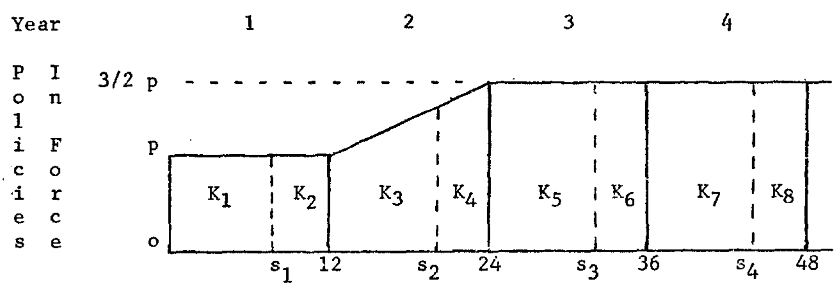
DISCUSSION BY CHARLES F. COOK

This actuarial note lives up to Mr. Simon's reputation for pertinent and handy mathematical models. In addition to providing a workable solution for a specific problem, it clearly illustrates the general technique of projection by geometric areas. This is a common method of modeling a variety of small actuarial problems, but one which is easy to mishandle in practice. Those actuaries who infrequently do this kind of thing should find it beneficial to follow the pattern of mathematical development in this paper, but substitute the precise values and formulas for their own problems as they go. For students, this paper is a "must."

When a new statistical plan or line of business is introduced, individual companies can usually develop proper IBNR reserves by combining data under a new statistical plan with those under the old plan until the new data base is matured. If a new line of business is a package, the same thing can be accomplished by temporarily combining package business with a proper mixture of closely related old lines. In the package case, the same procedure may sometimes be possible for a bureau or statistical agent. Such a treatment, which has long been used, is perfectly reasonable, and Mr. Simon's method is not required in these cases. Combination procedures, however, can only be applied if comparable prior data are available. They generally do exist for an individual company (except in the case of a completely new line of business), but at the bureau level this situation is less likely. Comparable prior data are absent not only for new bureaus, new policies, or new statistical plans, as cited by Mr. Simon, but more generally any time there is a dramatic increase in the volume of data being reported. If, for instance, several new companies began reporting one-year term business on January 1 of year 2, with a volume of policies equal to one-half of the previous total, we could sketch the situation in the notation of this paper as:



Here K_4/K_3 is equal to L_6/L_5 in Figure 2 of Mr. Simon's paper, so $B_{36:24}$ would be equal to his $B_{48:36}$ with $q=0$. The value of $B_{36:12}$, however, would be B_{12}, because $K_2/K_1 = K_6/K_5$, and the formula for $B_{24:12}$ would have to correct for a distortion *opposite* to the one treated in the paper. The point of this little example is to show that this paper has usefulness beyond its declared purpose of solving a single specific problem. A formula for $B_{24:12}$ in the case above, which arises from a different source and has an opposite bias, could easily be determined using Mr. Simon's problem as a guiding model, following his procedure but substituting the appropriate formulas for *K's* and *s's* determined from the drawing of this problem.

Other Types of IBNR Base

I have a strong personal preference for earned premium or premiums in force (rather than reported losses) as an IBNR base, because they have less random variation. My preference has been reinforced by this paper for a new reason. If the IBNR factor B_d' were defined as the ratio of unreported losses as of the close of the accounting period, to either the premiums in force at the end of the period or the premiums earned in the final quarter of the period, reference to the drawings makes it clear that the distortion would be far less. The correction of the distortion would also be easier. Let us assume, for example, that we are trying to evaluate B_{36}' and that we have determined that the unreported losses were U and the reported losses R. Then $s = \sqrt{\dfrac{R}{2(R+U)}}$. If the premiums in force were P_1 at the beginning of the year and P_2 at the end of the year, they averaged

$$[P_1 + P_2 + s(P_2 - P_1)] \div 2$$

during the same period, from s to the end of the year, as the unreported claims were incurred under Mr. Simon's assumptions. Then

