JOINT UNDERWRITING AS A REINSURANCE PROBLEM

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The reading requirements for the Associateship and Fellowship examinations have included the theory and the functions of reinsurance. Yet in recent years there have been no papers in the *Proceedings* dealing with this aspect as being practically applied by the actuarial profession. In the day to day dealings of the professional reinsurers, undoubtedly their actuaries are faced with unique reinsurance problems but this is probably not the case for the standard property and casualty actuary.

In the study that follows, the author was presented with the problem of developing a joint underwriting arrangement for two non-profit service plans. One organization was a well-established provider of hospital benefits; the other, a new organization incorporated to provide dental benefits.

To those unfamiliar with the term "service plan", a note of explanation is in order. A service plan contracts with the provider of benefits to deliver benefits (services) within the scope of its certificate (contract) to its members. Under this arrangement the service plan reimburses the provider of services directly. This is contrasted to the standard insurance company approach of reimbursing the contract holder who directly pays the provider of services.

A non-profit service corporation differs from a non-profit mutual in that the insured is not a policyholder and owner of the service organization. Generally, service organizations are regulated either under special legislation or interpretation by the state insurance commissioner. The legislation may prescribe that special reserves be established out of its surplus funds for contingencies. These arrangements may also stipulate that services be provided members even though funds may be lacking to pay the provider in full.

With this as a background, let us examine the problems of the newly formed Dental Service Plan. First of all, its initial available capital would not permit rapid growth without endangering its reserve position. To exist in the market place, it must have underwriting capacity yet maintain financial integrity and protect itself and its providers against a run of unexpected losses.

As a new organization, the start up cost to develop a viable organization could be prohibitive. The Dental Service Plan took these problems to the Hospital Service Plan seeking reasonable and realistic solutions. Inasmuch as all the functions necessary to operate a Dental Plan were the same as those already being performed by the Hospital Plan, it was agreed that the Dental Plan would not duplicate these functions but contract for these services to be performed by the Hospital Plan. The cost of this servicing arrangement was ultimately set as a percent of premium. The only expense that the Dental Plan would be directly responsible for would be for such items as legal, boards, bureaus, etc.

The Hospital Plan was agreeable to jointly underwrite business to provide capacity and maintain an adequate financial status. However, the Hospital Plan wanted to limit its loss to a stated amount within a specified time period. At that point, the Dental Plan would have to look to its providers for relief.

If we examine the various problems and limitations of the two service plans, planning to jointly underwrite, we see that they are similar to those commonly handled by a reinsurance company.

Some basic decisions had to be made by the Dental Plan as regards its underwriting policies and its reserve or surplus position. The use of the term reserve is in the sense that the reserve is surplus and not a loss or expense reserve. The Plan decided to write two types of accounts — 1) underwritten or premium, and 2) cost plus. A premium account represents that business underwritten on a guaranteed rate basis. A cost plus account pays as income its claims or losses (with no limit) plus an administrative charge for claims handling and other expenses. Inasmuch as there was no exposure to risk on cost plus business, the reserve requirements would be related to premium business where, of course, an underwriting loss would have to be offset by any accumulated or available reserve.

The two Plans had to mutually agree upon a reserve position, as in the final analysis, this would determine the degree to which each Plan would share in the gains or losses of the jointly underwritten operation. Further agreement had to be reached as to the maximum amount of underwriting loss the Hospital Plan would sustain during the agreed upon term of the arrangement, so as to determine when joint underwriting would terminate.

At the outset, it was established that the reserve should bear a specific ratio to incurred losses and that the joint underwriting agreement would guarantee this ratio up to the point the Hospital Plan reached the maximum underwriting loss it would sustain. All calculations would be based on the gross underwriting results for each calendar year with interim calculations being based upon twelve months ending gross data. The basic data required to develop the formula is contained on page 4 of the Annual Statement Blank.

To facilitate the calculation and provide a somewhat more accurate allocation of the underwritten business to be jointly shared, the income is split between premium business and cost plus business. Expenses are further categorized as to those purchased via a service contract with the Hospital Plan and those directly incurred by the Dental Plan. The nature of the conditions of the joint underwriting agreement precludes any prospective calculation of the pro rata distribution. The crucial point of the agreement is that the proportion ceded is determined retroactively at the end of the year so as to guarantee the predetermined ratio of reserve at the end of the year to losses incurred.

