ELEMENTS OF TIME-SERIES ANALYSIS IN LIABILITY AND PROPERTY INSURANCE RATEMAKING

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INTRODUCTION

Importance of the Subject. A chain of changes since the Southeastern Underwriters Association decision has made adequate methods of statistical time-series analysis increasingly important in non-life insurance. Narrowing of safety and profit margins in rates; the steady inflation of the dollar, an *adverse* trend contrasting sharply with the *favorable* trend in mortality that has underlain life insurance ratemaking; a highly probable understatement of loss data used in ratemaking, at least for liability insurance, due both to gradual and conscious erosion of safety margins in company loss reserves and also to actual *un*intentional understatement of reserves by many companies whose methods of estimation have not met the needs imposed by changing conditions; changes in coverages and in combinations of coverages; and doubtless many other changes; have all combined to make time-series analysis important.

In many current rate filings the use of time-series adjustments accounts for as much as, or more than, the proposed allowances for profit and contingencies. Use of such adjustments or failure to use them, and their accuracy or lack of sufficient accuracy, can mean the difference between rates that are within a suitably close range of the target and rates that are either materially inadequate or materially excessive. Such use can also mean the difference between profit and loss for the majority of insurers over any extended period of time. This difference can be due not only to the direct results of the time-series adjustments, but also indirectly to the effect that the degree of acceptance they win among rate regulatory personnel has on the speed with which rate filings are approved.

As ratemaking procedures are gradually being changed to reflect (on the basis of statistical evidence) a greater number of variables, more accurate methods of measuring the effects on loss costs of these variables, and more accurate methods of distinguishing the effects of one variable from another, are required. Continually improved methods of statistical analysis will have to be employed if the ratemaking procedures and adjustments applied to some variables are not to be overloaded or distorted in order to compensate for errors resulting from insufficient procedures used with other variables. Methods of time-series analysis can on occasion fall into either of the two groups. Use of adequate methods of time-series analysis can therefore on occasion point to the need for revision or perhaps improvement of procedures used to handle other variables.

Existing Contributions. The *Proceedings* of this Society do not appear to contain many papers on time-series analysis. A review of the indexes going back to Volume I was made under the headings of "time-series analysis," "trends," "cycles," and "seasonal," and for mention of these or similar terms in titles under the heading "ratemaking." The earliest references found were in some of J. H. Woodward's[1] and T. F. Tarbell's[2,3] interesting presidential addresses. Paul Benbrook[4] and Frank Harwayne[5] covered some methods of trend adjustment in papers primarily devoted to other topics. John W. Clarke[6] devoted a complete paper to seasonal fluctuations in automobile liability loss ratios, while David A. Tapley[7] also discussed such fluctuations in a paper on loss reserves. The most recent *Recommendations for Study*[8] contains no references to texts covering time-series analysis. It therefore seems that a paper on the subject can be useful in several respects.

The purpose of this paper is to show how methods of time-series analysis that have long been generally accepted among statisticians in all non-insurance fields where economics plays a role (e.g. all other types of business, government, and education) can be usefully employed in property and liability insurance. It will also be shown:

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[1] Woodward, J. H., "The Effect of Inflation on the Business of Insurance," *PCAS*, VI, p. 1.

[2] Tarbell, T. F., "Business Cycles and Casualty Insurance," *PCAS*, XVIII, p. 253.

[3] Tarbell, T. F., "The Effect of Changes in Values on Casualty Insurance," *PCAS*, XIX, p. 1.

[4] Benbrook, Paul, "The Advantages of Calendar-Accident Year Experience and the Need for Appropriate Trend and Projection Factors in the Determination of Automobile Liability Rates," *PCAS*, XLV, p. 20.

[5] Harwayne, Frank, "Some Further Notes on Estimating Ultimate Incurred Losses in Auto Liability Insurance," *PCAS*, XLVI, pp. 59, 312.

[6] Clarke, John W., "Seasonal Fluctuation in Loss Ratios for Automobile Bodily Injury Coverage," *PCAS*, XXXVI, p. 63.

[7] Tapley, David A., "Month of Loss Deficiency Reserves for Automobile Bodily Injury Losses Including Reserves for Incurred But Not Reported Claims," *PCAS*, XLIII, p. 166.

