IS "PROBABLE MAXIMUM LOSS" (PML) A USEFUL CONCEPT?

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Purpose of this Paper. The term "PML" or "probable maximum loss" is one of the most widely used terms in property insurance underwriting. But it represents one of the least clear concepts in all insurance. This fact is reflected by the results of a four-year study that involved collecting the personal and company definitions of PML from over one hundred underwriters and underwriting executives. No two of their definitions fully agree.

In the absence of a clear and specific meaning, the term can be a true invitation to disaster, because it thus provides a foundation of sand for the quantitative part of risk selection. The Lake Charles, Louisiana, oil refinery and McCormick Place, Chicago, fires of the 1960's dramatically demonstrated this fact to several insurers. On the other hand, if buttressed by a clear and specific definition and if based on properly collected and analyzed facts, the term can be an extremely useful and valuable tool. The purpose of this paper is to show how it can be made such a tool by suggesting (1) a precise definition, (2) how accuracy of PML estimates is related to the stability of a portfolio of risks, and (3) methods of measurable accuracy for determining the PML of a risk.

DEFINITION

The following definitions are suggested:

> The probable maximum loss *for a property* is that proportion of the total value of the property which will equal or exceed, in a stated proportion of all cases, the amount of loss from a specified peril or group of perils.
>
> The probable maximum loss *under a given insurance contract* is that proportion of the limit of liability which will equal or exceed, in a stated proportion of all cases, the amount of any loss covered by the contract.

In more familiar statistical language, that is more clearly related to credibility criteria for example, the insurance definition may be restated:

> The probable maximum loss under a given insurance contract is that proportion [100(m+k)%] of the limit of liability which with probability P is greater than or equal to any loss covered by the contract,

where m is the mean or "expected" proportion of loss.

The first of these two definitions is pertinent to the insured and his risk manager, while the second definition is of course more directly pertinent to the underwriter, since it is tied directly to his underwriting results. The first definition requires four pieces of information and the second calls for three pieces. These merit a closer look.

The first datum required for the property definition is the value of the property. The second required datum is a proportion of that value. These are definite, measurable quantities. The first can be expressed as a monetary amount, and the second either as a monetary amount or as a percentage of value. The fourth required datum is the peril or group of perils that is being considered. Since there are apt to be considerably different PML's for the different major perils, it is usually wise to determine these PML's separately and then to select the largest for use. For the insurance definition, the amount of insurance is needed instead of the value of the property, and the second needed datum differs correspondingly. The fourth datum is not needed explicitly for insurance.

The third datum is the major essential which is missing from existing definitions of PML. Unless we state in specific numerical terms the degree of probability which we desire, PML cannot have a clear or precise meaning. This probability must be factually based and should be measured as accurately as possible, not just pulled from the air or based on unaided judgment. The probability should also be selected on the basis of factual criteria that suitably link it to the objective underlying its selection: a definite degree of stability in underwriting results.

Benckert and Sternberg have secured evidence that the distribution by size *(monetary amount)* of fire losses to dwellings follows a Paretoan curve.[1] Mandelbrot has given a theoretical justification why all fire losses should be so distributed.[2] It is reasonable to assume therefore that the distribution of

[1] Benckert, L-G. and Sternberg, I., "An Attempt to Find an Expression for the Distribution of Fire Damage Amount," *Transactions XVth International Congress of Actuaries* Vol. II, p. 288, New York, 1957.

[2] Madelbrot, B., "Random Walks, Fire Damage Amount and Other Paretoan Risk Phenomena," *Operations Research*, Vol. XII, p. 582, 1964.

losses by *proportion of value* from any peril for a group of similar risks — or over a very long period of time for the same risk — also follows the Paretoan distribution, as indicated in Figure 1. The use of the variance and similar statistics related to such a curve, especially in determining probabilities or setting confidence intervals, accordingly requires some discretion.

It is easier to develop a confidence interval by transforming the relative frequency distribution into a cumulative or ogive form, which coincides with the "greater than or equal to" form of our definition of PML. This has been done in Figure 2.

