Thus, Chebyshev's theorem states that:

$$Pr\left[\,|t| \leq \frac{kE(T)}{\sigma_T}\right] \geq 1 - \frac{\sigma_T^2}{k^2E(T)^2}$$

regardless of the form of the distributions. Resolving this yields the following standard for full credibility:

$$\Lambda = \frac{A}{k^2(1-P)}$$

This estimate is much more conservative; for example, in comparison to the usual standard of 1,084 claims, the same parameters produce the value $\Lambda = 4{,}000$ for claim frequency and $4{,}000A$ for the pure premium. There are more elaborate Chebyshev-type relations, involving higher moments, which could be used to reduce this upper bound. From a practical standpoint, however, these are not useful since the required moments are not available.

DISCUSSION BY LEROY J. SIMON

This fine paper is providing a new stimulus to the thinking of actuaries on the important subject of credibility. A primary purpose of this review is to place additional information before the Society relating to another line of business, namely fire.

The Actuarial Bureau of the National Board of Fire Underwriters and, more recently, the National Insurance Actuarial and Statistical Association have assembled, under the direction of Dr. J. H. Finnegan, statistical data on fire losses in the United States. The latest compiled information was for 1964 and the results are shown in the accompanying table. The data were derived from "Adjusters' Loss Reports" which are forms completed by adjusters upon the settlement of each claim. The reports reflect the payment made to all involved companies as a combined total. Thus, if a claim were split among ten companies the entry would be made as one entry for the full amount and not as ten separate reports for shares of the amount. For our purposes, the method of compilation in the accompanying table is much better than the usual compilation of data in the fire field where split losses would be reported separately and never pulled together into a single combined total.

In many instances an adjuster's report represents the total damage sustained in a fire, but if the insured had one group of policies on his building and a second set of policies on contents, two separate adjustments would be made and two separate adjuster's loss reports would be submitted. Hence, it is proper to think of adjuster's loss reports as being on a claim basis, but not always representing the total loss from a single fire. The data are also deficient in that not all losses are reported with 100% completeness and in that all companies in the industry do not participate in preparing adjuster's loss reports.

The table is a composite of adjuster's loss reports for all states for amounts \$250 and over. Reports for amounts from the first dollar up were available only for Oregon and the Oregon data were used to approximate the countrywide figures for claims under \$250. Hence, the distribution is an approximation but an examination of the data indicates that this approximation is probably of much less importance than the effect that sampling fluctuation would be expected to have upon the various moments of this highly skewed distribution.

Having thus obtained a distribution of fire losses for combined buildings and contents losses and combined dwelling and commercial properties, we proceeded to calculate the various moments of the distribution. The moments were as follows:

$$\mu = 2191.56$$
$$\mu_2 = 208{,}557{,}000$$
$$\mu_3 = 224{,}875{,}000{,}000{,}000$$

If we assume the number of claims has a Poisson distribution, formula (F) (in the Mayerson paper) using $k = .05$ produces $\lambda = 53{,}435$. If, instead, we solve equation (E) which only involves two moments, we obtain $\lambda = 48{,}075$. The use of the third moment of the claim amount distribution increases the number of claims needed for full credibility by 11%.

The 1921 standard profit formula for fire insurance provided that only the first million dollars of loss would be chargeable to the state in which it originated. In 1949 the formula was modified to allow more to be charged to the state up to 10% of the annual fire insurance premium volume of the state. If the amounts in the table were limited to one million dollars the moments would be as follows:

$$\mu = 2169.75$$
$$\mu_2 = 139{,}970{,}000$$
$$\mu_3 = 55{,}928{,}400{,}000{,}000$$

FIRE LOSSES BY SIZE

Countrywide, 1964

Size of Payment		No. of Claims	Losses Paid
0	249	160,986	10,887,863
250	499	63,619	22,235,666
500	999	42,568	29,852,177
1,000	1,999	31,601	43,662,219
2,000	4,999	35,866	111,977,907
5,000	9,999	19,112	127,622,989
10,000	14,999	6,372	75,875,011
15,000	19,999	2,611	44,268,678
20,000	24,999	1,588	34,880,190
25,000	49,999	2,665	90,884,964
50,000	74,999	784	47,161,040
75,000	99,999	363	31,210,793
100,000	149,999	280	33,776,642
150,000	199,999	141	24,086,211
200,000	249,999	57	12,738,263
250,000	299,999	40	11,030,300
300,000	349,999	24	7,668,749
350,000	399,999	24	8,968,778
400,000	449,999	11	4,667,100
450,000	499,999	8	3,815,203
500,000	549,999	8	4,256,955
550,000	599,999	5	2,913,404
600,000	649,999	4	2,490,542
650,000	699,999	2	1,323,443
700,000	749,999	1	701,898
750,000	799,999	2	1,523,046
800,000	849,999	1	802,729
850,000	899,999	1	855,722
950,000	999,999	1	959,781
1,000,000	and over	7	15,042,833
Total		368,752	808,141,096

Substituting these values in equations (F) and (E) we obtain $\lambda = 35{,}287$ and 33,258 respectively. The introduction of the third moment here increases the credibility requirement by 6%.

The number of claims required for full credibility under the assumptions above is strikingly reduced by the introduction of the million dollar limitation. The values of λ in fire lines certainly contrast sharply with those in automobile and workmen's compensation found in the original paper. In closing, let me emphasize that the fire loss distribution data found herein is an approximation and should not be considered a precise nor final answer on this subject.

DISCUSSION BY CHARLES C. HEWITT, JR.

This review will have two principal parts:

(1) A focusing of attention upon the recent general definition of credibility by Buhlmann (1), and

(2) A commentary upon the true meaning of "full credibility" in view of the insight that Buhlmann's generalization provides.

(1) Partial Credibility — the Buhlmann Definition

Buhlmann restates the familiar

$$z = \frac{n}{n+K}$$

when n is the number of observations, but goes on to prove that

$$K = \frac{\textit{Expected value of the process variance}^{*}}{\textit{Variance of the hypothetical means}}$$

* This conclusion was reached with respect to both the Gamma-Poisson process and the Beta-Binomial process in Mayerson's earlier work (2) on a Bayesian treatment of credibility, but was not recognized in this most general form by either Mayerson or the author of this current review in his earlier review of Mayerson's Bayesian approach (3). In the latter review this author even went to the trouble of pointing out Albert W. Whitney's fifty-year-old statement (4) of this formulation for the (essentially) Beta-Binomial situation without achieving the insight contained in Buhlmann's analysis. (In failing to recognize K in the Buhlmann format, this reviewer was fooled by his own constant dependence on the Gamma-Poisson process and the coincidence that the mean and variance in the Poisson process are identical.) Finally (for those who prefer numerical values attached to ideas) the Appendix includes an application of the Buhlmann definition to Canadian private passenger auto statistics.