One may take but minor exceptions to the present paper. One exception is to eliminate the N from assumption "(b) that the random variables $N, X_1, X_2, \ldots$ are independent . . . ," since once that X_N is chosen, N is determined. Secondly, it would be clearer if it were stated whether the automobile property damage data were for total limits or were truncated to a standard limit. It would also be useful to mention the need, due to inflationary and other temperal changes, of frequently recalculating the size of N for each type of insurance and for at least some breakdowns or subclasses thereof. This might have to be done as often as yearly in order to produce an actual variance as small as that postulated from prior data.

The minor nature of these points simply tends to confirm an opinion that recognizes these authors' paper as a scholarly, clearly presented, and very important contribution.

DISCUSSION BY KENNETH L. McINTOSH

Assuming that necessary data could be made available, the only argument against recognition of claim cost variation in credibility calculations seems to be one advanced by Mr. Perryman himself. He noted that the resulting "great increases in credibility requirements could not very well be made in practice under present day conditions for they would greatly limit the employment of local data."[1] Mr. Perryman's "present day conditions" of 1932 are not, however, the "present day conditions" of 1969. Messrs. Mayerson, Jones, and Bowers have refocused attention upon the question, and perhaps the argument will bear re-examination. The data problem should not prove insoluble if it once is decided that the hidden cost of deficient credibility standards exceeds the out-of-pocket incident to data collection and processing.

When full credibility is defined by $P = 90\%$, $k = 0.05$, it is doubtful that retention of the 3rd and higher claim cost moments results in any significant increase in accuracy, except possibly in extreme cases. Assuming $\lambda_3 = \lambda_2 = \lambda$, then Eq. (E) of the paper becomes:

$$\lambda = \frac{Z_e^2}{k_1^2}\left[1 + \frac{\mu_2}{\mu^2}\right] \qquad (1)$$

[1] Perryman, F. S., "Some Notes on Credibility," *PCAS* Vol. XIX (1932), p. 73.

and solving Eq. (F) of the paper for k as a function of λ, we have:

$$k_2 = \frac{Z_e}{\lambda^{1/2}} \left[1 + \frac{\mu_2}{\mu^2} \right]^{1/2} + \frac{Z_e^2 - 1}{6\lambda} \cdot \frac{1 + 3\frac{\mu_2}{\mu^2} + \frac{\mu_3}{\mu^3}}{1 + \frac{\mu_2}{\mu^2}} \tag{2}$$

Substituting into Eq. (2) the value of λ from Eq. (1), after simplification the result is:

$$k_2 = k_1 + \frac{k_1^2 (Z_e^2 - 1)}{6Z_e^2} \cdot \frac{1 + 3\frac{\mu_2}{\mu^2} + \frac{\mu_3}{\mu^3}}{\left[1 + \frac{\mu_2}{\mu^2}\right]^2} \tag{3}$$

Setting $k_1 = \pm 0.05$ and $Z_e = \pm 1.645$, Eq. (3) becomes:

$$k_2 = \pm 0.05 + 0.000263 \, \frac{1 + 3\frac{\mu_2}{\mu^2} + \frac{\mu_3}{\mu^3}}{\left[1 + \frac{\mu_2}{\mu^2}\right]^2} \tag{3.a}$$

whence if:

$$\frac{1 + 3\frac{\mu_2}{\mu^2} + \frac{\mu_3}{\mu^3}}{\left[1 + \frac{\mu_2}{\mu^2}\right]^2} \leqslant 3.80$$

then by Eqs. (3) and (3.a) we have $k_2 - k_1 \leqslant 0.001$. It will be found that actually we have $k_2 - k_1 = 0.0007$ for the automobile data given in the paper, and for either set of workmen's compensation data we have $k_2 - k_1 = 0.0005$. It also should be noted that neglect of the third moment does not change the width of the confidence interval, but merely displaces it by a very small amount.