$$B' = \frac{2U}{P_1 + P_2 + s(P_2 - P_1)}$$

is an IBNR factor which can be applied to *any* mix of one and three year terms, at *any* point in time, on the base $P_1 + P_2 + s(P_2 - P_1)$ where P_1 and P_2 are defined as above, for the year to which B' is to be applied. This formula will *exactly* reproduce Mr. Simon's results. Now we know that in fact the average date of occurrence for our unreported claims is earlier than $(s + 12) \div 2$, as we assumed above. If we have found (or estimated) it to be A, then $B' = \dfrac{U}{P_1 + A(P_2 - P_1)}$ will be a *better* IBNR factor. This alternative but similar procedure does not take anything away from Mr. Simon's solution, because it is not compatible with a purely pure premium ratemaking method. (It makes total incurred losses partially dependent on the level of premium charged.) However, for calculating reserves at the company level, for a loss-ratio type ratemaking procedure, or for instances where Mr. Simon's assumptions are not valid, it does have some virtues worth considering.

Test of the Procedure

Mathematically, the procedure given by the author is exact to the extent his assumptions hold true. He has avoided critical assumptions very well, only one being significant — the assumption that the IBNR consists of the latest claims in the accident year. Mr. Simon gave a clear discussion of this point, concluding that: "This is not felt to be a limiting assumption." In order to test this assumption independently of the random variations which always occur in real-world applications, I compared the predictions of Mr. Simon's model against "artificial" results built by a model essentially identical to his except that the average actual[1] distributions of accident month by report month for the United Services Automobile Association for 1969 were applied uniformly to each month of the model. The build-up of the in-force followed his model exactly, and the loss distributions were adjusted to eliminate growth. This model was then run through a computer, yielding a set of IBNR factors at 12, 24, 36, and 48 months. Mr. Simon's formulas were then applied to "predict" the later IBNR factors from the earlier ones, and his projections were compared to the "actual" results. Because the loss distributions were constant over time and all other assumptions were identical, random and systematic prediction errors were eliminated. Thus the only source of "error" in this test was the bias resulting from disregarding the actual distribution of IBNR accident dates. The results of three tests for Homeowners for various proportions of three-year business are shown in Tables 1, 2, and 3.

Careful consideration of Mr. Simon's Figures 1 and 2 leads to the conclusion that these results can be generalized beyond the specific data used. He has treated all IBNR losses as occurring in the latest possible time period, when the policies in-force are at their maximum. Because the ratio of

[1] Only claim frequency was considered, for stability. All claims not reported by the twelfth month after the close of the accounting period were assumed to be reported in the eighteenth month.

TABLE 1

Homeowners with $q = 0.2$

Test Item	Test Value	True Value	Pct. Error
$B_{24:12}$	1.1678	1.1879	−1.69
$B_{36:24}$	1.1874	1.1875	−0.01
$B_{48:36}$	1.1812	1.1824	−0.10
$B_{36:12}$	1.1674	1.1875	−1.69
$B_{48:24}$	1.1812	1.1824	−0.10

TABLE 2

Homeowners with $q = 0.5$

Test Item	Test Value	True Value	Pct. Error
$B_{24:12}$	1.1801	1.1989	−1.57
$B_{36:24}$	1.1952	1.1959	−0.06
$B_{48:36}$	1.1794	1.1824	−0.25
$B_{36:12}$	1.1767	1.1959	−1.61
$B_{48:24}$	1.1787	1.1824	−0.31

