DISCUSSIONS OF PAPER PUBLISHED IN VOLUME LVII

CREDIBILITY FOR SEVERITY

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DISCUSSION BY HANS U. GERBER*

The credibility formulae discussed in this paper may be satisfactory from an *experience rating point of view,* where the premium of a particular risk is only influenced by the *total amount of its claims* experienced in the past. Thus a risk with 10 claims of $1,000 each is rated the same as a risk with just one claim of size $10,000.

However, from a *statistical point of view,* these credibility formulae seem to be oversimplified because they fail to distinguish between the credibility of the claim severity and the credibility of the claim frequency experienced. This simplification may be the reason why Hewitt observes a "reduction in credibility." In the sequel we shall present a credibility formula which is able to distinguish between the credibility of the severity and the one of the frequency.

To establish the terminology, we assume that the claims of each individual risk (described by its two parameters λ, θ) form a compound Poisson process with Poisson parameter λ (expected number of claims per unit time) and distribution $F^{(\theta)}(x)$ of the single claim amounts. With $\mu(\theta)$ and $\sigma^2(\theta)$ we denote the expected value and the variance, respectively for the claim amount of a given risk.

The distributions of λ and θ are supposedly known. However, we need only the values of:

$$m = E[\mu(\theta)] \quad , \quad Var[\mu(\theta)] \quad , \quad E[\sigma^2(\theta)]$$

$$k = E[\lambda] \quad , \quad Var[\lambda]$$

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From now on we consider a particular risk (for which we don't know the parameter values). If this risk showed n claims up to time t, let:

$$\overline{m} = \frac{S_1 + S_2 + \ldots S_n}{n}$$

be the average claim size observed and:

$$\bar{k} = \frac{n}{t}$$

the average claim frequency observed.

A credibility formula is an expression which estimates the (Bayesian) conditional expectation:

$$E[\lambda\mu(\theta) \mid \bar{k}; S_1, S_2, \ldots, S_n]$$

Let us consider credibility formulas of the form:

$$akm + b\bar{k}m + ck\overline{m} + d\bar{k}\overline{m}$$

(rather than of the form $akm + b\bar{k}\overline{m}$, as Hewitt does). According to Bühlmann's concept, we determine $a,b,c,$ and d in order to minimize the expected squared deviation of the credibility premium from $E[\lambda\mu(\theta) \mid \bar{k}; S_1, S_2, \ldots, S_n]$. Assuming that λ and θ are independently distributed, one finds:

$$a = \left(\frac{\dfrac{k}{Var[\lambda]}}{t + \dfrac{k}{Var[\lambda]}}\right) \cdot \left(\frac{\dfrac{E[\sigma^2(\theta)]}{Var[\mu(\theta)]}}{n + \dfrac{E[\sigma^2(\theta)]}{Var[\mu(\theta)]}}\right)$$

$$b = \left(\frac{t}{t + \dfrac{k}{Var[\lambda]}}\right) \cdot \left(\frac{\dfrac{E[\sigma^2(\theta)]}{Var[\mu(\theta)]}}{n + \dfrac{E[\sigma^2(\theta)]}{Var[\mu(\theta)]}}\right)$$

$$c = \left(\frac{\dfrac{k}{Var[\lambda]}}{t + \dfrac{k}{Var[\lambda]}}\right) \cdot \left(\frac{n}{n + \dfrac{E[\sigma^2(\theta)]}{Var[\mu(\theta)]}}\right)$$

$$d = \left(\frac{t}{t + \dfrac{k}{Var[\lambda]}}\right) \cdot \left(\frac{n}{n + \dfrac{E[\sigma^2(\theta)]}{Var[\mu(\theta)]}}\right)$$

Thus the credibility premium may be written as the product:

$$\{(1 - z_1)k + z_1\bar{k}\} \ \{(1 - z_2)m + z_2\bar{m}\}$$

with:

$$z_1(t) = \frac{t}{t + \dfrac{k}{Var[\lambda]}}$$

$$z_2(n) = \frac{n}{n + \dfrac{E[\sigma^2(\theta)]}{Var[\mu(\theta)]}}.$$

We notice that the two credibilities are properly distinguished now.

For Hewitt's numerical example we find:

$$z_1(t) = \frac{t}{t + 30.1}$$

$$z_2(n) = \frac{n}{n + \dfrac{e^{\sigma^2} - 1}{e^{S^2} - 1} e^{S^2}} = \frac{n}{n + 54.9}$$

Finally, we remark that the assumption of independence between λ and θ is not necessary for the construction of the above described credibility premium. However, in the general case, it will not be possible to write the credibility premium:

$$akm + b\bar{k}m + ck\bar{m} + d\bar{k}\bar{m}$$

in product form (as it was possible in the case of independence).

DISCUSSION BY HANS BÜHLMANN*

This is an inspiring paper very clearly written and well presented. I hope that the point made by Mr. Hewitt comes home, namely that credibility is theoretically justifiable and eminently practical. The main contribution of this paper is the explicit application of general credibility techniques to the

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