix}\text{Developed average}\\ \text{cost according to}\\ \text{case reserves}\end{pmatrix} = \frac{\begin{pmatrix}\text{\$ paid loss}\\ \text{to date}\end{pmatrix} + \begin{pmatrix}\text{\$ net case}\\ \text{reserves}\end{pmatrix}}{\text{No. reported}} \text{ or } \frac{\begin{pmatrix}\text{Report year incurred}\\ \text{loss to date}\end{pmatrix}}{\begin{matrix}\text{Report year No.}\\ \text{of claims}\end{matrix}}$$

Sampson's paper shows that this method was not as accurate as the Payment Development Method for his company's liability claims.

Incurred But Not Reported Reserves

Runoff

Like the runoff of the reserve for known claims, this is an estimate of a past reserve. It can be performed at any subsequent date. For example,

$$\begin{pmatrix}\text{IBNR Runoff}\\ \text{as of 12/31/70}\\ \text{based upon}\\ \text{development}\\ \text{through 6/30/72}\end{pmatrix} = \begin{pmatrix}\text{Paid loss during the}\\ \text{period 1/1/71—6/30/72}\\ \text{on all claims incurred}\\ \text{prior to 12/31/70 and}\\ \text{reported subsequent to}\\ \text{12/31/70}\end{pmatrix} + \begin{pmatrix}\text{Remaining reserve}\\ \text{as of 6/30/72}\\ \text{on all claims incurred}\\ \text{prior to 12/31/70 and}\\ \text{reported subsequent to}\\ \text{12/31/70}\end{pmatrix}$$

If the test permits too little time for development, it will definitely underestimate the required IBNR since many late reported claims will still be unreported. The minimum amount of time required for a reasonable development varies from company to company and from line to line. Three months might be enough in auto physical damage, whereas more than five years might be needed for excess general liability. After the passage of sufficient time, all claims incurred prior to 12/31/70 will be reported and settled and the test will become fully accurate.

IBNR Reserve as a Percentage of a Base

The IBNR reserve can be estimated as a certain percentage of a selected base. The base is chosen on the assumption that it is directly proportional to the IBNR. The percentage may come from a retrospective study of past IBNR, from another company, or from judgment. The base used might vary from line to line within a company. The percentage will certainly vary from line to line.

Almost every conceivable base is recommended in the literature.

Premiums in force.
Earned premium.
Written premium.
Incurred loss.
Paid loss.
Late reported incurred loss during a specified brief period after the close of the year.
(Calendar period number of claims) × (average cost per claim), where the average cost is based upon past averages, but the number of claims is the actual number for the calendar period.
Number of claims reported.
Net case reserve.
Gross case reserve.
Number of open claims.

A particular use of this method appears in fidelity and surety, where a special formula fixed by the United States Treasury requires a fidelity IBNR of at least 10% of the premiums in force and a surety IBNR of at least 5% of the premiums in force.

Modify Last Year's IBNR for Growth

This method measures the growth in IBNR by the growth in some indicator. The assumption of this method is that the percentage change in the IBNR will equal the percentage change in the indicator. This assumption is algebraically equivalent to the assumption of the Percent of Base Method, but the operation is a bit different, as the following example will show.

Suppose that the method used in 1969 to set the commercial multiple peril IBNR was to take 10% of earned premium, and the following figures were available at the close of 1970.

1969 earned premium	$10,000,000
1970 earned premium	$12,000,000
Estimated 12/31/69 IBNR based upon runoff or other retrospective test	$ 950,000

Using the Percent of Base Method, we would say that the test confirms that 10% is still a good percentage, so we would recommend a 12/31/70 IBNR of $1,200,000. Using this method, the 10% would never be explicitly mentioned. We would simply recommend a 12/31/70 IBNR of:

$$\frac{12{,}000{,}000}{10{,}000{,}000} \times 950{,}000 = \$1{,}140{,}000.$$

In effect, the Percent of Base Method says:

$$\begin{pmatrix}\text{Current estimate}\\ \text{of IBNR}\end{pmatrix} = \begin{pmatrix}\text{estimated prior IBNR as of}\\ \text{prior evaluation date}\end{pmatrix} \times \begin{pmatrix}\text{growth}\\ \text{factor}\end{pmatrix}$$

$$(\$1{,}200{,}000) = (\$1{,}000{,}000) \times (1.2)$$

(although, the estimated prior IBNR is reviewed using current data).

In contrast to the Percent of Base method, this method says:

$$\begin{pmatrix}\text{Current estimate}\\ \text{of IBNR}\end{pmatrix} = \begin{pmatrix}\text{estimated prior IBNR as of}\\ \text{current evaluation date}\end{pmatrix} \times \begin{pmatrix}\text{Growth}\\ \text{factor}\end{pmatrix}$$

$$(\$1{,}140{,}000) = (\$950{,}000) \times (1.2)$$

The Percent of Base method is more stable and this method is more responsive.

Tarbell Method

Tarbell's method of estimating IBNR also modifies the prior years' reserve for growth.[13] He has two basic formulas:

$$\begin{pmatrix}\text{Estimated IBNR}\\ \text{at the end}\\ \text{of year } y\end{pmatrix} = \frac{N^{y}_{10\text{-}11\text{-}12}}{N^{y-1}_{10\text{-}11\text{-}12}} \times \frac{C^{y}_{10\text{-}11\text{-}12}}{C^{y-1}_{10\text{-}11\text{-}12}} \times I^{y-1}_{(1)\ldots(12)}$$

$$\begin{pmatrix}\text{Estimated IBNR}\\ \text{at the end of month } n\\ \text{of year } y+1\end{pmatrix} = \frac{N^{y+1}_{(n-2)\text{-}(n-1)\text{-}n}}{N^{y}_{10\text{-}11\text{-}12}} \times \frac{C^{y+1}_{(n-2)\text{-}(n-1)\text{-}n}}{C^{y}_{10\text{-}11\text{-}12}} \times I^{y}_{(1)\ldots(n)} \times P_n$$

Where: N = No. of claims

C = Average incurred cost per claim

I = Amount of IBNR runoff

Superscripts designate calendar year

Subscripts designate calendar month

$I^{y}_{(1)\ldots(n)}$ = An n month runoff of the year end y IBNR

$I^{y-1}_{(1)\ldots(12)}$ = A 12 month runoff of the year end (y-1) IBNR

P_n is the factor, based upon experience, necessary to project

$I^{y}_{(1)\ldots(n)}$ to an ultimate basis, since an n month runoff may underestimate the IBNR.

Tarbell starts with the estimated IBNR as of the past year end, as indicated by the runoff, and increases it by the percentage increase in the three-month incurred loss. He points out that the three-month period is arbitrary and recommends varying the length of the period by line.

[13] T. F. Tarbell, "Incurred but not Reported Claim Reserves", *PCAS* Vol. XX, 1933.

Although $N \times C$ = incurred loss, Tarbell separates the two factors in order to amend the average cost factor by eliminating abnormal claims where necessary. Since the first formula requires an assumption that a 12-month runoff is fully developed, Tarbell recommends applying a projection factor if necessary.

