ACTUARIAL NOTE ON WORKMEN'S COMPENSATION LOSS RESERVES

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"Not only is there but one way of doing *things rightly, but there is only one way of* seeing *them, and that is, seeing the whole of them."*

— John Ruskin

In the calculation of tabular reserves for long term pension type awards special care must be used when an excess of loss reinsurance coverage is involved. In this situation some or all of the parties interested in the transaction — ceding company, reinsurer, and regulator — frequently do not understand, or sometimes are not even aware of, the proper way to calculate the ceded reserve and, of course, this usually means that the net reserve is also incorrect.

If, for example, a case involving a permanently disabled individual aged 45 with a life pension award of $7,142 a year, $137.34 weekly, is presented for reserving, and assuming that the liability is to be discounted for interest (3%) and for mortality using the Survivorship Annuitants' Table of Mortality, and further assuming that a reinsurance contract providing coverage excess of $50,000 retention is in effect, the reserves are often incorrectly calculated as follows:

Direct Reserve: \$7,142 $\bar{a}_{45}$ or \$129,280, since $\bar{a}_{45} = 18.101$ from the last column of the accompanying table (which column incidentally is found as Table XI in the State of New York Workmen's Compensation Board's Bulletin No. 222). The correct annuity factor is actually $\ddot{a}^{(m)}_{45}$ but since m is fairly large, 52, when weekly payments are involved $\bar{a}_{45}$ is often used since $\lim_{m \to \infty} \ddot{a}^{(m)}_{x} = \bar{a}_x$, and $\bar{a}_x$ is often approximated by $a_x + \frac{1}{2}$ or $\ddot{a}_x - \frac{1}{2}$. In terms of commutation functions this becomes $\frac{N_{x+1}}{D_x} + \frac{1}{2}$ which can be written as $\frac{N_{x+1} + \frac{1}{2} D_x}{D_x}$ or $\frac{1}{2} \frac{(N_x + N_{x+1})}{D_x}$ and this approximation appears to be incorporated in the New York Board's Table.

Having calculated the direct reserve of $129,280 one might then conclude that since a $50,000 retention is operating on this loss, the net reserve must be $50,000 and therefore the ceded reserve must be $79,280 (i.e. $129,280 — $50,000).

The direct reserve above is correct but the ceded and net reserves are not. The error arises in part from the fact that assuming a net reserve equal to the retention of $50,000 ignores the possibility that the annuitant may not survive long enough to exhaust the $50,000 retention. What must happen to cause the $50,000 retention to be exhausted? The annuitant must collect for seven years $\frac{\$50,000}{\$\ 7,142}$ and the probability of his surviving seven years is obviously less than one. Therefore, the expected value of this obligation is less than $50,000; in fact, it must be $7,142 $\bar{a}_{45:\overline{7|}}$ or $43,885. The only time the net incurred can be $50,000 is when the ceding company has actually paid benefits in the amount of $50,000. It is interesting to note that when $7,142 $\bar{a}_{45:\overline{7|}}$ is evaluated at 0% interest (*i.e.*, discounting for mortality only) the reserve becomes $48,523.

To put it another way, if the ceding company has one hundred similar cases, some annuitants would collect for eight or more years and the ultimate net incurred would be $50,000 on each of those cases, but some annuitants will survive only one year and have an ultimate incurred of $7,142, some will survive two years and have an ultimate incurred of $14,285 *etc.* It then is obvious that the average ultimate net incurred loss must be less than $50,000.

In addition to failing to take mortality into account, the above reserving method presents a similar problem as respects interest discounting. Even if it were certain the annuitant would survive seven years the present value of this obligation would be less than $50,000, since the funds set aside as a reserve would be augmented in this case by the assumed 3% investment income. The amount needed would be $7,142 $\bar{a}_{\overline{7|}}$ or $45,161.

The correct way to calculate the various reserves is to break the gross or direct reserve into its component pieces. The net reserve must be based on a temporary life annuity, thus taking into account both the mortality and interest discounting discussed above. The ceded reserve is based on a deferred annuity — deferred by the number of years needed to exhaust the ceding company retention.