I will now define in general algebraic terms the elements from page 4, as modified, which represent the contributions to reserve for the year and which then can be translated to the desired ratio of reserve to incurred losses from premium accounts. The subscript 'p' will indicate the data from premium business, the subscript 'c' for cost plus and no subscript for the combined results of premium and cost plus business.

W = desired ratio of reserve to incurred losses for premium written business;

X = portion, in terms of percent, that Dental Plan will retain;

P = premium earned;

L = losses incurred;

E = indirect expense via service contract;

D = expected direct expense provided for in premium;

A = actual direct expense;

U = underwriting gain or loss;

I = income from investment and other sources;

R_B = reserve at beginning of period;

Using the above terms, the reserve at the end (R_E) of the period would be expressed as follows:

$$R_E = R_B + P - L - (A + E) + I$$

Using the same basic approach, we can develop a formula from which we can derive X. In the joint underwriting arrangement the Hospital Plan would return to the Dental Plan that portion of the premium representing the expected direct expenses of the Dental Plan.

$$W = \frac{R_B + X(P_p - L_p - E_p) - A_p + (1 - X)D_p + U_c + I}{XL_p}$$

Solving for X:
$$X = \frac{R_B - A_p + D_p + U_c + I}{L_p(1 + W) + D_p + E_p - P_p}$$

The calculation of X will only occur when $\dfrac{R_E}{L_P}$ is less than W. When $\dfrac{R_E}{L_P} = W$, the equation reduces to unity.

There remains now the development of the maximum loss ratio which the Hospital Plan would sustain to limit its cumulative underwriting loss to the stated maximum.

The joint underwriting in its simplest form becames a form of pro rata reinsurance on a total portfolio of business. The underwriting gain or loss of the Hospital Plan is in direct ratio to that of the Dental Plan. Since X represents the portion of premium business to be underwritten by the Dental Plan then $(1 - X)$ represents the percent to be handled by the Hospital Plan. The underwriting gain or loss incurred by the Hospital Plan for a period would be expressed as $(1 - X)(P_p - L_p - E_p - D_p)$. This sum would be added to any cumulative underwriting results from prior periods. This gives us the basis to determine the maximum allowable loss ratio at which point the joint underwriting arrangement ceases and beyond which the Dental Plan must look to other sources for relief.

If we define Z as the maximum cumulative loss and H_B as the cumulative loss or gain for prior periods we can develop the basic equation from which to develop the maximum loss ratio which we will define as Q.

$$Z = H_B + (1 - X)(P_p - L_p - E_p - D_p)$$

Before substituting for X, let us group certain terms and introduce substitutions and equivalences to simplify the equation.

$$Y = R_B - A_p + D_p + U_c + I$$

$$C = (1 + W)$$

$$L_p = P_pQ$$

$$K = 1 - \frac{(D_p + E_p)}{P_p}$$

$$KP_p = P_p - D_p - E_p$$

Inserting these in the equation for Z produces the following:

$$Z = H_B + \left(1 - \frac{Y}{CP_pQ - KP_p}\right)(KP_p - P_pQ)$$

$$(Z - H_B)(CP_pQ - KP_p) = (CP_pQ - KP_p - Y)(KP_p - P_pQ)$$

$$(Z - H_B)CP_pQ - (Z - H_B)KP_p = CKP_p^2 Q - K^2P_p^2 - KP_pY - CP_p^2 Q^2 + KP_p^2 Q + YP_pQ$$

Dividing by P_p

$$(Z - H_B)CQ - (Z - H_B)K = CKP_pQ - K^2P_p - KY - CP_pQ^2 + KP_pQ + YQ$$

Rearranging the terms and setting the equation to zero produces:

$$CP_pQ^2 + Q[C(Z - H_B) - P_p(CK + K) - Y] + K[KP_p + Y - (Z - H_B)] = 0$$

This is conveniently a quadratic equation which for our use we will define as $aQ^2 + bQ + c = 0$ and further as

$$a = CP_p$$

$$b = C(Z - H_B) - P_p(CK + K) - Y$$

$$c = K[KP_p + Y - (Z - H_B)]$$

$$Q = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Logic would suggest and require that Q, the loss ratio, be positive. An analysis of the coefficient 'b' indicates that its sign is always negative thereby guaranteeing the numerator to be positive.