TOTAL LOSS RESERVES

Runoff

The runoff of total loss reserves is the sum of the runoffs of the reserve for known claims and IBNR, since it is meant to test the total of the two reserves.

$$\begin{pmatrix}\text{Runoff of total loss}\\ \text{reserve as of 12/31/70}\\ \text{based upon development}\\ \text{through 6/30/72}\end{pmatrix} = \begin{pmatrix}\text{Paid loss during the}\\ \text{period 1/1/71-6/30/72}\\ \text{on all claims incurred}\\ \text{prior to 12/31/70}\end{pmatrix} + \begin{pmatrix}\text{Remaining reserve}\\ \text{as of 6/30/72 on}\\ \text{all claims incurred}\\ \text{prior to 12/31/70}\end{pmatrix}$$

Schedule 0 of the Annual Statement is a runoff of total loss reserves although there is a special treatment of salvage and non-ledger reinsurance (described in Special Topics). Column (16) shows a one year development of the total loss reserve for the previous year end, and column (17) shows a two-year development of the total loss reserve for the second previous year end.

Total Loss Reserves as a Percentage of a Base

Total loss reserves are sometimes tested by representing them as a percentage of a selected base. The magazine *U. S. Investor* annually shows total loss and loss expense reserves as a percentage of earned premium and as a percentage of written premium by major line.

Ruth Salzmann[14] has recommended testing total loss and loss expense reserves by the ratio

$$\frac{\begin{pmatrix}\text{Total loss and loss}\\ \text{expense reserves 12/31/n}\end{pmatrix}}{\begin{pmatrix}\text{Total loss and}\\ \text{loss expense}\\ \text{reserves 12/31/n-1}\end{pmatrix} + \begin{pmatrix}\text{Premiums}\\ \text{earned}\\ \text{calendar year n}\end{pmatrix} - \begin{pmatrix}\text{Paid loss}\\ \text{calendar}\\ \text{year n}\end{pmatrix}}$$

[14] R. Salzmann, "How Adequate Are Loss and Loss Expense Liabilities?", *PCAS* Vol. LIX, 1972.

Although these tests are fallible, they do have the advantage of convenience.

Runoff of Cumulative Incurred Loss

Schedule P, Part 3 tests the total loss reserve by means of a runoff of cumulative incurred loss. It compares the estimated incurred loss for all accident years prior to a certain date (referred to as "Cumulative Total") with later evaluations of the same figure. In this schedule, evaluation date is referred to as "Reserve Date". For an example, look at the 1971 and 1972 Schedule P, Part 3C tables on the attached exhibit.

The 1972 Schedule P, Part 3C shows that the cumulative total through accident year 1971 as of 12/31/71 was \$536,468, but after one year's reserve development the new estimate was \$538,082. This indicates an apparent reserve deficiency of \$1,614. The cumulative 1971 total of \$536,468 can also be obtained from the 1971 Schedule P by adding the last two figures in the column "12/31/71".

This method of reserve development is shown to be equivalent to a runoff of the total loss reserve by subtracting out the cumulative paid loss as of December 31, 1971. The cumulative paid loss as of December 31, 1971 is the sum of the last two numbers in the last column of the 1971 Schedule P—Part 3C: \$413,748 + \$15,001 = \$428,749.

$$\begin{pmatrix}\text{Cumulative incurred loss}\\ \text{as of 12/31/71}\end{pmatrix} - \begin{pmatrix}\text{Cumulative paid loss}\\ \text{as of 12/31/71}\end{pmatrix} = \begin{pmatrix}\text{Carried total loss}\\ \text{reserve as of}\\ \text{12/31/71}\end{pmatrix}$$

$$(\$536{,}468) \quad - \quad (\$428{,}749) \quad = \quad (\$107{,}719)$$

This figure is analogous to 1972 Schedule O, Column (16) "Estimated liability on unpaid losses December 31, 1971 per column 5 Part 3A, 1971."

$$\begin{pmatrix}\text{One year developed}\\ \text{cumulative incurred loss}\\ \text{as of 12/31/71}\end{pmatrix} - \begin{pmatrix}\text{Cumulative}\\ \text{paid loss}\\ \text{as of 12/31/71}\end{pmatrix} = \begin{pmatrix}\text{One year runoff of}\\ \text{total loss reserve}\\ \text{as of 12/31/71}\end{pmatrix}$$

$$(\$538{,}082) \quad - \quad (\$428{,}749) \quad = \quad (\$109{,}333)$$

This one-year runoff of the total loss reserve as of 12/31/71 is analogous to 1972 Schedule O, Column (14) "Total Losses Incurred to December 31 of Current Year on Losses Incurred Prior to 1972". Of course, simple alge-

SCHEDULE P—PART 3C—DEVELOPMENT OF INCURRED COMPENSATION LOSSES SUMS OF COLUMNS (3) AND (10), SCHEDULE P, PART 2

1971

Policy Years	Years In Which Losses Were Incurred	RESERVE DATE 12/31/66	12/31/67	12/31/68	12/31/69	12/31/70	12/31/71	Cumulative Loss Payments As Of December 31, Current Year
	•	•	•	•	•	•	•	•
	•	•	•	•	•	•	•	•
	•	•	•	•	•	•	•	•
	Sub-Total	XXX	XXX	XXX	$430,719	$430,176	$433,995	$383,912
	1970	XXX	XXX	XXX	XXX	54,340	50,998	29,836
	Sub-Total	XXX	XXX	XXX	XXX	484,516	484,993	413,748
	1971	XXX	XXX	XXX	XXX	XXX	51,475	15,001

SCHEDULE P—PART 3C—DEVELOPMENT OF INCURRED COMPENSATION LOSSES SUMS OF COLUMNS (3) AND (10), SCHEDULE P, PART 2

1972

Policy Years	Years In Which Losses Were Incurred	RESERVE DATE 12/31/67	12/31/68	12/31/69	12/31/70	12/31/71	12/31/72	Cumulative Loss Payments As Of December 31, Current Year
	•	•	•	•	•	•	•	•
	•	•	•	•	•	•	•	•
	•	•	•	•	•	•	•	•
	Cumulative Total	XXX	XXX	$430,719	$430,176	$433,995	$436,298	$392,376
	1970	XXX	XXX	XXX	54,340	50,998	52,814	37,197
	Cumulative Total	XXX	XXX	XXX	484,516	484,993	489,112	429,555
	1971	XXX	XXX	XXX	XXX	51,475	48,970	29,478
	Cumulative Total	XXX	XXX	XXX	XXX	536,468	538,082	459,033
	1972	XXX	XXX	XXX	XXX	XXX	56,296	14,960

bra shows that this runoff of total loss reserve must indicate the same reserve deficiency as the runoff of total incurred loss.

$$\begin{pmatrix}\text{One year runoff of}\\ \text{total loss reserve}\\ \text{as of 12/31/71}\end{pmatrix} - \begin{pmatrix}\text{Carried total loss}\\ \text{reserve}\\ \text{as of 12/31/71}\end{pmatrix} = \begin{pmatrix}\text{Estimated total loss}\\ \text{reserve deficiency}\\ \text{as of 12/31/71}\end{pmatrix}$$

$$(\$109{,}333) \quad - \quad (\$107{,}719) \quad = \quad (\$1{,}614)$$

This figure is analogous to 1972 Schedule O, Column (18) "Change in Such Estimated Liability December 31, 1971".