TABLE 3

Homeowners with $q = 0.8$

Test Item	Test Value	True Value	Pct. Error
$B_{24:12}$	1.1989	1.2159	−1.40
$B_{36:24}$	1.2032	1.2054	−0.18
$B_{48:36}$	1.1775	1.1824	−0.41
$B_{36:12}$	1.1872	1.2054	−1.51
$B_{48:24}$	1.1755	1.1824	−0.58

policies in-force to the 12-months-earlier policies in-force decreases monotonically in the model, this leads to projecting the *minimum* possible increase in IBNR and the *maximum* possible increase in reported cases. Any deviation from this — any IBNR loss which is earlier in time than *any* reported loss — should get a higher "leverage" in its projection to a later date than it gets from Mr. Simon's model. Therefore, although they are reasonably close, his IBNR factors are biased downward. That is, IBNR will tend to be consistently, although slightly, underestimated. In the case of projections

$B_{c:24}$ and $B_{c:36}$, the magnitude of the distortion remaining after correction by the Simon formulas is negligible, but for the factors $B_{c:12}$ it is sufficient to be disturbing. An underestimate in incurred losses of about 1.5% translates into a lot of dollars in rates. It would therefore be reasonable, if permitted, to *add .01 to the formulas for* $B_{24:12}$ *and* $B_{36:12}$. This is sufficiently overjustified by USAA data (15,581 claims), which indicate an adjustment of +.015, so that one can be confident that it would be *at least* .01 for broader-based accident month/report month distributions.

Scope of Application

This paper was oriented specifically to personal property lines, for an accident year valued as of 12 months (immediately at the close of the year). It is interesting to investigate whether the procedure will work acceptably well in other lines or at other valuation dates. I repeated the same test discussed above four more times — for automobile liability (BI and PD combined) and for automobile physical damage valued at 12 months, and for automobile physical damage and homeowners valued at 15 months (considering losses for a calendar-accident year as IBNR only if they were still unreported as of the following March 31). The results of these tests are shown in Tables 4 and 5.

TABLE 4

$B_{24:12}$ $(q = 0)$

Test Item	Test Value	True Value	Pct. Error
Auto liability	1.1099	1.1360	−2.30
Auto physical damage	1.1043	1.1226	−1.63
Auto physical damage ($B_{27:15}$)	1.0131	1.0177	−0.45

TABLE 5

Homeowners with $q = 0.5$
Evaluated at 15 months

Test Item	Test Value	True Value	Pct. Error
$B_{24:12}$	1.0257	1.0331	−0.72
$B_{36:24}$	1.0325	1.0328	−0.03
$B_{48:36}$	1.0301	1.0315	−0.14
$B_{36:12}$	1.0252	1.0328	−0.74
$B_{48:24}$	1.0298	1.0315	−0.16

These results confirm the position that Mr. Simon's procedure is sensitive to delays in reporting claims which increase the "spread" of IBNR accident dates. This is especially clear in the 15-month valuation tests. Such later valuations eliminate the vast majority of IBNR claims, which are reported reasonably promptly after occurrence. The remaining IBNR have a much less compact distribution over time and as a result, although the total errors in incurred losses are reduced because there is less total IBNR, the error becomes quite large when compared to the IBNR itself. On this basis the error for automobile physical damage is -26.0% and for $B_{24:12}$ in homeowners it is -22.4%.

Conclusion

Mr. Simon has produced an adequate procedure for controlling the distortion in IBNR factors for personal property insurance during the period following introduction of a new policy, bureau, or statistical plan. He has also provided a set of very simple linear equations to predict IBNR factors under the conditions for the specific problem at hand i.e., a 50-50 mix of one and three-year term policies which eliminate the need for any further significant effort to solve the particular problem.

The results are not perfect. Based on a partially simulated and partially real model, the estimates appear to have a downward bias. In the case of prediction from first-year reports it is about -1.5% which might be considered a significant understatement of losses. The mixture of policy terms makes this particular problem rather complex, however, and his result is certainly far better than the results of other, simpler procedures.

The apparent inability of Mr. Simon's soundly conceived, rather complex procedure to eliminate distortion more completely only highlights the need for better models of the loss development phenomenon. Certainly these test results detract nothing from the paper or the author's workmanship. It is a significant step toward more sophisticated actuarial forecasting of ultimate loss levels.