An empirical approach was used to determine that the positive value of the radical must be used to produce results that would occur in the real world. If the negative value is used, X becomes greater than unity and negative underwriting or cessions are developed.

I have worked out an example illustrating how this approach might be applied as a reinsurance vehicle. In the example, when the maximum loss by Reinsurer A is attained, Reinsurer A is no longer considered as the prime reinsurer and the reduction in the amount of loss (incurred losses) necessary to produce the maximum underwriting loss for Reinsurer A is absorbed by the ceding company or another reinsurer.

In the illustration that follows, assumptions were made in regards to the various elements in the formulas:

$$E = .15P$$
$$D = .03P$$
$$R_B = \$25,000$$
$$W = .10$$
$$Z = \$125,000$$

Also for convenience, let H_E equal the reinsurer's cumulative gain or loss at the end of the period under study. If H_E exceeds Z then Q must be calculated. The ending reserve (H_E) of a period becomes the beginning reserve (H_B) for the next period.

The expenses of the reinsurer (ceded to) are based upon an indirect expense of 15% and direct expense of 3% which would be considered as commission to or a return of the expenses of the direct insurer. The term of the agreement is for a three year period.

In the first year of the program the experience on a direct basis produced an underwriting and operational gain. The ratio of the ending reserve to incurred losses for direct premium business was below the desired ratio of 10% necessitating a cession, the net of which would produce 10%. After the cession the ratio of the net reserve R_E (\$105,200) to net loss incurred (\$1,052,000), equals 10%. The net reserve becomes the beginning reserve for 1972. At this point, the reinsurer has a gain of \$33,700 (H_E) which becomes H_B for 1972.

The second year's experience produced underwriting and operational losses. The reserve ratio dropped below the 10% level requiring a cession. The net ending reserve for the ceding company at this point is $133,820 with the reinsurer's cumulative experience showing a loss of $54,070.

Adverse results for the third year dropped the reserve ratio below 10%. The initial calculation for the cession produced a cumulative loss to the reinsurer in excess of $125,000. Q, the maximum allowable loss ratio to be incurred to limit the loss to $125,000 was calculated. This loss ratio was introduced into the calculation to develop the pro rata amounts to be shared to produce a maximum cumulative loss of $125,000.

	Cost Plus$(_c)$	Direct Business Premium $(_p)$	Total	Premium Accounts Ceded	Retained
Premium (P)	$1,000,000	$3,000,000	$4,000,000	$1,685,000	$1,315,000
Losses (L)	787,400	2,400,000	3,187,400	1,348,000	1,052,000
Expenses:					
Indirect (E)	157,480	450,000	607,480	252,750	197,250
Direct (A)	26,220	75,000	101,220	50,550	24,450
Total	183,700	525,000	708,700	303,300	221,700
Income:					
Net Underwriting Results (U) ...	28,900	75,000	103,900	33,700	41,300
Investment Income (I)			10,000	—	38,900*
Total			113,900	33,700	80,200
Reserve at Beginning of Period			25,000[R_B]	— [H_B]	25,000[R_B]
Reserve at End of Period			138,900	33,700[H_E]	105,200[R_E]
Ratio of Ending Reserve to Losses on Premium Business			5.8%	2.5%	10.0%

*Investment Income and Income from Cost Plus

$$\% \text{ to be ceded} = (1.00 - X) = 1.000 - \left\{ \frac{R_B + (I + U_c) - A_p + .03P_p}{1.1L_p - .82\,P_\rho} \right\}$$

$$= 1.000 - \left\{ \frac{\$25,000 + \$38,900 - \$75,000 + \$90,000}{\$2,640,000 - \$2,460,000} \right\}$$