Accident Year Loss Development

The total loss reserve is related to the accident year incurred loss. For example, assume that all claims are settled within ten years. Then:

12/70 required total loss reserve =

accident year 1961 contribution to the total loss reserve
+ accident year 1962 contribution to the total loss reserve
+ . . . . .
+ accident year 1970 contribution to the total loss reserve.

$$\begin{pmatrix}\text{Accident year y}\\ \text{contribution to}\\ \text{the total loss}\\ \text{reserve}\end{pmatrix} = \begin{pmatrix}\text{Accident year y}\\ \text{ultimate incurred}\\ \text{loss}\end{pmatrix} - \begin{pmatrix}\text{Accident year y}\\ \text{paid loss to date}\\ \text{as of 12/31/70}\end{pmatrix}$$

There is a similar formula using policy year, with the added complication that only half of the final policy year is used.

12/70 required total loss reserve =

policy year 1961 contribution to the total loss reserve
+ . . . . .
+ policy year 1969 contribution to the total loss reserve
+ policy year 1970/accident year 1970 contribution to the total loss reserve.

Both the accident year and policy year formulas are exact. The problem lies in estimating the ultimate incurred loss. A number of methods recommended in the readings are described in the following section.

Loss Ratio Method

The Loss Ratio Method assumes that a line of business will always produce a certain loss ratio. This ratio is multiplied by the policy year earned premium to obtain the estimated policy year incurred loss or is multiplied by the calendar year earned premium to obtain the estimated accident year incurred loss.

Schedule P, Parts 1 and 2 use the Loss Ratio Method to estimate a minimum total loss and loss expense reserve for liability and workmen's compensation. Part 1 assumes a 60% loss and loss expense ratio for liability and Part 2 assumes a 65% loss and loss expense ratio for workmen's compensation, both for the last three accident years.

Accident Year Incurred Loss Development

Accident year incurred loss can be developed by means of loss development factors as in ratemaking. These factors are obtained by observing the rate of development of older accident years and assuming that newer accident years will develop at a similar rate. Note that Schedule P, Part 3 provides the figures necessary to perform such a loss development for liability and workmen's compensation.

The incurred development assumes that accident year losses will be reported and reserved consistently. Rate of payments is irrelevant since the payments do not affect the gross reserves. An inaccurate reserve for known claims will not invalidate the incurred development provided that it is consistent from one accident year to the next.

Bornhuetter and Ferguson recommend an IBNR method which combines the Loss Ratio Method and Accident Year Incurred Loss Development.[15] The accident year incurred loss age-to-age factors are used to obtain an expected losses IBNR factor which is $1.0 - 1.0/\text{ultimate factor}$, for each accident year.

$$\begin{pmatrix}\text{Accident year y}\\ \text{contribution to}\\ \text{the IBNR}\end{pmatrix} = \begin{pmatrix}\text{Expected Losses}\\ \text{IBNR Factor for}\\ \text{accident year y}\end{pmatrix} \times \begin{pmatrix}\text{Earned premium}\\ \text{calendar}\\ \text{year y}\end{pmatrix} \times \begin{pmatrix}\text{Expected}\\ \text{loss}\\ \text{ratio}\end{pmatrix}$$

It should be noted that this formula estimates a gross IBNR which includes provision for the redundancy in the reserve for known claims.

[15] Bornhuetter and Ferguson, *op. cit.*

The Bornhuetter—Ferguson approach results in a compromise between incurred loss development and the Loss Ratio Method. The relationship can be seen in the following examples which pertain to a partially developed accident year:

		Example A	**Example B**
1.	Earned premium	2,000	2,000
2.	Expected loss ratio	.50	.50
3.	Expected loss	1,000	1,000
4.	Ultimate factor	1.25	1.25
5.	Expected loss to date (3) ÷ (4)	800	800
6.	Incurred loss to date	800	900
	Estimated subsequent development according to:		
7.	Incurred loss development [(4) − 1.0] × (6)	200	225
8.	Loss Ratio Method (3) − (6)	200	100
9.	Bornhuetter—Ferguson [(4) − 1.0] × (5)	200	200

In Example A the incurred loss to date equals expected loss to date so all three methods agree. In Example B the incurred loss to date is higher than the expected loss to date. The incurred loss development assumes that the increase is due to worsening experience, so the same percentage increase will apply to the subsequent development. The Loss Ratio Method assumes that the increase is due to accelerated loss reporting or strengthened case reserves, so the increase will be offset by an equal dollar reduction in the subsequent development.

The Bornhuetter—Ferguson approach ignores the incurred loss to date and produces an estimate in between the other two.

Accident Year Paid Loss Development

Accident year paid loss can similarly be developed by means of age-to-age factors. The process is analogous to the report year paid loss development described in the Projection Method for reserving for known claims. Of course, the ultimate value of the accident year paid loss equals the ultimate value of the accident year incurred loss.

Note that Schedule P—Part 4 provides the figures necessary to determine an accident year paid loss and loss expense development for liability and compensation.

The paid development requires that accident year losses be paid at a consistent rate. Accuracy of the reserve for known claims is irrelevant.

Prospective Test of Reserves

Schedule P, Part 4 provides a test of the current total loss and loss expense reserve. It is called a prospective test because the reserve date and evaluation date are equal. The test works by comparing the accident year contributions to the current carried reserve with the accident year contributions to past estimated reserves, relative to paid loss and loss expense to date and relative to the calendar year earned premium. For each accident year we start with loss and loss expense incurred from Schedule P, Parts 1 and 2. Loss and loss expense incurred is the sum of paid loss and loss expense to date plus reserve for known claims plus that accident's year portion of the IBNR and loss expense reserves.

$$\begin{pmatrix}\text{Current estimated loss \&}\\ \text{loss expense reserve}\\ \text{as of a point in time t}\end{pmatrix} = \begin{pmatrix}\text{Loss and}\\ \text{loss expense}\\ \text{incurred}\end{pmatrix} - \begin{pmatrix}\text{Paid loss to}\\ \text{date}\\ \text{as of time t}\end{pmatrix}$$

In order to test the current reserve some assumptions must be made. There are two simple assumptions which enable us to use Schedule P, Part 4.

First, for any number of years of development, there is a fairly constant ratio of accident year total loss and loss expense reserve required to accident year loss and loss expense paid through that number of years. This reserve to paid-to-date ratio does not vary from one accident year to another. This assumption is equivalent to the assumption of consistent accident year paid loss age-to-age factors.

