DISCUSSION BY DAVID J. GRADY

Credibility is the foundation stone of casualty actuarial science. To the theoretician it offers endless opportunities to advance the mathematical basis of our art; to the practioner it provides a means for charting a course between the twin requirements of insurance pricing: stability and responsiveness. To assign too much credibility to an insurer's experience is to court insolvency; to give too little is to risk adverse selection and a declining portfolio. A prime determinant of the appropriateness of a credibility procedure is the level at which full credibility is established.

Mr. Hansen's paper is a concise exploration of the problem of setting the level of full credibility for estimating claim frequency. He traces a clear path through the current difficulties and proposes a rather elegant solution. I would like to make the path which Mr. Hansen has cleared somewhat broader by commenting on the distributions and assumptions employed in his presentation.

Five probability distributions are utilized in the paper. The claim frequencies for individual insureds are assumed to obey independent Poisson processes. The normal distribution is brought into the paper by means of the Central Limit Theorem. The relationships among the means and variances of the exponential, gamma and negative binomial distributions lead directly to Mr. Hansen's choice for a structure function. Since a knowledge of these measures is fundamental to an understanding of this choice, the table presented below may be of some help in following the author's analysis.

Distribution	*Density Function*	*Mean*	*Variance*
Exponential	$f(\lambda) = \alpha e^{-\alpha\lambda}$	$\frac{1}{\alpha}$	$\frac{1}{\alpha^2}$
Gamma	$f(\lambda) = \frac{\alpha^\beta}{\Gamma(\beta)} e^{-\alpha\lambda} \lambda^{\beta-1}$	$\frac{\beta}{\alpha}$	$\frac{\beta}{\alpha^2}$
Negative Binomial	$p(m) = \binom{m+\beta-1}{m}\left(\frac{\alpha}{\alpha+1}\right)^\beta \left(\frac{1}{\alpha+1}\right)^m$	$\frac{\beta}{\alpha}$	$\frac{\beta}{\alpha}+\frac{\beta}{\alpha^2}$

Thus, the means of the gamma and negative binomial distributions are identical. The variance of the negative binomial distribution is equal to the sum of the mean and variance of the gamma distribution.

Since the coefficient of variation is the standard deviation divided by the mean, the coefficient of variation for the exponential distribution equals one. Similarly, the coefficient of variation for the gamma distribution is $\frac{1}{\sqrt{\beta}}$. Since gamma distributions having increasing failure rates require $\beta > 1$, the coefficient of variation for this class of gamma distributions is bounded above by that of the exponential distribution. Chart I shows two members of the class of gamma distributions with increasing failure rates ($\beta = 2$ and $\beta = 10$) and their limiting exponential ($\beta = 1$). A member of the class of gamma distributions with decreasing failure rates ($\beta = 0.5$) is indicated by a dotted line since this class was disqualified by the author. Since the distributions in Chart I were constructed using a fixed mean, the primary purpose of the graph is to provide visual confirmation for Mr. Hansen's statement that an exponential structure function maximizes the variance for a given value of the mean.

The author dismisses the homogeneity assumption underlying current credibility tables as totally unrealistic. In its place he proposes two new assumptions:

a. The class of gamma distributions with increasing failure rate provides a reasonable set of structure functions for the Poisson parameter.

b. The actuary is able to select an appropriate upper bound for expected claim frequency.

The first assumption appears reasonable from two standpoints:

a. The class of gamma distributions under consideration has considerable flexibility.

b. Fairly good results have been obtained in fitting the negative binomial distribution to actual claim data.

The second assumption appears quite innocuous since such knowledge lies at the heart of our profession. However, the key to this problem lies in the closeness of the upper bound to the actual expected claim frequency.

Mr. Hansen's method of determining full credibility for expected claim frequency consists of two basic steps:

1. Maximize the mean.

2. Maximize the variance associated with that mean.

The author provides us with a method for obtaining the least upper bound for the variance, but we are left to our own devices to find a corresponding methodology for obtaining a least upper bound for the mean itself.