Considering the uncertainty in the observed values of the higher moments and remembering that truncation error will result in any case from chopping the expansion at a given number of terms,[2] errors of the magni-

[2] It would have been helpful had the paper included some indication of error bounds to be associated with the Cornish-Fisher expansion.

tude of those calculated above seem negligible for all practical purposes. Comparable results might be expected, on the basis of the Central Limit theorem, when the assumption that $\lambda_3 = \lambda_2 = \lambda$ is abandoned, although the calculations have not been made.

Partial credibilities present a different picture when the expected number of claims drops below about *100*. Noting that $Z_{.5} = 0$, and that from the definition of t_e it follows that $T_e = E(T) \longrightarrow t_e = 0$, using only the first two terms of the expansion, we find, after some algebra, that:

$$\frac{T_{.5}}{E(T)} = 1 - \frac{\mu_3\lambda + 3\mu_2\mu\lambda_2 + \mu^3\lambda_3}{6\mu\lambda(\mu_2\lambda + \mu^2\lambda_2)} \tag{4}$$

and that if Z_a is the solution of:

$$Z + \frac{\mu_3\lambda + 3\mu_2\mu\lambda_2 + \mu^3\lambda_3}{6(\mu_2\lambda + \mu^2\lambda_2)^{3/2}}(Z^2 - 1) = 0 \tag{5}$$

then:

$$Pr\{T < E(T)\} = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{Z_a} e^{-1/2Z^2}dZ \tag{6}$$

Using the automobile claim cost data from the paper and assuming $\lambda_3 = \lambda_2 = \lambda$, values of $T_{.5}/E(T)$ and of $Pr\{T < E(T)\}$ were calculated for several values of λ selected to have arithmetically convenient square roots. Some of the results are listed in the following table:

λ	$\frac{T_{.5}}{E(T)}$	Z_a	$Pr\{T < E(T)\}$
$\geqslant 400$	$\geqslant 0.995$	$\leqslant 0.049$	$\leqslant 0.520$
225	0.991	0.065	0.526
100	0.980	0.096	0.538
64	0.969	0.119	0.548
36	0.945	0.157	0.562
16	0.876	0.229	0.591
9	0.780	0.294	0.616

$$\frac{T_{.5}}{E(T)} = 1 - 1.984\lambda^{-1} \qquad Z_a = (0.2688\lambda + 1)^{1/2} - 0.5184\lambda^{1/2}$$

In all probability the sampling mean, as an unbiased estimator, will have zeroed in upon true expectation by the end of all eternity, but classification rates normally rest upon at most only five or six years of experience. When at most only five or six observations are to be used in a given rate calculation, a probability of 0.55 or more that a single observation will fall below expectation, and a probability of 0.5 that it will fall below 95% of expectation, would seem significant even though credibility will be low in such cases. Where the rating formula ultimately rests upon truncated distribution,[3] the effects of skewness will be minimized, and perhaps may be reduced to negligible proportions. Nevertheless, the matter seems worth investigation.

Entirely apart from the present application specifically to the credibility problem, the Cornish-Fisher expansion seems to offer a simple technique whereby empirical distributions of loss may be developed readily, either when a theoretical distribution cannot be fitted, or when a theoretical distribution, if fitted, is too complex for routine practical calculation. Although the estimation of annuity costs as such may be of little interest to most casualty actuaries, as an example of techniques readily applicable to casualty problems, Mr. Bowers' paper[4] cited by the present authors will repay study by anyone interested in actuarial methods.

It is to be hoped that Messrs. Mayerson, Jones, and Bowers will not rest with their present significant contribution, and that additional data will become available to permit practical application of their results.

DISCUSSION BY DALE NELSON

In their paper, the authors present a distribution-free approach to the problem of evaluating the full credibility standard for a specific block of business, after having briefly reviewed the customary approach. Their motivation stems from two principal concerns:

(i) the usual derivation is based on the distribution of the number of claims and, generally, ignores the distribution of claim amounts

[3] E.g., when, as in private passenger automobile, basic limits experience, rather than total limits experience, is used.

[4] Bowers, Newton L., Jr., "An Approximation to the Distribution of Annuity Costs," *TSA* Vol. XIX (1967), p. 295.