It is also worth noting that the probability pertinent to PML involves only one tail — the upper end — of the relative frequency distribution of claims, as shown in Figure 2. With respect to PML we are only interested in adverse fluctuations, those *above* the PML value. This differs from most ratemaking situations, in which both upward and downward fluctuations about the mean or some other statistic must be considered.

PML AND THE STABILITY OF A PORTFOLIO

PML is used in at least two types of situations. Its primary uses is in the quantitative part of underwriting or risk selection. Here it is used as the basis for attempting to secure an adequate spread of risk, by limiting the amount of an insurer's liability to loss from a single occurrence. It is used primarily in connection with the fire peril, and to a lesser extent in connection with other perils giving rise to localized losses, for example sprinkler leakage, water damage, and explosion. It is still less used in connection with windstorm, earthquake, and similar loss to individual properties. It is used very little and with extreme imprecision in connection with catastrophic exposures that give rise to losses to several insured properties at the same time. With respect to the financial soundness of insurers, however, a precise use in connection with the catastrophic exposure is its potentially most important type of employment.

The term is also used in connection with engineering inspection of existing properties, and engineering analysis for safety and loss prevention of proposed building designs. Its present use in these connections, however, is just as imprecise as in connection with underwriting.

The immediate purpose of determining the PML for any specific property or risk is to provide a basis for selecting the maximum amount of

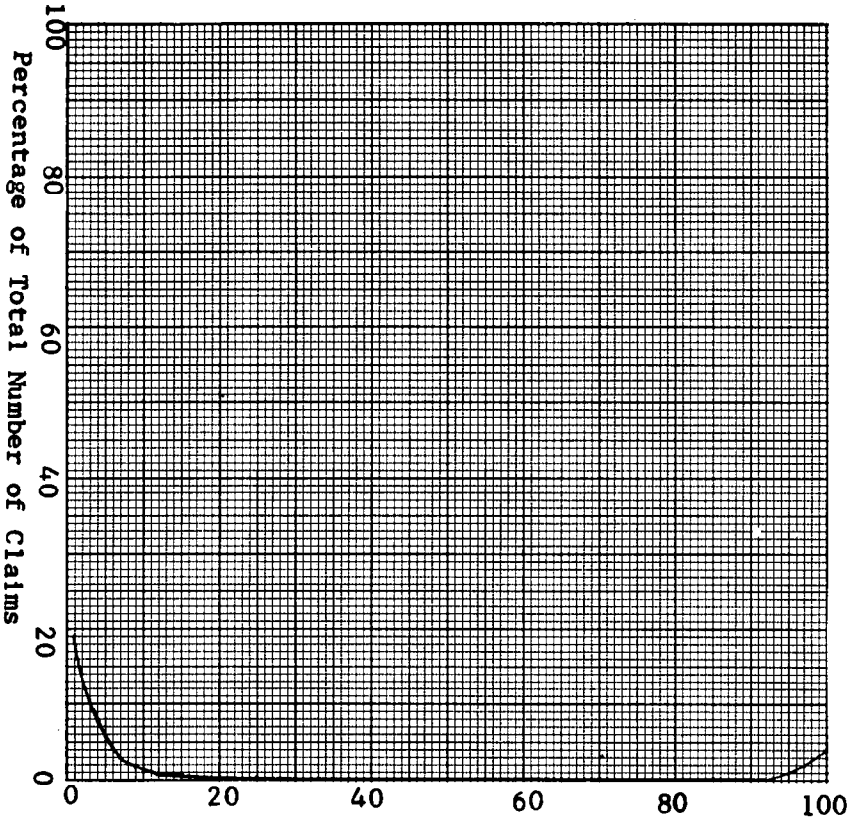

Figure 1.- Shape of a Relative Frequency Distribution of Property Claim Amounts as Percentages of Insured Amounts