[8] Casualty Actuarial Society, *Recommendations for Study* (1969 Syllabus), 16th ed., 1968.

(1) how the concept behind the statistical quality control chart can be adapted to creat a rapid and simple method of adjusting for cyclical variation,

(2) that the same methods are applicable to all lines of property and liability insurance, a fact whose recognition and use could greatly simplify the problems of ratemakers in making sufficiently accurate time-series adjustments,

(3) that the prevalent practice in property and liability insurance of adjusting only for trend, while ignoring the other types of temporal fluctuations, can and does materially reduce the accuracy of results,

(4) that the economic statistician's technique of making an index number can help solve two of the actuary's problems. One of them, previously unsolved, is how to combine in one meaningful time series partly disparate data such as those arising from use of different deductible amounts. The second problem is how to overcome the sparsity of data and lessened stability of results that arises from subdividing data by type of deductible and using the subdivisions separately,

(5) how, as a result of difficulties met in applying the techniques to existing or available data, some improvements in the form and quality of data collected for ratemaking purposes can be made that will also improve the results of time-series analysis based on them.

Organization of the Paper. With the object of going from the simpler to the more complex, there are discussed in order liability coverage, automobile property coverage, and coverage on fixed-location properties. Prior to specific applications, some basic considerations applicable to all lines of insurance are reviewed.

Of the four major types of temporal movements or variations over time, seasonal adjustments will not be considered in this paper. The available data are all in yearly form. Rates are seldom reviewed or changed more frequently than one per year. It is therefore not essential at present to adjust for this type of change. At a later time, however, a paper covering methods of seasonal adjustment for internal budgeting, loss reserving, and perhaps even interim rate adjustments should prove valuable.

SOME BASICS OF TIME-SERIES ANALYSIS

Purpose and Nature. The purpose and nature of time-series analysis are well and succinctly stated by Riggleman and Frisbee:

> "One of the chief problems in modern business is that of estimating what the future changes in business conditions will be. This makes it necessary to analyze data over a period of time. If the data are merely descriptive of a situation at a certain time, the methods . . . of frequency distributions, averages, and dispersions may be all that are necessary in making an analysis. But, if the data represent changes that are taking place over a period of time, it is necessary to use special methods which will describe change or progress as well as describe a static situation. Data representing change over a period of time are known as time series, and . . . specialized methods . . . are necessary in time-series analysis."[9]

The examples they cite of practical problems met a generation ago in non-insurance industries, due to lack of such analysis, show interesting parallels to current insurance problems.[10]

The major types of movements in time series are generally considered to be:

(1) basic or long-time trend,

(2) cycles (irregular periodic variations), i.e. wavelike changes over periods of somewhat irregular length,

(3) seasonal (regular periodic) variations, i.e. wavelike changes over periods of fixed length,

(4) irregular, random, or erratic fluctuations.[11]

Trend may be defined as a long-term movement, usually measured over decades, reflecting a tendency either to grow or to decline.[12] *Cycles* reflect

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[9] Riggleman, J. R., and Frisbee, I. N., *Business Statistics*, McGraw-Hill Book Co., Inc., New York, 1938, p. 270.

[10] *Ibid.*, pp. 270–273.

[11] *Ibid.*, pp. 275 ff.; see also Croxton, F. E., and Cowden, D. J., *Applied General Statistics*, Prentice-Hall, Inc., New York, 1939, pp. 363–376. Flaskaemper, Paul, *Allgemeine Statistik*, Vol. I, Richard Meiner Publishers, Hamburg, 1949, pp. 133–137, characterizes another basic temporal relationship: constancy. This amounts to a flat trend, or one with zero slope. The automobile collision trend reported here approximates this relationship.

[12] Riggleman and Frisbee, *op. cit.*, p. 276; Croxton and Cowden, *op. cit.*, pp. 364–367.

"the persistent tendency for business to prosper, decline, stagnate, recover, and prosper again, in apparently never-ending sequence."[13] *Seasonal* or *periodic variation* is a well-defined movement repeated each year[14] (or each month, week, day, or similar fixed period). *Irregular* or erratic *fluctuations* are those remaining in time series after the effects of the other three types have been removed.[15] Major irregular movements such as a change in the price of gold, a general war, or a widespread natural calamity such as a severe and prolonged drought, must be specifically taken into account in time-series analysis. The remainder are usually considered only to the extent that they affect the size of a calculated standard error of estimate or similar measure.