Direct	Net	Ceded
$\$7{,}142\ \bar{a}_{45}$	$= \$7{,}142\ \bar{a}_{45:\overline{7}\rceil}$	$+ \$7{,}142\ {}_{7}\vert\bar{a}_{45}$

or

$$\$7{,}142\,\frac{\overline{N}_{45}}{D_{45}} = \$7{,}142\,\frac{\overline{N}_{45} - \overline{N}_{52}}{D_{45}} + \$7{,}142\,\frac{\overline{N}_{52}}{D_{45}}$$

To calculate the above, the $\overline{N}_x$ and D_x values underlying New York's Table XI are needed. These values are shown in the accompanying table, but the New York Special Bulletin does not show the N_x and D_x values which underlie the $\bar{a}_x$ values, and since this booklet is widely used, the lack of data has no doubt contributed to the confusion that seems to exist concerning proper reserving techniques. The $\overline{N}_x$ and D_x values are derived from the Survivorship Annuitants' Mortality Table for ages 15 through 95. The Survivorship Annuitants' Mortality Table is a "Makehamized" Table (*i.e.*, the l_x values observed were graduated using Makeham's formula $l_x = ks^x g^{c^x}$) for ages 15 through 95 with constants of $\log s = -.0022402$, $\log c = .04579609$, $\log g = -.000093999$, and $\log k = 5.0226717$, all to the base 10. For these ages, the values shown in the accompanying table are consistent with the $\bar{a}_x$ values shown in Table XI in the State of New York Workmen's Compensation Board's Special Bulletin No. 222.

For ages over 95, the $\bar{a}_x$ values in New York's Bulletin No. 222 are not based on the Survivorship Annuitants' Mortality Table. Accordingly the $\overline{N}_x$ and D_x values shown in the table for ages over 95 were calculated to be consistent with New York's $\bar{a}_x$ values for ages over 95 and with the N_x and D_x values for age 95. For ages under 15 the values were calculated directly from the Makeham formula since neither the New York Workmen's Compensation Board's Bulletin No. 222 nor the Survivorship Annuitants' Mortality Table extends below age 15. It is recognized that the values shown for under 15, and possibly the young adult ages, are not entirely satisfactory since no attempt was made to modify the Makeham formula or adjust the constants. Since the Makeham formula has no minima or maxima or points of inflection, it cannot (without modification) accurately portray some of the peculiarities often observed in the mortality curve below the age of 25.

Following are several examples of correct and incorrect reserves assuming an annual pension of $7,142 (with weekly payments).

Retention $50,000

	Correct		Incorrect	
Age	Ceded	Net	Ceded	Net
45	$85,395	$43,885	$79,280	$50,000
55	60,812	42,969	53,781	50,000

Retention $100,000

	Correct		Incorrect	
Age	Ceded	Net	Ceded	Net
45	$52,312	$76,968	$29,280	$100,000
55	30,713	73,068	3,781	100,000

It is clear from the above examples that the difference between the correct and incorrect reserves can be quite significant. Actuaries might well inquire as to how such matters are handled in their own companies.

LIFE AWARDS ON PERMANENT DISABILITY CASES

Present Value of $1 per annum Payable Until Death, Based on Survivorship Annuitants' Mortality Table and Interest at 3%

(This table does not provide for remarriage factors; but values for ages after 65 can be used for valuing widows' pensions)

Age(x)	D_x	$\overline{N}_x$	$\bar{a}_x$
0	105,336.000	2,781,362.447	26.405
1	101,739.806	2,677,824.544	26.320
2	98,265.624	2,577,821.829	26.233
3	94,909.341	2,481,234.346	26.143
4	91,667.875	2,387,945.738	26.050
5	88,537.303	2,297,843.149	25.953
6	85,513.005	2,210,817.995	25.854
7	82,591.396	2,126,765.794	25.750