Tables I and II and the result of an attempt to investigate the effects of possible errors in estimating expected claim frequency. These tables are merely an expansion of the table in the original paper. The values in the column for the exponential distribution *($\beta = 1$)* may be compared with any of the "true" values above them to obtain a measure of the effect of selecting a mean which is too high. The selected value may also be compared with any of the "true" values above and to the right of it in order to determine the compound effect of maximizing both mean and variance.

A graphical analysis of the problem is presented in Charts II and III. The importance of obtaining a least upper bound for expected claim frequency is especially evident in Chart III. This graph points up the fact that the expected number of claims is a linear function of the selected upper bound for claim frequency.

Hence, even using Mr. Hansen's method, the practitioner still is torn between the alternatives of stability versus responsiveness. Although I have pointed up the fact that an overconservative insurer utilizing the author's approach may find the competition running away with rather large chunks of its portfolio, my sympathies actually lie with Mr. Hansen's treatment of the overall problem. In the hierarchy of requirements which an insurance company must meet, solvency must outweigh competitiveness.

Chart I

The Gamma Distribution for a Fixed Mean ($\mu = 0.10$) and Selected Values of β

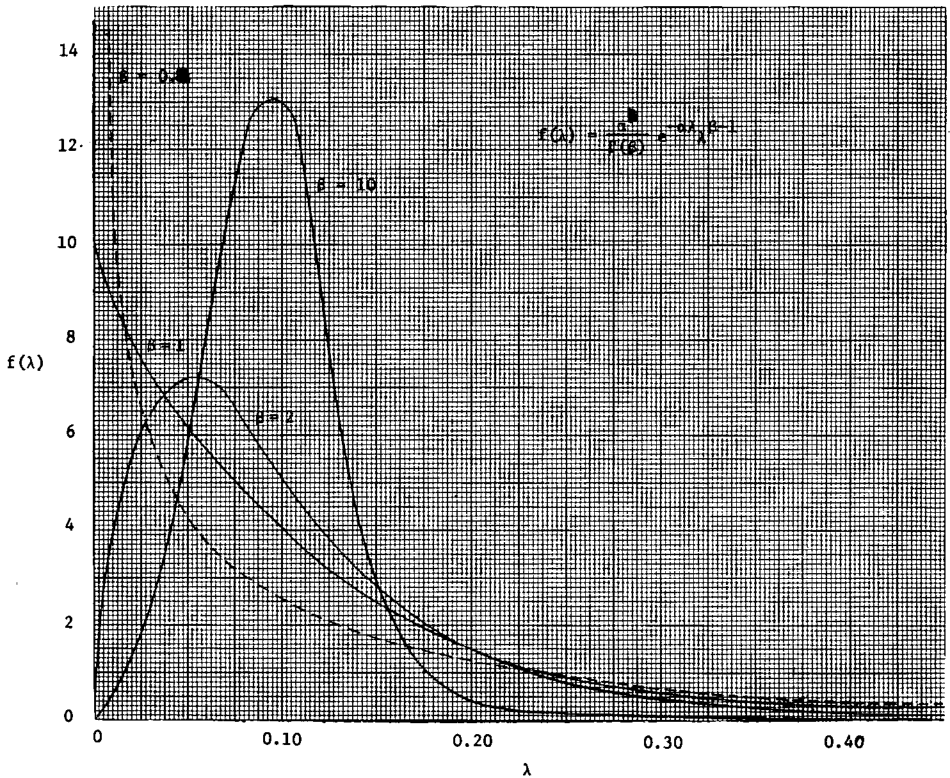

TABLE I

Required Sample Size: Number of Exposure Units

Full Credibility Standards with a Tolerance of Error of 5% and 90% Confidence

Upper Bound for Claim Frequency	*Required Sample Size when β is Equal to:* 0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00	3.00	4.00	5.00
0.05	25,978	23,813	23,091	22,731	22,514	22,370	22,267	22,189	22,009	21,919	21,865
0.10	15,154	12,989	12,267	11,907	11,690	11,546	11,443	11,365	11,185	11,095	11,041