Measuring Trend. Since trend is a long-term movement, measuring it with reasonable accuracy requires data for a relatively large number of years. Ideally, the term covered by the data should extend over the periods of at least two or three of the longest cycles. This is clearly necessary to avoid mistaking some cyclical movements for trend movements. As a practical matter it is not usually possible at the outset to secure a consistent and long enough series of precisely pertinent data. Ten years' data are mandatory as a minimum for reasonably reliable results, and in many cases will not suffice.[16] Insufficient data can on occasion be buttressed by a longer series of similar type, but the statistical correlation between the two series should be measured to determine the suitability of the match. In judging the amount of data required it is usually desirable to plot what are available, both on arithmetic and on semi-logarithmic graph paper, to get an over-all perspective. Descriptions of the types of curves available for fitting as trend lines[17] and criteria for selecting one curve from among these fitted[18] are readily available in standard texts.

The reason underlying the authorities' insistence on measuring trends with time-series of adequate length is well illustrated by difficulties inher-

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[13] *Ibid.*, p. 279.

[14] *Ibid.*, p. 277.

[15] *Ibid.*, p. 279.

[16] *Ibid.*, p. 289. They also caution, "It is always possible to obtain close-fitting so-called trends, by fitting lines or curves over an unrepresentative short period of time." See also Croxton and Cowden, *op. cit.*, p. 408.

[17] Cf. Croxton and Cowden, *op. cit.*, pp. 395–457, and Riggleman and Frisbee, *op. cit.*, pp. 295–310.

[18] Cf. Croxton and Cowden, *op. cit.*, pp. 418–419 and 461–462; Riggleman and Frisbee, *op. cit.*, pp. 288–290.

ent in using data for the short time periods commonly employed in both United States and Canadian ratemaking. These periods have commonly been 3¾ years in the United States and either 3 or 4 years in Canada. The data used in the United States have been twelve-month moving averages of fiscal-accident-year mean claim severity, spaced at quarterly intervals. This is equivalent to weighting the first and last quarters once, the next-to-first and next-to-last quarters twice, those second-from-first and -last thrice, and all other quarters four times. In Canada, unaveraged policy year pure premiums have been used.

State data going back far enough could not be obtained, but Figures 1, 2, and 3 illustrate the point with Canadian data. Figure 1 shows the pure premiums for private passenger liability insurance in Ontario from 1945 through 1966, and a straight trend line fitted to these data. The trend line passes reasonably close to the center of the cycles or waves in the actual data. Figure 2 shows all the different trend lines that would have been fitted to the data, starting with 1954 when the minimum of ten years' data were available, if all the data available each year had been used. By way of contrast, Figure 3 shows all the different trend lines that would have been fitted to the data had the method presently employed in Canada (with four years' data each time) been used since 1948.

It can be seen from comparing Figures 2 and 3 that the long-period trend lines have the stability that is desired for ratemaking. They overlap each other to a very high degree, because they use all the available data and use enough data. The four-year trend lines, on the other hand, go in widely different directions that give no perspective on the real long-term direction of the data.

Another means of comparison is to examine the range in estimates by the two sets of trend lines at various dates. Let us take as example a comparison of the actual pure premium index number with the short-term and long-term estimates for 1945 and 1966, the two end years, and for 1955, in the middle of the period. The table below shows the far greater differences in maximum and minimum estimates produced by the short-term trend lines. The long-term trend lines gave rise to more accurate and more stable results.

Source of Trend	Year Estimated	Minimum Estimate	Actual	Maximum Estimate	Range of Difference between Estimates
Figure 2	1945	.3456	.4049	.4275	.0819
Figure 3	1945	−1.5947	.4049	.7313	2.3260
Figure 2	1955	.8374	.8133	.8926	.0552
Figure 3	1955	.0777	.8133	.9682	.8905
Figure 2	1966	1.2884	1.6115	1.4942	.2058
Figure 3	1966	1.0828	1.6115	1.9174	.8346

Table 1. Errors of Estimate of Various Trend Lines

Measuring Cycles. Cycles are the most difficult of the various types of economic fluctuations to measure, because they are periodic and also because they fluctuate both in amplitude (height of peaks and depth of troughs) and in period (horizontal distance from peak to peak or from trough to trough) at the same time.