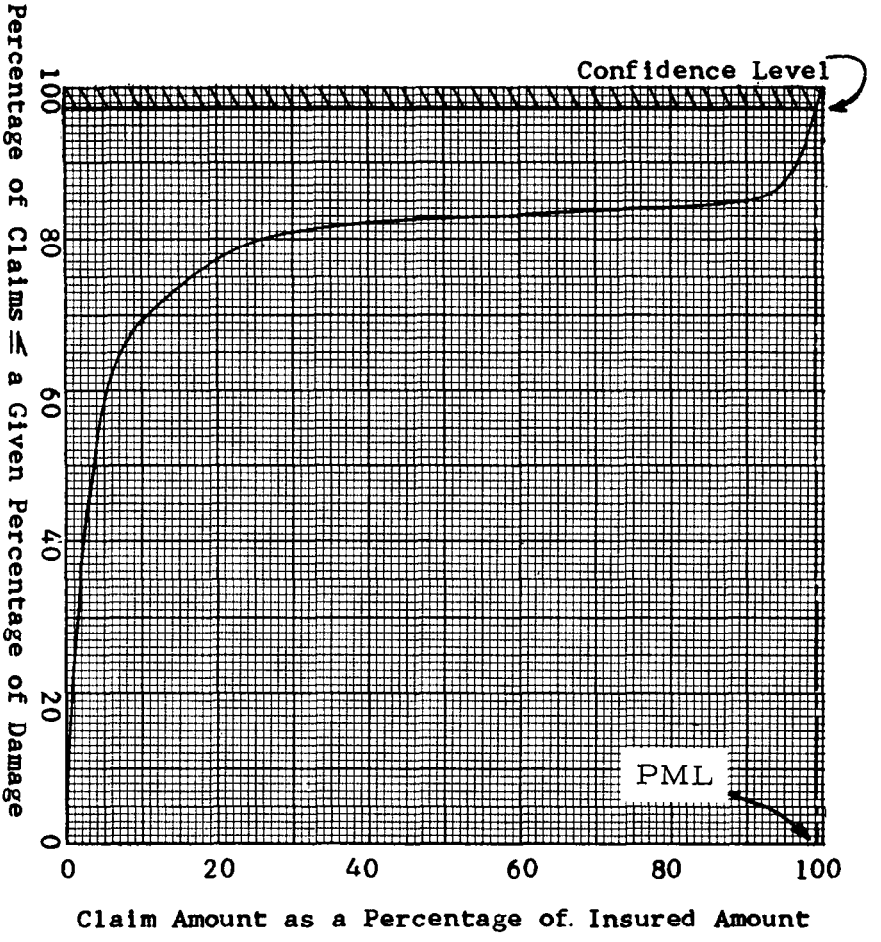

Figure 2.- Shape of a Cumulative Relative Frequency Distribution of Property Claim Amounts as Percentages of Insured Amounts

insurance that an insurer should retain on the risk for its own account. This amount is commonly called the insurer's "net retention." PML is a tool to be used in achieving a particular result — the retention — not an end in itself. Parallel to determining the company's own retention or exposure to loss on a particular risk, the maximum amount to which an insurer wishes to expose its treaty reinsurers on the same risk is also based on the underwriter's assessment of the PML.

In turn, the purpose of setting underwriting retentions is to stabilize an insurer's experience so that one or more large individual losses will not adversely affect its over-all underwriting result by more than a specified amount during any one year.

The ultimate objective for determining the PML of an individual risk is therefore to help stabilize the over-all claim results of a portfolio or group of risks during each year or other accounting period. Most insurers set a goal each year of a specific monetary amount of claims. This may be done explicitly, or it may be done implicitly by stating a target premium volume and a target loss ratio.

The stability objective is, then, to experience an *actual* total amount of claims, C_a, no greater than the target ("expected") amount, C_e, plus k, a constant. $C_a - C_e = k$ can be equated either with the accumulated amount of unexpended catastrophe loadings to all premiums received since a certain starting date, or with a certain proportion of surplus designated as a catastrophe reserve.