Suppose the upper left-hand portion of Schedule P, Part 4A looks like this:

1971 SCHEDULE P–PART 4A (000 omitted)

	1965	1966	1967	1968	1969	1970	1971
Summary Data from Schedule P—Part 1A							
1. Premiums Earned	25,000	27,000	30,000	33,000	35,000	37,000	42,000
2. Loss & Loss Exp. Inc'd.	15,000	16,500	18,000	19,500	21,000	22,500	26,000
Loss & Loss Expense through 1 year							
3. Paid	10,000	11,000	12,000	13,000	14,000	15,000	16,000
4. Reserve (2) − (3)	5,000	5,500	6,000	6,500	7,000	7,500	10,000

For older accident years, when the estimated reserve at age 1 year is known to be very close to the required reserve, there is a reserve to paid-to-date ratio of 0.5. In fact, this example was constructed so that the 0.5 ratio holds for each accident year except the current one, 1971. Following the first assumption, we suspect that the accident year 1971 portion of the December 1971 loss and loss expense reserve is redundant by $2,000,000. This cannot be a definite conclusion because the increased reserve to paid-to-date ratio might have had causes such as a slowdown in claims settlement or a change in reinsurance or a new bookkeeping method.

Second, for any number of years of development, there is a fairly constant ratio of required accident year loss and loss expense reserve to calendar year earned premium. This reserve to earned premium ratio does not vary from one accident year to another.

Suppose the upper right-hand portion of Schedule P, Part 4A looks like this:

1971 SCHEDULE P–PART 4A
Percentages

		1965	1966	1967	1968	1969	1970	1971
	Summary Data from Schedule P—Part 1A							
1.	Premiums Earned	100.0	100.0	100.0	100.0	100.0	100.0	100.0
2.	Loss & Loss Exp. Inc'd.	64.0	65.0	63.0	62.0	65.0	64.0	57.0
	Loss & Loss Expense through 1 year							
3.	Paid	23.0	24.0	22.0	21.0	24.0	23.0	23.0
4.	Reserve (2)—(3)	41.0	41.0	41.0	41.0	41.0	41.0	34.0

Following the second assumption, we suspect that the accident year 1971 portion of the December 1971 loss and loss expense reserve is deficient by 7% of the 1971 earned premium. This cannot be a definite conclusion because the decreased reserve to earned premium ratio might have had causes such as an improved loss ratio or a speedup of claims settlement.

Lorah Method[16]

This is an accident month loss development based upon separate estimates of number of claims and average claim. For all but the last two accident months, the number of claims is estimated by projecting the number reported to date.

$$\begin{pmatrix}\text{Estimated number}\\ \text{of claims}\end{pmatrix} = \begin{pmatrix}\text{number reported}\\ \text{to date}\end{pmatrix} \times \begin{pmatrix}\text{development}\\ \text{factor}\end{pmatrix}$$

The average cost is based upon claims closed in the most recent 12 calendar months. This is referred to as the claims disposed of (C.D.O.) cost.

$$\begin{pmatrix}\text{C.D.O.}\\ \text{cost}\end{pmatrix} = \frac{\text{gross amount paid on closed claims}}{(\text{number closed with payment}) + (\text{number closed without payment})}$$

[16] J. W. Lorah, "Loss Reserves—Case and Incurred but not Reported, Auto Lines Only", *Insurance Accounting and Statistical Association Proceedings*, 1961.

For the most recent two accident months, an alternate formula is used.

$$\begin{pmatrix}\text{Estimated accident}\\ \text{month incurred loss}\end{pmatrix} = \begin{pmatrix}\text{Estimated incurred loss}\\ \text{for the accident month}\\ \text{one year prior}\end{pmatrix} \times \begin{pmatrix}\text{growth}\\ \text{factor}\end{pmatrix}$$

The growth factor can be based upon number of policies or amount of premium.

Lorah splits his accident month estimated incurred loss into reported and unreported portions.

$$\begin{pmatrix}\text{Accident month}\\ \text{reported losses}\end{pmatrix} = \begin{pmatrix}\text{No. losses}\\ \text{reported}\end{pmatrix} \times \begin{pmatrix}\text{C.D.O.}\\ \text{cost}\end{pmatrix}$$

$$\begin{pmatrix}\text{Accident month}\\ \text{contribution to IBNR}\end{pmatrix} = \begin{pmatrix}\text{Estimated No. of}\\ \text{unreported losses}\end{pmatrix} \times \begin{pmatrix}\text{C.D.O.}\\ \text{cost}\end{pmatrix}$$

Summing these contributions provides an estimated reserve for known claims and an estimated IBNR.

This method of reserving requires the usual assumptions of consistency in loss frequency, reporting lag and claims settlement policies. In order to split the reserve into IBNR and reserve for known claims, Lorah assumes that the amount of a claim is independent of the reporting lag. In utilizing the average closed claim to predict an average accident month claim, he assumes that these two averages are equal.

Under certain common circumstances this final assumption cannot be expected to hold. If for a certain line of business large claims settle more slowly than small claims and this line is growing, then the average claim closed in a given calendar year will be less than the average accident year claim. The reason is that the claims closed in a given calendar year will include a proportionately larger number of recent claims than old claims because of growth, and these recent claims are below average in size. An example will clarify this situation.

Assume that all claims in an accident year will close within the first three years of development. There is growth in the number of claims, but there is no change from one accident year to the next in the distribution of claims by size or duration. The closing pattern is indicated in the table.

ACCIDENT YEAR CLOSING PATTERNS

Accident Year	Closing in First Year Number	Closing in First Year Average	Closing in Second Year Number	Closing in Second Year Average	Closing in Third Year Number	Closing in Third Year Average
1968	2	$100	2	$200	2	$300
1969	3	100	3	200	3	300
1970	4	100	4	200	4	300

The average claim for each accident year is $200. The average claim closed in 1970 is $178. $\left(\frac{4 \times \$100 + 3 \times \$200 + 2 \times \$300}{4 + 3 + 2}\right)$

Observe also that the average gross case reserve on claims open as of December 31, 1970 is $264. $\left(\frac{4 \times \$200 + 4 \times \$300 + 3 \times \$300}{4 + 4 + 3}\right)$

Average Value Method

This method, as described by Scheibl, consists of making separate estimates of number of claims and average size of claim by accident year.[17] The count is subject to development. Scheibl recommends an accident year projection of number of claims by means of age-to-age factors.

He mentions four different approaches to estimating the average accident year claim.

a. Estimate the percentage change in average claim for the current accident year based upon the percentage change in average claim for past accident years.

b. Estimate the dollar change in average claim for the current accident year based upon the dollar change in average claim for past accident years.

c. Estimate the second differences in average claim for the current accident year based upon the second differences in average claim for past accident years.

d. Estimate the ultimate average loss for the current year based upon the change in the average paid loss to date over prior years.

[17]Scheibl, *op. cit.*

(The Payment Development Method follows an analogous approach using report year instead of accident year.)

These assumptions can be stated algebraically if we introduce some notation. Let

AI_t^y = average incurred loss for accident year y after t years of development.

AP_t^y = average paid loss for accident year y after t years of development.

