DISTORTION IN IBNR FACTORS

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The purpose of this actuarial note is to set forth the reasoning and the results involved in a small actuarial problem which occurs from time to time. By establishing a record in our *Proceedings* it is hoped that other actuaries will not have to go through the process of solving the problem independently each time it arises.

When a new line of business is introduced, policies are put on the books in a manner which cumulatively places more and more business in force. We would expect losses to be incurred roughly in proportion to the number of policies in force and, since there are more policies in force at the end of a given accounting period than at the beginning of the period, a factor which measures the incurred but not reported (IBNR) losses will be influenced (that is, distorted) by the relatively heavier weight of policies in force at the end of the accounting period. This same effect also occurs when one changes statistical plans and begins putting business on the books under a new program. It is in this latter vein that these investigations were conducted and this paper is written.

Figure 1 illustrates the build-up of one-year policies from the start of the new program (assumed to be January 1 of a year) until the entire block of business has been converted to the new program at the end of twelve months. The scale "Policies in Force" has been labeled to reach p at the end of the twelve-month period where p represents the proportion of all policies which are on a one-year basis. Figure 2 illustrates the build-up of three-year policies from the start of the new program until the entire block of business has been converted to the new program by the end of thirty-six months. The scale here has been labeled to reach q at the end of the three-year period. We are assuming that only one-year and three-year policies are involved so that $p + q = 1$. Note that it is also assumed that there are a very large number of policies which go on the books in a smooth and regular fashion.

Let us now define the IBNR factor, B, as all incurred losses of a given accident year evaluated at some subsequent date divided by the incurred losses for such accident year reported as of the close of the accounting period. This factor will be subscripted to denote the close of the accounting

FIGURE 1

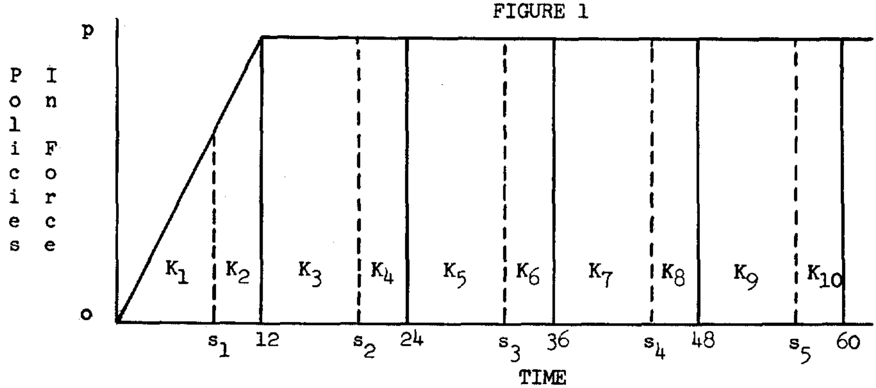
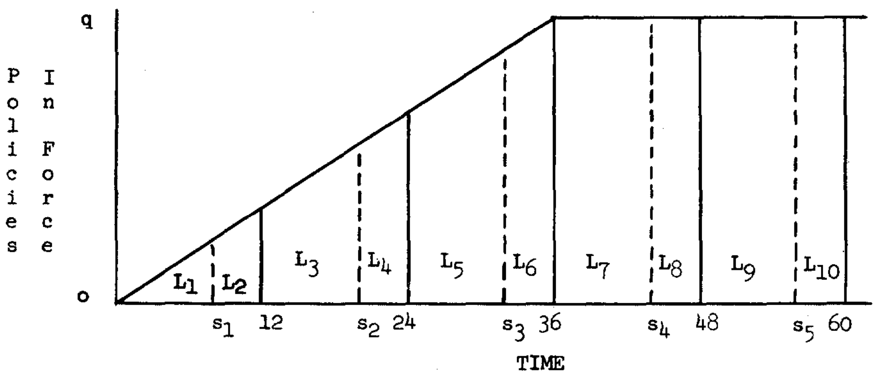


FIGURE 2



period such that B_{12} is the IBNR factor when accounts are closed after the twelfth month.

Our objective is to convert the factor B into an equivalent time period representing the portion of time that is missing from the losses. For example, a factor $B = 1.14$ will represent, under certain conditions, the loss of 1.5 months of incurred losses from the data. We will then assume that during some subsequent accounting period the same conditions will maintain and 1.5 months of losses will be missing from that accounting period also. Note that we are treating the 1.5 figure as a sharp line of demarcation whereas we know from experience that unreported losses will be distributed about this line to some extent. This is not felt to be an inhibiting assumption. These demarcation points are shown along the Time axis of the graphs and are labeled "s." Next we will set forth the formulas which express the area of each of the sections of the two graphs. Following this we will set forth formulas for IBNR factors in terms of these areas.

$$K_1 = ps_1^2/24$$

$$K_1 + K_2 = 6p$$

$$\left.\begin{aligned} K_i &= p[s_{(i+1)/2} - 6(i-1)] \\ K_i + K_{i+1} &= 12p \end{aligned}\right\} \quad i = 3, 5, 7, 9, \ldots.$$

$$L_1 = qs_1^2/72$$

$$L_1 + L_2 = 2q$$

$$L_3 = q(s_2^2 - 144)/72$$

$$L_3 + L_4 = 6q$$

$$L_5 = q(s_3^2 - 576)/72$$

$$L_5 + L_6 = 10q$$

$$\left.\begin{aligned} L_j &= q[s_{(j+1)/2} - 6(j-1)] \\ L_j + L_{j+1} &= 12q \end{aligned}\right\} \quad j = 7, 9, \ldots.$$