Realistically, some chance fluctuation (as well as fluctuation from other causes) above or below the targeted amount of claims must be expected. Any favorable fluctuation below the target is welcome and requires no defense. But any adverse fluctuation, above the target, must be limited in accordance with the financial resources available to the insurer to absorb it. The size of an insurer's surplus, and the relative size of its surplus and the targeted amount of claims, determine how much of an adverse fluctuation the insurer can safely absorb and how high a probability it requires that a selected maximum allowable adverse fluctuation will not be exceeded.

Even if the PML's on all of an insurer's risks are determined with great accuracy, however, adequate stability of results will not be achieved unless the insurer's retentions on the different classes of risks are appropriately graded. How to achieve these appropriate gradings lies outside the scope

of this paper, even though closely related to its subject. It needs emphasis, however, that unstable underwriting results can not properly be attributed to inaccurate determination of PML's unless the influence of an insurer's retention schedule (line sheet) and other pertinent factors is first examined and found to be favorable.

METHODS OF MEASURING PML

Methods now in use for determining PML's are necessarily based on sketchily informed judgment, since the degree of accuracy to which PML can be measured depends largely on the quality and quantity of pertinent statistical information that is available. It is not possible, for example, to determine the probabilities previously described without having facts on which to base them, and such facts are not presently being collected, except for dwellings, in the manner required.

It is therefore appropriate to examine what facts are needed to measure PML and then to investigate how and if these facts can economically be obtained. There are also different methods by which PML can be measured. These all deserve examination so that, even if at present only the simplest and least accurate is feasible to use, it can be seen whether at a later time more accurate methods can be substituted.

The simplest approach to measuring PML is to obtain the amount of claim and the amount of insurance on each risk that has sustained a loss during a given year, and to classify these paired figures by major statistical class (occupancy, construction, protection, and peril or coverage). Separation by major individual peril is to be preferred. The pairs of figures can be translated into loss percentages, a frequency distribution of these percentages made for each of the sub-classes described, and the maximum percentage of loss involved in 90, 95, 99, or some higher percentage of all the claims in each category determined. The use of data for more than one year would increase the spread and probably the stability of these results.

An adjustment to reflect the different proportions of insurance to value would materially improve accuracy. This could take the form of a further subdivision of data by type of average or coinsurance clause. It would be a four-way or five-way split (none, 80%, 90%, and 100%, or all these plus 70%) that would further fragment the data. It might alternatively be simplified into a two-way split (i.e., with or without an average clause) by

multiplying the loss percentage of each risk insured with an average clause by the percentage of that clause. This would approximately put all the results from these latter risks on a 100 per cent average-clause basis, as Table I illustrates. It is clear from the table how the average clause achieves equity by holding claim payments to exactly the same percentage of the amount of insurance, whether or not the insured honors his commitment to purchase the specified amount of insurance. At the same time it avoids distortions in ratemaking from under-insurance.

(1) Average Clause Percentage (Insured's Commitment)	(2) Value of Property	(3) Amount of Insurance	(4) Amount of Loss	(5) Percentage of Insurance to Average Clause Commitment	(6) Amount of Claim	(7) Percentage of Claim	(8) Percentage of Claim × Average Clause Percentage
80	10,000	8,000	5,000	100.0	5,000	62.5	50.0
90	10,000	9,000	5,000	100.0	5,000	55.6	50.0
100	10,000	10,000	5,000	100.0	5,000	50.0	50.0
80	10,000	6,000	5,000	75.0	3,750	62.5	50.0
90	10,000	6,000	5,000	66.7	3,333	55.6	50.0
100	10,000	6,000	5,000	60.0	3,000	50.0	50.0
80	10,000	4,000	5,000	50.0	2,500	62.5	50.0
90	10,000	4,500	5,000	50.0	2,500	55.6	50.0
100	10,000	5,000	5,000	50.0	2,500	50.0	50.0

Table 1. Adjustment of Average-Clause Results to a Full-Insurance Basis

A further step towards increased accuracy would be to analyze the total results of all six sub-classes at one time by multiple correlation. The effects of differences between the different years during which data were collected, between states and other geographical subdivisions, and effects of other variables included in the statistical collecting plan should be included in the correlation model. This step could also be taken with currently collected statistics if corresponding claim amounts and insurance amounts were kept together.