Then the four assumptions can be restated as:

a. $\dfrac{AI_{t+1}^y}{AI_t^y}$ is independent of y.

b. $AI_{t+1}^y - AI_t^y$ is independent of y.

c. $(AI_{t+1}^y - AI_t^y) - (AI_t^y - AI_{t-1}^y)$ is independent of y.

d. $\dfrac{AP_{t+1}^y}{AP_t^y}$ is independent of y.

More generally, Scheibl suggests looking for any consistent pattern relating AI_{t+1}^y to AI_t^y or relating AP_{t+1}^y to AP_t^y. An examination of the data at hand should demonstrate which pattern is most consistent.

SPECIAL TOPICS

RESERVE FOR REOPENED CLAIMS

A problem in settling claims is that closed cases may be reopened because of developments not foreseen by the claims adjuster. This problem is particularly acute in workmen's compensation. A provision for closed claims which will be reopened must be included within the reserve for known claims. This can be accomplished in several ways.

Estimate Loss Reserves by a Method Which Includes Reopened Claims

Methods which do include a provision for reopened claims are Notice-Average Method, Average Value Method (for reserve for known claims), Case Reserve Runoff, Correct Case Reserve for Bias, Report Year Loss Development, Projection Method, Payment Development Method, Runoff of Total Loss Reserves, Runoff of Cumulative Incurred Loss, Accident Year Loss Development, and Policy Year Loss Development. Methods which do not include a provision for reopened claims are Individual Case Estimate, Fast Track Reserves, Tabular Value, and Average Value Method (for total loss reserves).

Treat Reopened Claims like IBNR

If reopened claims are treated analogously to newly reported claims for the purpose of loss reserve calculations, with the reopened date taken in place of the reported date, then the claim can be treated like a late reported claim. Any of the IBNR methods can be used to calculate a reserve for reopened claims.

Balcarek Method[18]

This is a method for calculating a separate reserve for reopened claims based upon the number of claims closed. There are two steps to this approach. First, estimate the number of closed claims at the end of a particular year which will be reopened at a later date. Claims closed and reopened in the same calendar year are not included. Second, estimate the average incurred cost after reopening.

The number of claims that will reopen is estimated on the basis of the number of claims closed in the last eight years. The formula is the following:

Estimated number of claims that will reopen =

.00460 × no. of claims closed during the present year
+ .00114 × no. of claims closed during the first preceding year
+ .00051 × no. of claims closed during the second preceding year
+ . . .
+ .00002 × no. of claims closed during the eighth preceding year

[18]R. J. Balcarek, "Reserves for Reopened Claims on Workmen's Compensation", *PCAS* Vol. XLVIII, 1961.

We will refer to these reopening probabilities as r_k, so that

$$\begin{pmatrix}\text{Estimated no. of claims that}\\ \text{will reopen as of the}\\ \text{end of year } t\end{pmatrix} = \sum_{k=0}^{8} r_k \times \begin{pmatrix}\text{No. of claims}\\ \text{closed during}\\ \text{year } t\text{-}k\end{pmatrix}$$

The coefficients r_k were calculated by observing the probability of reopening in a given year based upon past experience and fitting this function to an exponential curve.(See Balcarek Table 2.) The fitted curve was used to find the cumulative probability of reopenings in a given year or later. These cumulative probabilities are used because the reserve for reopened claims covers closed claims which will be reopened in the first subsequent year or second subsequent year, etc. Balcarek's statistics show that only a negligible percentage of claims are reopened after the eighth subsequent year.

In estimating the average incurred cost, Balcarek did not assume that the average reopened claim would equal the average closed claim. Instead, he looked for a factor that would relate the two. He based his average reopened claim upon a developed figure rather than the original estimate. He discovered that for workmen's compensation claims in his company the ratio of average reopened claim to average closed claim was stable at about 4.5 This produced the formula:

Estimated reopened reserve at the end of year t

$$= \sum_{k=0}^{8} \begin{pmatrix}\text{Estimated number of claims closed}\\ \text{in year } (t\text{-}k) \text{ which will reopen}\\ \text{subsequent to year } t\end{pmatrix} \times \begin{pmatrix}\text{Average reopened}\\ \text{value of a claim}\\ \text{closed in year } (t\text{-}k)\end{pmatrix}$$

$$= \sum_{k=0}^{8} r_k \times \begin{pmatrix}\text{Number of claims}\\ \text{closed in year } (t\text{-}k)\end{pmatrix} \times 4.5 \times \begin{pmatrix}\text{Average claim}\\ \text{closed in year } (t\text{-}k)\end{pmatrix}$$

$$= 4.5 \sum_{k=0}^{8} r_k \times \begin{pmatrix}\text{Gross amount of loss on}\\ \text{claims closed in year } (t\text{-}k)\end{pmatrix}$$

Salvage and Subrogation

An insurer will sometimes settle a property loss by agreeing with the insured upon the sound value of the damaged goods, paying him a total loss, and then selling the damaged items for salvage. The amount of the sale is credited to the insurance company. The salvage is booked as a negative paid loss. The doctrine of subrogation gives the insurer whatever rights the insured possessed against responsible third parties. The amount recovered under the right of subrogation is limited by the amount of the loss payment which has been made to the insured. The insurer cannot make a profit by subrogating against the person who caused the loss. The amount realized through subrogation is credited to the insurance company and is booked as a negative paid loss.

At any point in time an insurance company can anticipate receiving a certain amount of salvage and subrogation on claims incurred, be they unreported, in the course of settlement, or closed. This outstanding salvage and subrogation resembles a credit loss reserve. There is a controversy over whether the anticipated salvage and subrogation should be used to reduce reserves. Reserves established with no anticipated salvage and subrogation are said to be Gross of Salvage. Reserves which include anticipated salvage and subrogation (and therefore are lower) are said to be Net of Salvage.

One of the primary functions of loss reserves is to aid in determining the company's financial security. This purpose favors setting reserves conservatively. Therefore statutory insurance accounting requires setting reserves gross of salvage rather than offsetting liabilities with probable but uncertain assets.

The other primary function of loss reserves is to produce accurate income statements for control of underwriting and rates. Net of salvage reserves are the more accurate because an ongoing company will normally collect salvage and subrogation outstanding. Losses and salvage are treated symmetrically by net of salvage reserving. Loss reserves are established on the basis of losses which the company anticipates paying; these reserves are offset by salvage and subrogation which the company anticipates collecting.

Case reserves are always established gross of salvage. The various reserving methods and tests described in this article can be used to set reserves gross of salvage or net of salvage depending upon whether salvage and subrogation are included in the data entering the calculation.

Schedule P of the Annual Statement is net of salvage, as is evident from the fact that it balances to Part-3A. Of course, there is no salvage in liability or workmen's compensation, but there is some subrogation in workmen's compensation.

The Schedule O test of reserves is a compromise between net and gross of salvage. The reserve test as of a year end is gross of salvage on claims paid prior to that year end and net of salvage on claims not paid prior to that year end. The principle followed is this: On claims for which both the loss and salvage are still uncertain, the anticipated salvage may be used to offset the anticipated loss. However, on claims which have already been paid, anticipated salvage may not be used to offset loss reserves since there are no reserves remaining on those claims.