Perhaps for this reason they are presently almost totally neglected in non-life insurance ratemaking. It will become apparent from the data shown later in this paper, however, that there are cycles that materially affect the accuracy of ratemaking. This fact has long been recognized.[19]

If sufficient data are available, it is possible to measure the cyclical component of time series by fitting a sine or other periodic curve to the data by harmonic analysis, after the influence of trend has been removed. But since data of adequate quantity and quality for this purpose (especially data that are consistently gathered over a long period) are infrequently available, it is common to make any allowance for cyclical influences in some other manner.[20] Common methods of deriving cyclical adjustments, in addition to harmonic analysis, are: (1) residual method, (2) direct method, and (3) method of cyclical averages.[21] But all of these methods are extremely time consuming to use. They are not easily adaptable to the need for speed in promptly processing collected insurance statistics into revised rates. A simple, rapid, and flexible method is needed for use with ratemaking pro-

[19] See Dean, A. F., *Fire-Rating As a Science,* J. M. Murphy, Chicago, 1901, chapters on "The Law of Rhythm" and "Law of the Wave of Fire Destruction," pp. 32–43, and graphs of fire insurance results on pp. 118–188.

[20] Croxton and Cowden, *op. cit.*, pp. 540, 571.

[21] *Ibid.*

Figure 1.- Indexes of Yearly Pure Premiums (Base Year = 1960) for Ontario Private Passenger Automobile Liability Insurance (All-Limits Data) and Straight Trend Line