0.15	11,545	9,381	8,659	8,298	8,082	7,938	7,835	7,757	7,577	7,487	7,433
0.25	8,659	6,494	5,773	5,412	5,196	5,051	4,948	4,871	4,690	4,600	4,546
0.35	7,422	5,257	4,536	4,175	3,959	3,814	3,711	3,634	3,453	3,363	3,309
0.50	6,494	4,330	3,608	3,247	3,031	2,886	2,783	2,706	2,526	2,435	2,381
0.75	5,773	3,608	2,886	2,526	2,309	2,165	2,062	1,984	1,804	1,714	1,660
1.00	5,412	3,247	2,526	2,165	1,948	1,804	1,701	1,624	1,443	1,353	1,299
1.50	5,051	2,886	2,165	1,804	1,588	1,443	1,340	1,263	1,082	992	938
2.00	4,871	2,706	1,984	1,624	1,407	1,263	1,160	1,082	902	812	758
3.00	4,690	2,526	1,804	1,443	1,227	1,082	979	902	722	631	577
5.00	4,546	2,381	1,660	1,299	1,082	938	835	758	577	487	433

Table II
Expected Number of Claims
Full Credibility Standards with a Tolerance of Error of 5% and 90% Confidence

Upper Bound for Claim Frequency	*Expected Number of Claims when β is Equal to:*										
	0.25	*0.50*	*0.75*	*1.00*	*1.25*	*1.50*	*1.75*	*2.00*	*3.00*	*4.00*	*5.00*
0.05	1,299	1,191	1,155	1,137	1,126	1,119	1,113	1,109	1,100	1,096	1,093
0.10	1,515	1,299	1,227	1,191	1,169	1,155	1,144	1,137	1,119	1,110	1,104
0.15	1,732	1,407	1,299	1,245	1,212	1,191	1,175	1,164	1,137	1,123	1,115
0.25	2,165	1,624	1,443	1,353	1,299	1,263	1,237	1,218	1,173	1,150	1,137
0.35	2,598	1,840	1,588	1,461	1,385	1,335	1,299	1,272	1,209	1,177	1,158
0.50	3,247	2,165	1,804	1,624	1,515	1,443	1,392	1,353	1,263	1,218	1,191
0.75	4,330	2,706	2,165	1,894	1,732	1,624	1,546	1,488	1,353	1,285	1,245
1.00	5,412	3,247	2,526	2,165	1,948	1,804	1,701	1,624	1,443	1,353	1,299
1.50	7,577	4,330	3,247	2,706	2,381	2,165	2,010	1,894	1,624	1,488	1,407
2.00	9,742	5,412	3,969	3,247	2,814	2,526	2,319	2,165	1,804	1,624	1,515
3.00	14,071	7,577	5,412	4,330	3,680	3,247	2,938	2,706	2,165	1,894	1,732
5.00	22,731	11,907	8,298	6,494	5,412	4,690	4,175	3,788	2,886	2,435	2,165

Chart II

Required Sample Size for Selected Values of β

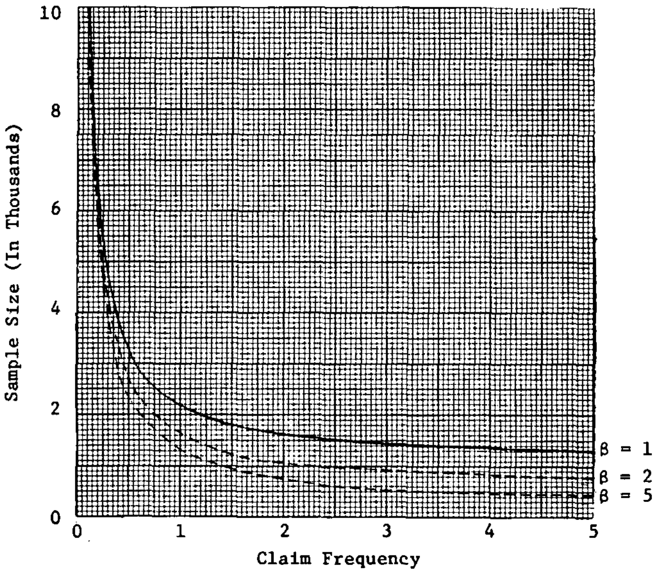

Chart III

Expected Number of Claims for Selected Values of β