The 1971 Schedule O can be used to illustrate the principle. For the 12/70 runoff, we have column (14) = column (3) + (4) + (11) + (12).

$$\begin{pmatrix}\text{One year runoff of 12/70 total loss reserve net of}\\ \text{salvage \& reinsurance on claims closed in 1971}\end{pmatrix} =$$

$$\begin{pmatrix}\text{Paid loss during 1971}\\ \text{on accident year 1970}\\ \text{net of salvage \& reinsurance}\\ \text{on claims closed in 1971}\end{pmatrix} + \begin{pmatrix}\text{Paid loss during 1971 on}\\ \text{accident year 1969 and prior}\\ \text{net of salvage \& reinsurance}\\ \text{on claims closed in 1971}\end{pmatrix}$$

$$+ \begin{pmatrix}\text{Total loss reserve carried}\\ \text{12/71 on accident year 1970}\\ \text{gross of salvage \& reinsurance}\end{pmatrix} + \begin{pmatrix}\text{Total loss reserve carried}\\ \text{12/71 on accident year 1969}\\ \text{\& prior gross of salvage \&}\\ \text{reinsurance}\end{pmatrix}$$

For the 12/69 runoff, we have column (15) = column (4) − (6) + (9) + (12)

$$\begin{pmatrix}\text{Two year runoff of 12/69 total loss reserve net of}\\ \text{salvage \& reinsurance on claims closed in 1970 and 1971}\end{pmatrix} =$$

$$\begin{pmatrix}\text{Paid loss during 1971 on}\\ \text{accident year 1969 and prior}\\ \text{net of salvage \& reinsurance}\\ \text{on claims closed in 1971}\end{pmatrix} - \begin{pmatrix}\text{Salvage \& reinsurance received}\\ \text{during 1971}\\ \text{on accident year 1969 and prior}\\ \text{on claims closed in 1970}\end{pmatrix}$$

$$+\begin{pmatrix}\text{Paid loss during 1970 on}\\ \text{accident year 1969 and prior}\\ \text{net of salvage \& reinsurance}\\ \text{on claims closed in 1971}\end{pmatrix} + \begin{pmatrix}\text{Total loss reserve carried 12/71}\\ \text{on accident year 1969 and prior}\\ \text{gross of salvage \& reinsurance}\end{pmatrix}$$

Relationship between Calendar Year and Accident Year Incurred Loss

This section will demonstrate the fact that the increase in loss reserve redundancy during a year equals the excess of the calendar year incurred loss over the accident year ultimate incurred loss.

To prove this theorem, we first show that

$$\begin{pmatrix}\text{accident year } y\\ \text{ultimate incurred loss}\end{pmatrix} =$$

$$\begin{pmatrix}\text{calendar year } y\\ \text{paid loss}\end{pmatrix} + \begin{pmatrix}12/31/y \text{ required}\\ \text{total loss reserve}\end{pmatrix} - \begin{pmatrix}12/31/(y\text{-}1) \text{ required}\\ \text{total loss reserve}\end{pmatrix}$$

Indeed,

$$\begin{pmatrix}12/31/y \text{ required}\\ \text{total loss reserve}\end{pmatrix} =$$

$$\sum_{t\leq y}\left[\begin{pmatrix}\text{accident year t}\\ \text{ultimate incurred loss}\end{pmatrix} - \sum_{s\leq y}\begin{pmatrix}\text{accident year t paid loss}\\ \text{during calendar year s}\end{pmatrix}\right].$$

$$\begin{pmatrix}12/31/(y\text{-}1) \text{ required}\\ \text{total loss reserve}\end{pmatrix} =$$

$$\sum_{t\leq y-1}\left[\begin{pmatrix}\text{accident year t}\\ \text{ultimate incurred loss}\end{pmatrix} - \sum_{s\leq y-1}\begin{pmatrix}\text{accident year t paid loss}\\ \text{during calendar year s}\end{pmatrix}\right].$$

$$\begin{pmatrix}\text{12/31/}y\text{ required}\\ \text{total loss reserve}\end{pmatrix} - \begin{pmatrix}\text{12/31/}y\text{-}1\text{ required}\\ \text{total loss reserve}\end{pmatrix} =$$

$$\begin{pmatrix}\text{accident year }y\\ \text{ultimate incurred loss}\end{pmatrix} - \begin{pmatrix}\text{calendar year }y\\ \text{paid loss}\end{pmatrix}.$$

Using this result, we see that

$$\begin{pmatrix}\text{increase in loss reserve}\\ \text{redundancy during year }y\end{pmatrix} =$$

$$\begin{pmatrix}\text{12/31/}y\text{ carried}\\ \text{total loss reserve}\end{pmatrix} - \begin{pmatrix}\text{12/31/}y\text{ required}\\ \text{total loss reserve}\end{pmatrix} -$$

$$\left[\begin{pmatrix}\text{12/31/}(y\text{-}1)\text{ carried}\\ \text{total loss reserve}\end{pmatrix} - \begin{pmatrix}\text{12/31/}(y\text{-}1)\text{ required}\\ \text{total loss reserve}\end{pmatrix}\right] =$$

$$\begin{pmatrix}\text{calendar year }y\\ \text{paid loss}\end{pmatrix} + \begin{pmatrix}\text{12/31/}y\text{ carried}\\ \text{total loss reserve}\end{pmatrix} - \begin{pmatrix}\text{12/31/}(y\text{-}1)\text{ carried}\\ \text{total loss reserve}\end{pmatrix} -$$

$$\left[\begin{pmatrix}\text{calendar year }y\\ \text{paid loss}\end{pmatrix} + \begin{pmatrix}\text{12/31/}y\text{ required}\\ \text{total loss reserve}\end{pmatrix} - \begin{pmatrix}\text{12/31/}(y\text{-}1)\text{ required}\\ \text{total loss reserve}\end{pmatrix}\right] =$$

$$\begin{pmatrix}\text{calendar year }y\\ \text{incurred loss}\end{pmatrix} - \begin{pmatrix}\text{accident year }y\\ \text{ultimate incurred loss}\end{pmatrix}.$$

As a corollary, a calendar year incurred loss will be the same as an accident year ultimate incurred loss, provided that the beginning and ending carried total loss reserves are at the proper level or provided that these two reserves are inaccurate by equal dollar amounts.

Loss Expense Reserves

Loss expense reserves are established for the purpose of covering all future expenses required to investigate and settle claims already incurred, whether reported or not. Loss expense is also called loss adjustment expense or claim expense. Allocated loss expenses are those which can be allocated to a specific claim, such as legal fees and outside claim adjusters' fees. Unallocated loss expenses are those which cannot be allocated to a specific claim, such as salaries and rent. Different methods are used to set the reserves for allocated and unallocated loss adjustment expense.

Allocated Loss Expense Reserve

Loss Reserve Methods

Since allocated loss expense payments are chargeable to specific claims, individual payments can be recorded in the same detail as the claims themselves. Line, class, accident date, reported date, policy year, state, territory etc. can all be captured. It follows that any method used to establish or test loss reserves can also be used to establish or test allocated loss expense (ALE) reserves. One common method is the establishment of a per case ALE reserve along with the per case loss reserve. Of course, this reserve must be supplemented by a reserve for anticipated ALE on incurred but not reported claims.