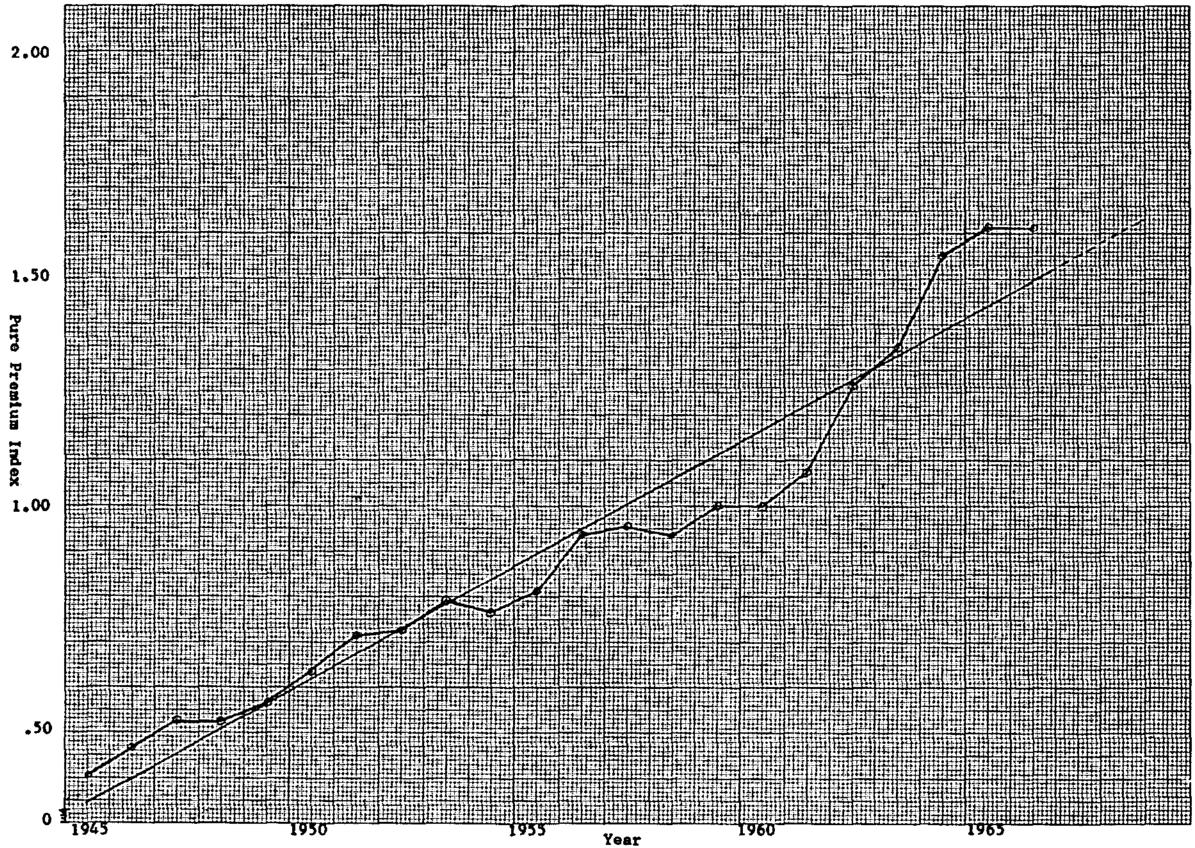


Figure 2.- Straight Trend Lines for the Periods 1945 - 1954 Through 1945 - 1966, Fitted to Indexes of Yearly Pure Premiums (Base