Age(x)	D_x	$\overline{N}_x$	$\bar{a}_x$
8	79,769.013	2,045,585.589	25.644
9	77,042.509	1,967,179.828	25.534
10	74,409.391	1,891,453.878	25.420
11	71,865.747	1,818,316.309	25.302
12	69,407.851	1,747,679.510	25.180
13	67,034.212	1,679,458.479	25.054
14	64,740.623	1,613,571.062	24.924
15	62,525.056	1,549,938.223	24.789
16	60,384.253	1,488,483.568	24.650
17	58,316.325	1,429,133.279	24.507
18	56,318.220	1,371,816.006	24.358
19	54,388.179	1,316,462.807	24.205
20	52,522.789	1,263,007.323	24.047
21	50,721.000	1,211,385.428	23.883
22	48,979.089	1,161,535.383	23.715
23	47,296.635	1,113,397.521	23.541
24	45,670.145	1,066,914.131	23.361
25	44,098,755	1,022,029.681	23.176
26	42,579.232	978,690.688	22.985
27	41,111.265	936,845.439	22.788
28	39,692.251	896,443.681	22.585
29	38,320.174	857,437.468	22.376
30	36,993.527	819,780.617	22.160
31	35,710.853	783,428.427	21.938
32	34,470.736	748,337.632	21.709
33	33,271.058	714,466.735	21.474
34	32,110.557	681,775.928	21.232
35	30,988.011	650,226.644	20.983
36	29,901.891	619,781.693	20.727
37	28,850.741	590,405.377	20.464
38	27,833.179	562,063.417	20.194
39	26,847.893	534,722.881	19.917
40	25,893.630	508,352.120	19.632
41	24,969.203	482,920.703	19.341
42	24,073.481	458,399.361	19.042
43	23,205.109	434,760.066	18.736
44	22,363.085	411,975.969	18.422

Age(x)	D_x	$\overline{N}_x$	a_x
45	21,546.459	390,021.197	18.101
46	20,753.554	368,871.190	17.774
47	19,984.072	348,502.377	17.439
48	19,236.243	328,892.220	17.098
49	18,509.384	310,019.407	16.749
50	17,802.389	291,863.520	16.395
51	17,114.453	274,405.099	16.034
52	16,444.609	257,625.568	15.666
53	15,791.751	241,507.388	15.293
54	15,155.069	226,033.978	14.915
55	14,533.617	211,189.635	14.531
56	13,926.531	196,959.561	14.143
57	13,333.206	183,329.692	13.750
58	12,752.366	170,286,906	13.353
59	12,183.735	157,818.855	12.953
60	11,626.207	145,913.884	12.550
61	11,079.286	134,561.137	12.145
62	10,542.363	123,750.312	11.738
63	10,014.735	113,471.763	11.330
64	9,495.9302	103,716.430	10.922
65	8,985.5276	94,475.7015	10.514
66	8,483.1555	85,741.3600	10.107
67	7,988.4857	77,505.5394	9.702
68	7,501.6349	69,760.4791	9.299
69	7,022.3176	62,498.5028	8.900
70	6,550.7915	55,711.9483	8.505
71	6,087.4103	49,392.8474	8.114
72	5,632.4882	43,532.8981	7.729
73	5,186.7672	38,123.2704	7.350
74	4,750.8981	33,154.4377	6.979
75	4,326.2143	28,615.8815	6.615
76	3,913.6716	24,495.9386	6.259
77	3,514.8155	20,781.6951	5.913
78	3,131.1876	17,458.6936	5.576
79	2,764.4087	14,510.8955	5.249
80	2,416.3392	11,920.5216	4.933
81	2,088.7550	9,667.9745	4.629

Age (x)	D_x	$\overline{N}_x$	$\bar{a}_x$
82	1,783.4301	7,731.8820	4.335
83	1,502.0312	6,089.1513	4.054
84	1,245.8651	4,715.2032	3.785
85	1,016.0746	3,584.2333	3.528
86	813.2519	2,669.5700	3.283
87	637.4287	1,944.2297	3.050
88	488.1464	1,381.4422	2.830
89	364.2336	955.2522	2.623
90	264.1173	641.0767	2.427
91	185.5462	416.2450	2.243
92	125.7632	260.5903	2.072
93	81.9761	156.7206	1.912
94	51.1329	90.1661	1.763
95	30.4014	49.3990	1.625
96	17.116	25.640	1.498
97	9.081	12.541	1.381
98	4.512	5.744	1.273
99	2.084	2.446	1.174
100	0.887	0.961	1.084
101	0.345	0.345	1.002
102	0.121	0.112	.928
103	0.038	0.033	.861

Reprints of Papers from Previous Proceedings

The following five papers are being reprinted in this volume of the *Proceedings* in order to make available to the membership and students a few of the pertinent papers published in previous *Proceedings*. Volumes of these *Proceedings* are presently out of print and are difficult to obtain. Since all of these papers are required reading in the examination syllabus and are still applicable to current practices, it is hoped these reprints will supply a readily accessible source for reference.

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Editor