Ratio Method

Although the ALE reserve could be based upon premiums, incurred loss, or any of the other bases used for IBNR, the only base recommended in the readings is the total loss reserve.

The simplest formula of this type is

$$\begin{pmatrix}\text{Estimated}\\ \text{ALE reserve}\end{pmatrix} = (\text{Factor}) \times \begin{pmatrix}\text{Reserve for known}\\ \text{claims plus IBNR}\end{pmatrix}$$

where the factor is the ratio of paid ALE to paid loss for a calendar period. *Examination of Insurance Companies* recommends a factor of

$\dfrac{\text{Paid ALE for 3 calendar years}}{\text{Paid loss for 3 calendar years}}$ for liability claims, provided that the ratio

of calendar year paid ALE to paid loss has remained fairly constant.[19] This simple formula depends upon three assumptions.

a. The loss reserves are accurate.

b. For an individual claim, the ratio of ALE to loss amount is independent of how long it takes to settle the claim.

c. Losses and ALE are paid out at the same rate.

Intuitively, it appears that assumptions (b) and (c) might not hold

[19] New York (State) Insurance Department, *op. cit.*

for legal expense. Slow settling liability claims are more likely to have gone to trial—requiring large amounts of legal expense. Quick settling liability claims are more likely to be settled out of court—requiring little or no legal fees. So, slow closing claims appear to have more legal expense per claim dollar than quick closing claims. Normally, a lawyer submits his bill after the case is settled and since legal expense generally is attached to the slower cases, legal expense would appear to be paid out slower than losses.

Slifka's figures show that assumptions (b) and (c) do not hold for the miscellaneous liability line in his company.[20] His Exhibit II, and Exhibit V show that losses are paid more quickly than ALE. For example, these two exhibits show that 50% of the ultimate loss will be paid within two and one-half years after the start of the accident year, but 50% of the ultimate ALE will not be paid until four years after the start of the accident year. The incremental line in Exhibit III shows that claims which settle later will tend to need more dollars of ALE per dollar of loss than claims which settle earlier. For example, claims settled during the first year have a ratio of ALE to loss which is about 5%. The ALE-to-loss ratio is 10% for claims settled during the second year, 25% for claims settled during the third year, 27% for claims settled during the fourth year, and 40% for claims settled during the fifth year. As Brian says, "The claims paid during a calendar year are heavily weighted by small easy to handle items. It is the severity in the outstanding losses that produce the major portion of the allocated loss expense."[21]

The direction in which these two assumptions fail to hold implies that the Ratio Method will underestimate the required ALE reserve. The longer it takes to settle a claim, the longer it remains in the loss reserves. It follows that the loss reserves include a disproportionately large share of slow settling claims which have a higher than average ratio of ALE to loss. It is also clear that if ALE is paid more slowly than losses, then the ratio of required ALE reserve to required loss reserve will be higher than the ratio of paid ALE to paid loss.

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[20] R. S. Slifka, "Testing of Loss Adjustment (Allocated) Expense Reserve", *Insurance Accounting and Statistical Association Proceedings*, 1968.

[21] R. E. Brian, "Formula Reserving for Loss Expenses", *Insurance Accounting and Statistical Association Proceedings*, 1967.

Brian Method[22]

R. E. Brian recommends a modification of the Ratio Method to correct the inappropriate factor. His formula is also

(ALE reserve) = (Factor) × (Reserve for known claims + IBNR)

but his factor is not a calendar period paid-to-paid factor. Instead, he assumes that there is some appropriate factor which is constant over time and sets out to find it. He determines the factor that would have been appropriate in the past by taking the ratio of ALE runoff to total loss reserve runoff for past year ends. The current ratio is based upon these estimated past ratios with consideration given to historical and trend development. As Brian says, "The above approach follows a complete cycle. The factors are developed on the basis of outstanding losses tc allocated expenses paid and are applied in the reverse manner."

Slifka Method[23]

This is an accident year calculation of ALE Reserve. The formula

$$\begin{pmatrix}\text{Accident year t contribution}\\ \text{to the ALE reserve}\end{pmatrix} = \begin{pmatrix}\text{Accident year t}\\ \text{ultimate ALE}\end{pmatrix} - \begin{pmatrix}\text{Paid ALE}\\ \text{to date}\end{pmatrix}$$

becomes, in Slifka's terminology,

$$\begin{pmatrix}\text{Accident year t}\\ \text{contribution to}\\ \text{the ALE reserve}\end{pmatrix} = \begin{pmatrix}\text{Accident year t}\\ \text{ultimate ALE}\end{pmatrix} - \begin{pmatrix}\text{Accident year t}\\ \text{ALE paid on}\\ \text{closed to date}\end{pmatrix} - \begin{pmatrix}\text{Accident year t}\\ \text{paid on}\\ \text{pending to date}\end{pmatrix}$$

For the most recent four accident years,

$$\begin{pmatrix}\text{Accident year}\\ \text{ultimate ALE}\end{pmatrix} = (\text{Factor}) \times \begin{pmatrix}\text{Accident year ultimate}\\ \text{incurred loss}\end{pmatrix}$$

This factor is based upon the ratio of paid ALE to paid loss for fully developed accident years. The factor is also affected by the paid ALE to date for the given accident year.

[22] Brian, *op. cit.*

[23] Slifka, *op. cit.*

For the fifth, sixth, and seventh prior accident years,

$$\begin{pmatrix}\text{Accident year t}\\ \text{contribution to}\\ \text{the ALE reserve}\end{pmatrix} = \begin{pmatrix}\text{No. of}\\ \text{open}\\ \text{claims}\end{pmatrix} \times \begin{pmatrix}\text{Average expected}\\ \text{total ALE payment}\\ \text{on all open claims}\end{pmatrix} = \begin{pmatrix}\text{Paid ALE to}\\ \text{date on}\\ \text{open claims}\end{pmatrix}$$

This formula makes no provision for unreported claims, but relatively few claims are reported more than four years late. The average expected total ALE payment on open claims is actually a weighted average by expected year of closing. That is, first an average ALE per claim is developed based upon the number of years required to close the claim. Then the claims open are assigned a year of closing based upon past patterns. These are multiplied together, summed, and divided by the total number of open claims to obtain the average ALE payment per open claim. For example, Exhibit VIII of Slifka's paper shows that for accident year 1960, 190 claims are open as of 12/66. He projects that 94 of them will close in 1967, 64 in 1968, and 32 in 1969. It is also projected that $1,800 will be the average ALE for those claims closing in 1967, $2,000 for those closing in 1970, and $2,200 for those closing in 1971. The average expected ALE payment for the 190 accident year 1960 claims open as of 12/66 is approximately $2,000 [(94 × $1,800 + 64 × $2,000 + 32 × $2,200) ÷ 190].