Year = 1960) for Ontario Private Passenger Automobile Liability Insurance (All-Limits Data)

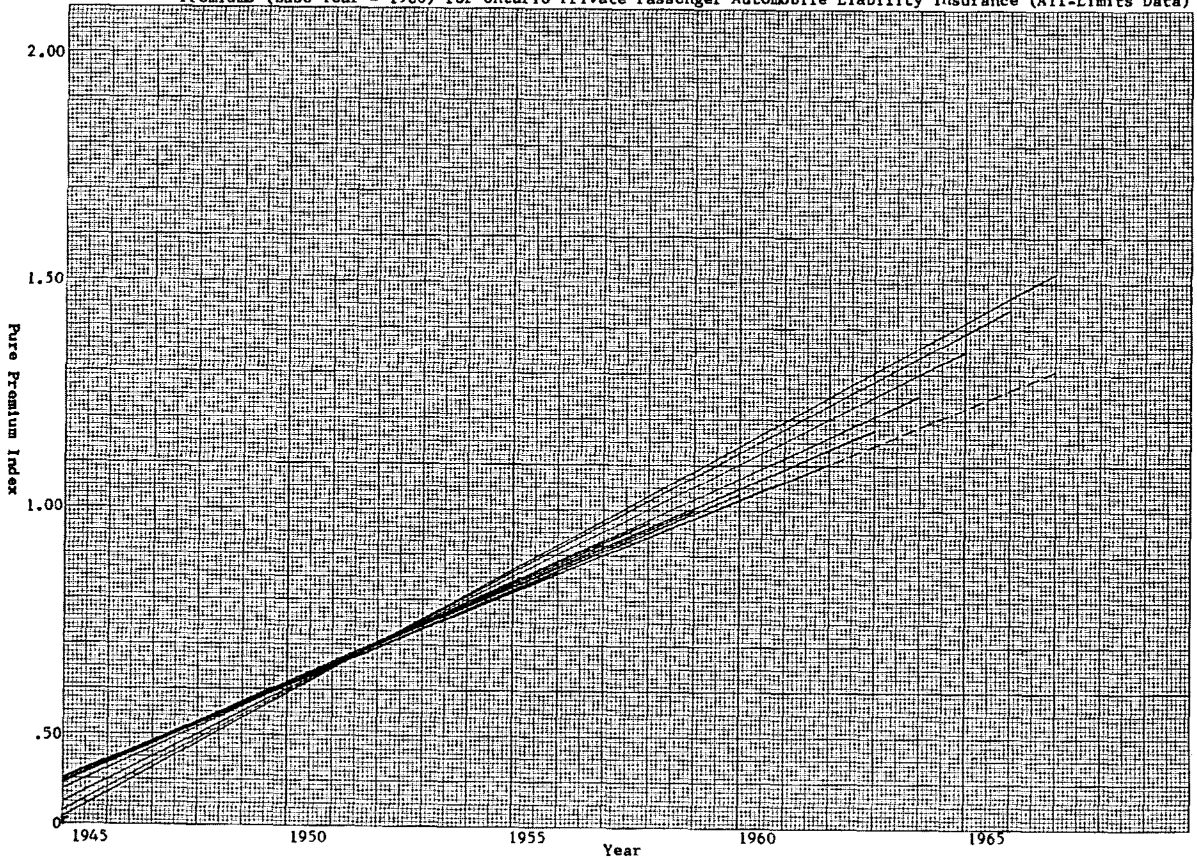
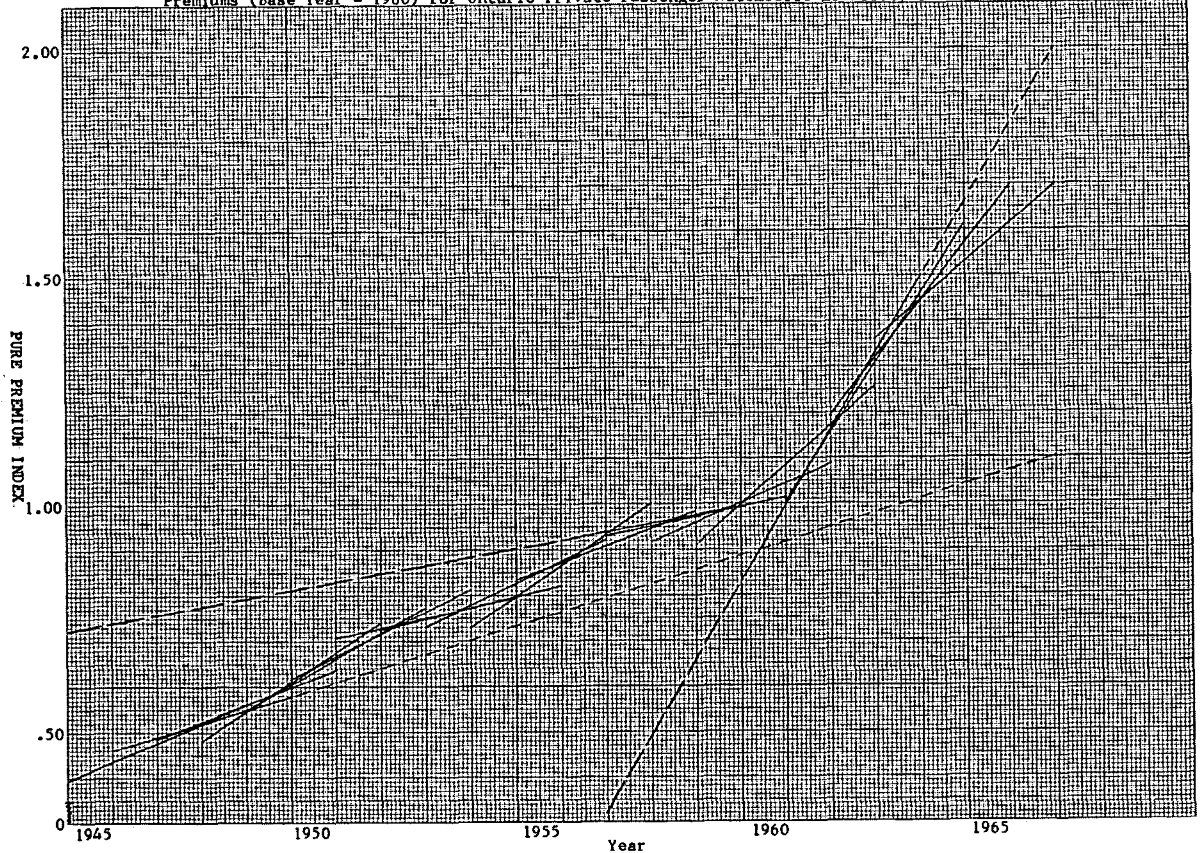