$$= 1.000 - \frac{\$\ 78,900}{\$180,000}$$

$$= 1.000 - .43833333$$

$$= .56166666$$

CALENDAR YEAR 1972

	Cost Plus($_c$)	Direct Business Premium ($_p$)	Total	Premium Accounts Ceded	Premium Accounts Retained
Premium (P)	$1,500,000	$4,500,000	$6,000,000	$2,925,652	$1,574,348
Losses (L)	1,181,100	3,825,000	5,006,100	2,486,804	1,338,196
Expenses:					
Indirect (E)	236,220	675,000	911,220	438,848	236,152
Direct (A)	39,330	112,500	151,830	87,770	24,730
Total	275,550	787,500	1,063,050	526,618	260,882
Income:					
Net Underwriting Results (U) ...	43,350	(112,500)	(69,150)	(87,770)	(24,730)
Investment Income (I)			10,000		53,350*
Total			(59,150).	(87,770)	28,620
Reserve at Beginning of Period				105,200[R_B] 33,700[H_B]	105,200[R_B]
Reserve at End of Period				46,050 (54,070)[H_E]	133,820[R_E]
Ratio of Ending Reserve to Losses on Premium Business				1.2%	10.0%

*Investment Income and Income from Cost Plus

$$\% \text{ to be ceded} = (1.00 - X) = 1.000 - \left\{ \frac{R_B + (I + U_c) - Ap + .03P_p}{1.1L_\rho - .82 \, P\rho} \right\}$$

$$= 1.000 - \left\{ \frac{\$105,200 + \$53,350 - \$112,500 + \$135,000}{\$4,207,500 - \$3,690,000} \right\}$$

$$= 1.000 - \frac{\$181,050}{\$517,500}$$

$$= 1.000 - .3498551$$

$$= .6501449$$

CALENDAR YEAR 1973

	Direct Business			Premium Accounts		Premium Accounts		
	Cost Plus (c)	Premium (p)	Total	Ceded	Retained	First Retention	Ceded	Retained
Premium (P)	$2,500,000	$7,500,000	$10,000,000	$5,295,044	$2,204,956	$7,500,000	$4,909,686	$2,590,314
Losses (L)	1,968,500	6,375,000	8,343,500	4,500,787	1,874,213	6,258,352	4,096,872	2,161,480
Expenses:								
Indirect (E)	393,700	1,125,000	1,518,700	794,257	330,743	1,125,000	736,453	388,547
Direct (A)	65,550	187,500	253,050	158,851	28,649	187,500	147,291	40,209
Total	459,250	1,312,500	1,771,750	953,108	359,392	1,312,500	883,744	428,756
Income:								
Net Underwriting Results (U)	72,250	(187,500)	(115,250)	(158,851)	(28,649)	(70,852)	(70,930)	78
Investment Income (I)			10,000		82,250*			82,250*
Total			(105,250)		53,601			82,328
Reserve at Beginning of Period				133,820[R_B]	(54,070)[H_B]	133,820[R_B]	(54,070)[H_B]	133,820[R_B]
Reserve at End of Period				28,570	(212,921)[H_E]**	187,421[R_E]	(125,000)[H_E]	216,148[R_E]
Ratio of Ending Reserve to Losses on Premium Business				0.4%		10.0%		10.0%

*Investment Income and Income from Cost Plus
**Exceeds Contractual Limit ($H_E > Z$) Calculate Q

Step #1 % to be ceded = $(1.00 - X) = 1.00 - \left\{ \dfrac{R_B + (I + U_c) - Ap + .03P_p}{1.1L\rho - .82P\rho} \right\}$

$= 1.000 - \left\{ \dfrac{\$133,820 + \$82,250 - \$187,500 + \$225,000}{\$7,012,500 - \$6,150,000} \right\}$

$= 1.000 - \dfrac{\$253,570}{\$862,500}$

$= 1.000 - .2939942$

$= .7060058$

Step #3 % to be ceded = $(1.00 - X) =$

$1.000 - \left\{ \dfrac{:\$133,820 + \$82,250 - \$187,500 + \$225,000}{\$6,884,187 - \$6,150,000} \right\} =$

$1.000 - \dfrac{\$253,570}{\$734,187} =$

$1.000 - .34537522$

$.65462478$

Step #2 $Q = \dfrac{\$13,246,593 + \sqrt{\$272,256,107,649}}{\$16,500,000} = \dfrac{\$13,768,375}{\$16,500,000} = .83444693$