UNALLOCATED LOSS EXPENSE RESERVES

Since unallocated loss expenses are not charged to specific claims, the individual payments cannot be assigned to line, class, accident date or reported date. The total paid unallocated loss expense (ULE) must be allocated to accident year and line based upon a time study or judgment. It is not possible to test the allocation retrospectively. Reserving methods based upon an allocation of ULE can be no more accurate than the allocation itself. For example, Parts 1, 2 and 4 of Schedule P test loss reserves including all loss expense. Schedule P itself cannot be used to determine whether the proper dollars of calendar year ULE were allocated to auto liability, general liability, and workmen's compensation. Nor can it be used to determine whether the allocation of the ULE to accident year was proper. To the degree that either of these allocations is inaccurate the reserve test will be inaccurate.

Ratio Method

$$\begin{pmatrix}\text{ULE} \\ \text{reserve}\end{pmatrix} = (\text{Factor}) \times \left[.50 \times \begin{pmatrix}\text{Reserve for} \\ \text{known claims}\end{pmatrix} + 1.00 \times (\text{IBNR}) \right]$$

where *Examination of Insurance Companies*[24] recommends a factor of $\frac{\text{Paid ULE for 3 calendar years}}{\text{Paid loss for 3 calendar years}}$. This formula depends upon the same three assumptions as does the ratio method for ALE reserves, as well as a fourth assumption: that 50% represents a reasonable estimate of the portion of investigation and adjustment already accomplished on open claims.

Projection Method—Accident Year Basis

In 1969 and 1970, Part 3 and 4 of Schedule P prescribed an allocation of ULE to accident year. Although these were dropped from the Statement in 1971, the reserving method which follows from them can still be used.

Part 4 of the 1970 Schedule P distributed the workmen's compensation ULE paid as follows

40% to the current accident year
45% to the first prior accident year
10% to the second prior accident year
5% to the third prior accident year

The table below shows how this distribution can be used to estimate a 12/70 ULE reserve. If we ignore growth, the table can be read as either a calendar year distribution of ULE paid by accident year or as an accident year distribution by calendar year.

DISTRIBUTION OF ULE PAYMENT PERCENTAGES BY CALENDAR YEAR AND ACCIDENT YEAR

Calendar Year	Accident Year 1968	1969	1970	1971	1972	1973
1971	5	10	45	40		
1972		5	10	45	40	
1973			5	10	45	40

[24]New York (State) Insurance Department, *op. cit.*

The reserve covers payments to be made 1971 and subsequent on claims occurring in 1970 and prior. If the calendar year paid ULE has been steady, we can take 80% (5% + 10% + 45% + 5% + 10% + 5%) of a typical calendar year's paid ULE as an estimated reserve.

The weakness of this method is that the percentage distribution is only an assumption.

Projection Method—Policy Year Basis

Prior to 1969, Part 3 and Part 4 of Schedule P prescribed an allocation of ULE to policy year. Part 4 of the 1968 Schedule P distributed the paid workmen's compensation ULE as follows:

40% to the current policy year
45% to the first prior policy year
10% to the second prior policy year
5% to the third prior policy year.

Each policy year contains two accident years. The 40% allocated to the current policy year all goes to the first accident year within that policy year, since the second accident year has not yet begun. Therefore, of the remaining 60%, 10% must be allocated to first accident years and 50% to second accident years, assuming that a calendar year's paid ULE is divided equally between first and second accident years within policy years. The reserve calculation will produce the same results regardless of how this ten-fifty split is achieved. For the purpose of illustration we assume here that the allocation percentage are these in the following table.

ASSUMED DISTRIBUTION OF CALENDAR YEAR PAID ULE

	First Accident Year	Second Accident Year	Total
Current policy year	40%	0%	40%
First prior policy year	7	38	45
Second prior policy year	2	8	10
Third prior policy year	1	4	5
	50%	50%	100%

This table can be restated as follows:

DISTRIBUTION OF ULE PAYMENT PERCENTAGES

BY POLICY YEAR, ACCIDENT YEAR AND CALENDAR YEAR

Accident Year	Policy Year 1968		1969		1970		1971		1972		1973	
Calendar Year	**68**	**69**	**69**	**70**	**70**	**71**	**71**	**72**	**72**	**73**	**73**	**74**
1971	1	4	2	8	7	38	40	0				
1972			1	4	2	8	7	38	40	0		
1973					1	4	2	8	7	38	40	0

We can take 30% (1% + 4% + 2% + 8% + 7% + 1% + 4% + 2% + 1%) of a typical calendar year's paid ULE as an estimated Reserve.

Dollar Method

Examination of Insurance Companies[25] recommends the application of a percentage to one year's paid ULE. They say, "Such a study requires a cost-accounting analysis of the time and effort spent in a year in servicing claims and distributing the cost to the years of occurrence of these claims." Presumably, the proper percentage is derived as in the Accident Year Projection Method, using the allocation of calendar year payments to accident year revealed by the time study rather than the one previously prescribed by Schedule P.

Brian Method[26]

Brian recommends a method based upon the allocation of the ULE Reserve to five types of loss transactions. The lag in each of these types of transaction is used to calculate the reserve amount.

A simple example will illustrate the basic principle. Suppose that for a certain line of business these are the figures for an average calendar month:

[25] New York (State) Insurance Department, *op. cit.*

[26] Brian, *op. cit.*

Average Calendar Month Loss Transactions

Type of Transaction	Number
Single Payments	60
New Claims	200
Re-openings	70
Closings	270
Outstanding Claims	400
TOTAL	1,000
Amount of unallocated loss expense paid	$10,000

In this illustration we will calculate the portion of the ULE reserve relating to single payments. The average unallocated cost per transaction is $10 ($10,000 ÷ 1,000). The persistence assumption states that if the company were to cancel all business December 31, 1971, the number of single payments in 1972 relating to accidents December 31, 1971 and prior would be

January	60
February	40
March	20
April and subsequent	0
	120

Therefore the single payment portion of the ULE reserve is $1,200 (120 × $10.).

Assuming a persistence pattern is equivalent to assuming a distribution of calendar period payments by accident period. In the case of single payments, the distribution of calendar month payments to accident month which is equivalent to Brian's assumed persistence pattern is

Current accident month	0
First prior accident month	1/3
Second prior accident month	1/3
Third prior accident month	1/3

This assumption permits us to calculate the single payment portion of the ULE reserve using an accident month projection.

DISTRIBUTION OF ULE PAYMENT FRACTIONS BY CALENDAR MONTH AND ACCIDENT MONTH

Calendar Month	Accident Month			
	September	October	November	December
January		1/3	1/3	1/3
February			1/3	1/3
March				1/3

The single payment portion of the ULE reserve is twice the average calendar month ULE paid, or \$1,200 (2 × 60 × \$10.). Of course, this result is the same as the one we obtained by using the persistence pattern.

The basic assumption of this method is that the persistence pattern derived by allocating all ULE payments among only five transactions in equal amounts per transaction is not too different from the true persistence pattern. This is a reasonable assumption, but the only way to test it is to use another method to estimate the persistence pattern and compare the two estimated patterns. The step that would bring the most improvement to ULE reserve estimation would be to devise a better method for estimating the persistence pattern.