Figure 3.- Straight Trend Lines for the Four-Year Periods 1945-1948 Through 1963-1966, Fitted to Indexes of Yearly Pure Premiums (Base Year = 1960) for Ontario Private Passenger Automobile Liability Insurance (All-Limits Data)



cedures, that are often not stable over long periods, and also for analysis of such procedures. No such method was found in a search of many standard statistics tests, so in the course of practical work it became necessary to devise one.

To meet the problem, a method was devised that incorporates ideas from both the theory of runs and the well-known statistical control chart. The method involves setting up a simple rule. Guide lines or limits are set up one standard error above and below the trend, so that roughly two-thirds (68 per cent) of all data points will fall between them. A rule such as this can then be adopted for projections:

(1) If the starting point (i.e., the last datum point) falls on the trend line or within one per cent, use only the trend adjustment.

(2) If the starting point falls between the trend line and a guide line, determine toward which of the two lines an arrow placed on the last two data points is aimed. Use a cyclical adjustment equal to half the vertical distance from the starting datum point to that line.

(3) If the starting point falls outside a guide line, use a cyclical adjustment equal to the vertical distance from the starting point to the guide line.

This rule was designed to dampen extreme swings in projections and rates, while still providing a response both to the relative positions of the last datum and the trend line and to the direction of the latest identifiable cyclical movement.

The guide lines may be set any number of standard errors from the trend, depending upon the level of probability (for example 75 per cent or 90 per cent rather than the 68 per cent used here) which it is felt provides an acceptable balance between adapting to large fluctuations and maintaining stable rates. The guide line interval can best be set after testing an individual user's actual data, to see what will avoid yearly swings in rates greater than 20 or 25 per cent, and after it has been determined whether a separate catastrophe adjustment procedure is needed to remove for separate handling the extreme parts of extra large fluctuations.

The topping out or bottoming out of a cycle corresponds to the end of a run.[22] The parallel between the guide lines and the customary statistical

[22] See Hoel, P. G., *Introduction to Mathematical Statistics,* John Wiley & Sons, New York, 1947, pp. 177–182.

control chart is obvious. This rule or one like it also introduces a self-correcting tendency overtime as to errors, a highly desirable characteristic for a forecasting or projecting procedure.

Irregular Fluctuations. Large irregular fluctuations in the data require special handling. The major recent irregular influence — World War II and its attendant driving, building, price, and other restrictions — did not necessitate any adjustment in the examples given here because the data do not go back that far. The greatest irregular fluctuations in the property insurance data are probably those due to data collection procedures (e.g., use of paid losses rather than incurred losses, and less than optimal accuracy in calculating earned extended coverage premiums) so that the truly random fluctuations are partly masked. Adjustments are not usually attempted for other than major irregular fluctuations.

What Should Be Measured. It is clearly preferable to measure time-linked changes in the precise data on which the rate level is based. If the rate level is based on pure premiums, time-series analysis should be applied to those same pure premiums. If a loss ratio ratemaking procedure is used, time-series analysis should be applied to the same series of loss ratios (accurately adjusted to a single rate level) to which the ratemaking procedure is applied. To do otherwise involves the disadvantages of ignoring the most directly pertinent data, thereby increasing the margin of error or variance, and increasing the amount of needed work. Work is increased by the need to measure the statistical correlation between the directly applicable series and any series to be used in its stead, since without very high correlation the substitute cannot be satisfactory. A valid reason for using a substitute series is to overcome sparsity of data in time. For example, suitably constructed indexes combining data for fire and allied coverages, residence theft coverage, and comprehensive personal liability coverage, for the period prior to introduction of the homeowners contracts, would permit valid extension backward in time of actual homeowners data.

Because conditions differ markedly from one section of the country to another, geographic identity of data used for rate level adjustment and for time-series analysis is also important. Even though a long series of data be required, it seems unlikely that averaging state time-series results with concurrent countrywide results is ever appropriate. In addition, absence of one or more of the types of consistency mentioned in the next section of this paper makes almost all countrywide series of questionable value for this purpose. Averaging results from small volume states with those of neighboring states having similar conditions may sometimes be helpful in stabilizing indications for the former, but is an otherwise undesirable indirection. Unless the standard error about the state trend line exceeds ten, or perhaps even fifteen, per cent of the current trend value,[23] there is no apparent need for introduction of outside data. If such a large variation is noted, it is appropriate first to check for and try to eliminate or reduce fluctuations due to use of less than optimal procedures for collecting and processing the data before resorting to less directly pertinent data.

A split, rather than a substitution, may permit more precise analysis and thereby be advantageous. Separate analysis of the mean severity and relative frequency components of pure premiums may reveal facts hidden by the combination. But use only of one of the two components introduces distortions that partly defeat the purpose of the analysis. Separation and special handling of catastrophe data can markedly reduce the level of fluctuations in the remaining data, while still permitting needed reflection in the rates of catastrophe losses. A split of data according to the different forms of homeowners contracts promises some increase in accuracy over a single combined series for two or more forms. A split of such data according to perils or coverages having materially different characteristics promises even greater advantage. An example would be separation of data for (1) windstorm and hail, (2) theft, (3) fire and other property perils, and (4) liability perils. Such a split would be analogous to the longstanding split of workmen's compensation experience into medical expense, partial disability benefits, and death and permanent total disability benefits.