A third stage would be to include in a correlation model all of the variables included in the schedules and other rating plans. This would involve making available to a central statistical agency the schedule-rating makeups for individual risks that are now kept at the state level by the individual rating and inspection bureaus.

At present probably only the first stage is possible. While this would probably produce PML estimates with a wide variance, they would still be a major improvement because they would be fact-based and because the variance would be known. Nothing required for measuring PML's on a class basis is not already required for accurate ratemaking. Indeed, establishment of such fact-based PML's could be a step in improving ratemaking accuracy. Once the third stage described above is reached, a suitable multiple-correlation model would be made available to insurers for transfer from underwriters to a computer the determining of PML's for individual risks of any degree of complexity. Such a model would also permit the complex retention guides or line sheets of property insurers to be based directly and precisely on factual data.

Judging Underwriters' Performance in Estimating PML. Only if there is feedback to underwriters that shows them which estimates are good and which are poor can they and their superiors hope for improvement in PML estimates. Also, the superiors cannot soundly judge this aspect of job performance without such information. For these two internal purposes it is therefore useful for an insurer to secure regularly from its statistical records a summary of PML performance for each underwriter, yearly or perhaps more often.

This can be accomplished by recording the insurance PML percentage for each risk estimated by an underwriter, by similarly recording the actual percentage of loss to insured amount for each claim on such risks during a unit time period, by calculating the error of estimate (actual percentage minus estimated percentage) for each claim, and by calculating the mean and variance of the whole group of these errors of estimate for each time period.

It might be desirable to weight the errors of estimate by the amounts of insurance involved, since a small percentage error on a large risk could affect an insurer's results as much as large percentage errors on several small risks. Although errors in both directions are to be avoided (too conservative PML's lead to wastefully high reinsurance purchases and excessive reinsurance processing costs, while too liberal PML's lead to an excessive number of unstabilizing large claims) any error would preferably be in a conservative direction. It is therefore important to consider the sign of the mean error as well as its size.

For each time period, the mean error and variance of each underwriter could be compared with the over-all company mean and variance, or with the over-all mean and variance of underwriters handling the same types of risks. Separate consideration of results with family risks and with business risks would be the minimum split needed if underwriters are specialized on that basis in the company. A review and analysis of the largest percentage errors from each underwriter's results could lay the foundation for better results in succeeding periods. A comparison of the mean errors and variances over time, both for individuals and for the company as a whole, could keep management abreast of whether the desirable downward trend was present in each case and of which underwriters needed help in improving their results.

DISCUSSION BY ROBERT L. HURLEY

There is much that the reader may find remarkable in the paper, "Is Probable Maximum Loss (PML) a Useful Concept?" The term, itself, is believed one of those esoteric symbols of the underwriting fraternity whose members must, in turn, sometimes find certain actuarial arcana a bit mystifying. It is not possible that PML can convey to the actuary the associations (not necessarily all pleasant) that these letters can suggest to the experienced fire underwriter. Presented with the McGuinness warnings on large fire losses, an underwriter may well reflect that there have been fire catastrophies before McCormick place, which he, incidentally, might not regard as likely destined to be the last of such disasters. Nevertheless, a lifelong schooling not to hazard, needlessly, an undue portion of his company's assets in a single occurrence would typically dissuade the underwriter from placing any significant reliance upon a purely fatalist approach to risk evaluation. Moreover, he could not help being at least a bit curious about any such approach as Dr. McGuinness's which might be construed as showing the underwriter how much he could safely write on the risks offered to him. The actuary, too, would have more than a passing interest in any such demonstration, although, understandably, the underwriter would be the most immediate beneficiary of any such mathematical solution to the age old problem of determining PML.

But before attempting to evaluate the McGuinness proposal, it may be helpful to identify his mathematical sources since they stem more from the economics and sociological than from the actuarial literature. About the