It is now possible to calculate the ratio of areas which represent the various IBNR factors and to solve the equations for the points of demarcation, s_i. The first case below is set forth more completely but for the other cases merely the results are stated.

$$\begin{aligned} B_{12} &= (K_1 + K_2 + L_1 + L_2)/(K_1 + L_1) \\ &= (6p + 2q)/(ps_1^2/24 + qs_1^2/72) \\ &= 144/s_1^2 \end{aligned}$$

$$s_1/12 = 1/\sqrt{B_{12}}$$

$$B_{24} = 432(2p + q)/(qs_2^2 + 72ps_2 - 144q - 864p)$$

$$s_2/12 = 1 + 1/B_{24} \quad \text{for } q = 0$$

$$s_2/12 = 3(1 - 1/q) + \sqrt{(-2 + 3/q)^2 + 3(-1 + 2/q)/B_{24}} \text{ otherwise}$$

$$B_{36} = 144(6p + 5q)/(qs_3{}^2 + 72ps_3 - 576q - 1728p)$$

$$s_3/12 = 2 + 1/B_{36} \quad \text{for } q = 0$$

$$s_3/12 = 3(1 - 1/q) + \sqrt{(-1 + 3/q)^2 + (-1 + 6/q)/B_{36}} \text{ otherwise}$$

$$B_i = (12p + 12q)/\{p[s_{i/12} - (i - 12)] + q[s_{i/12} - (i - 12)]\}$$

$$= 12[s_{i/12} - (i - 12)]$$

$$s_{i/12}/12 = (i - 12)/12 + 1/B_i \quad \text{for } i = 48, 60, \ldots.$$

Now let us define $B_{c:d}$ as the factor which one should apply to data evaluated at time "c" when the latest available IBNR factor is B_d. First we will consider the case where $c - d = 12$. This will be the condition when you have had an opportunity to evaluate the IBNR factor for a given year sometime during the subsequent year and are thus relatively current. (We will later consider the case of an organization which examines its data only once a year and, under the mode of operation, has a twenty-four-month difference between c and d.) The general procedure is illustrated by $B_{24:12}$ where one takes the above equation for B_{24}, puts $s_2 = s_1 + 12$, substitutes for s_1 its value in terms of B_{12} and simplifies. Under these conditions it is found that, given B_{12} one would use at month 24 the following factor:

$$B_{24:12} = 3(2 - q)B_{12}/[q + 2(3 - 2q)\sqrt{B_{12}}]$$

Similarly:

$$B_{36:24} = (6 - q)/[2\sqrt{(3 - 2q)^2 + 3q(2 - q)/(B_{24}} - 2(3 - 2q) + 3(2 - q)/B_{24}]$$

$$B_{48:36} = B_{36} \quad \text{for } q = 0$$

$$B_{48:36} = 1/[(1 - 3/q) + \sqrt{(1 - 3/q)^2 + (-1 + 6/q)/B_{36}}] \text{ otherwise}$$

For higher values of $B_{c:d}$, simply use B_d; that is, no distortion is present thereafter.

Finally, let's consider the case illustrated by a statistical agency which runs its data once a year and has need of the IBNR factor (corrected for distortion) prior to the running of the current year's data. This places $c - d = 24$ and results in the following:

$$B_{36:12} = (6 - q)B_{12}/[(6 - 2q)\sqrt{B_{12}} + q]$$

$$B_{48:24} = B_{24} \quad \text{for } q = 0$$

$$B_{48:24} = 1/[(2 - 3/q) + \sqrt{(2 - 3/q)^2 + 3(-1 + 2/q)/B_{24}}] \quad \text{otherwise}$$

$$B_{60:36} = B_{36} \quad \text{for } q = 0$$

$$B_{60:36} = 1/[(1 - 3/q) + \sqrt{(1 - 3/q)^2 + (-1 + 6/q)/B_{36}}] \quad \text{otherwise}$$

For higher values of $B_{c:d}$, use B_d.

To illustrate the magnitude of the functions and their behavior, we assumed $q = ½$ (which is approximately true for certain personal property lines during 1966-1969) and evaluated $B_{24:12}$, $B_{36:24}$, $B_{48:36}$, $B_{36:12}$, $B_{48:24}$ and $B_{60:36}$ as shown below:

B_d	$B_{24:12}$	$B_{36:24}$	$B_{48:36} = B_{60:36}$	$B_{36:12}$	$B_{48:24}$
1.00	1.0000	1.0000	1.0000	1.0000	1.0000
1.10	1.0543	1.0982	1.0916	1.0533	1.0899
1.20	1.1062	1.1963	1.1831	1.1042	1.1797
1.30	1.1560	1.2943	1.2745	1.1531	1.2694
1.40	1.2039	1.3924	1.3659	1.2001	1.3589

Over this range, these values are nearly linear and could be represented by

$$\hat{B}_{24:12} = .53\, B_{12} + .47$$

$$\hat{B}_{36:24} = .98\, B_{24} + .02$$

$$\hat{B}_{48:36} = \hat{B}_{60:36} = .92\, B_{36} + .08$$

$$\hat{B}_{36:12} = .51\, B_{12} + .49$$

$$\hat{B}_{48:24} = .90\, B_{24} + .10$$

To summarize, we have established the simple formulas $\hat{B}$ above which permit one to remove the distortion caused in IBNR factors computed on an accident year basis when a new line of business or a new statistical program is introduced.