How Far Data Should Be Projected. Figure 4 shows the differences in six types of yearly accumulations of data used in insurance. The accumulations with "calendar" in their names extend from January through December. The rough borders of some of the accumulations reflect the changes in reserves for prior years' losses that are inaccurately assigned to the year in which an accounting change is made instead of being assigned to the year in which the underlying accident occurred. The accumulations or years with "policy" or "accident" in their names have all losses and reserve changes assigned back to the policy or accident year (i.e. exposure period) in which each loss occurred, hence are considerably more accurate for ratemaking purposes. The graph makes it easier to visualize the center of each type of accumulation of data.

[23] Extended coverage and other catastrophe-involved data are likely to have larger standard errors, as evidenced by the illustration given later.

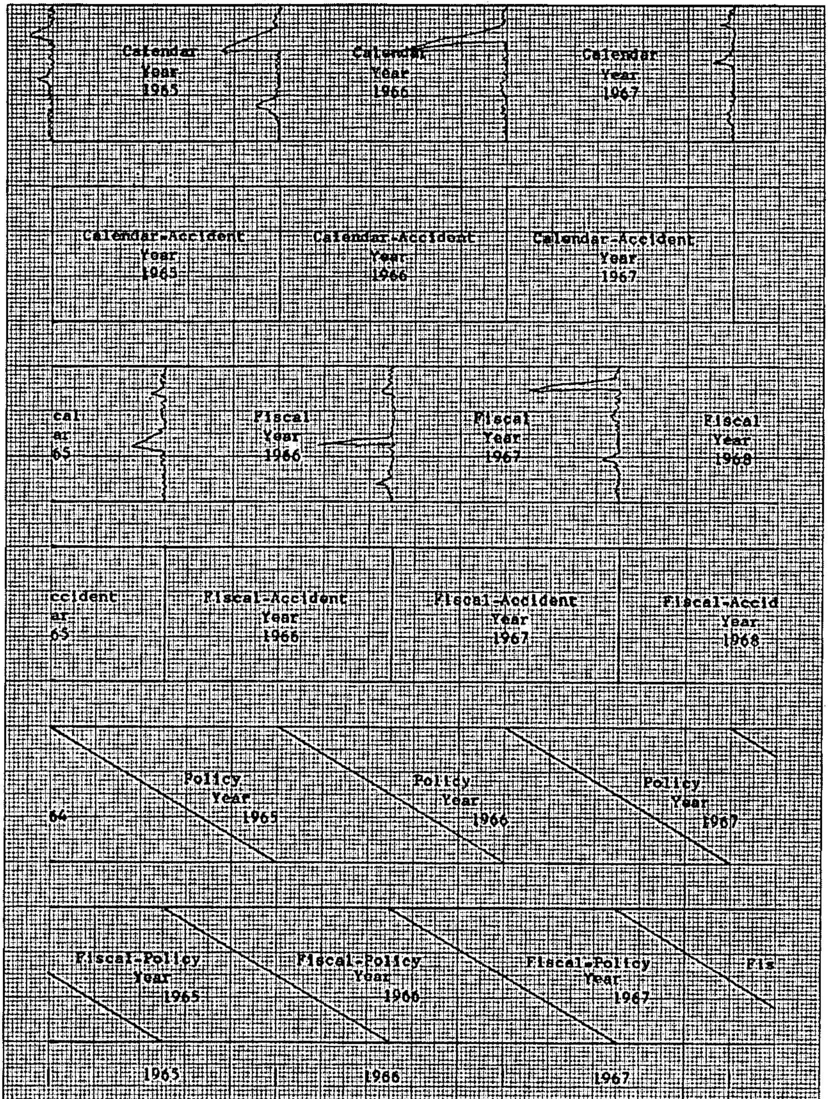


Figure 4. - Periods Covered by Different Yearly Groupings of Data

The latest actual experience data reflect results at the midpoint of the period during which they were gathered. Calendar year data reflect an average result as of midnight, 30 June. Policy year data reflect an average result as of midnight, 31 December. If data for two or more years are used jointly, both the weights ascribed to each and their respective temporal midpoints must be used to determine the effective midpoint of the combined mass of data. It is from this effective midpoint that the time-series adjustment must be projected. Determination and reflection of these midpoints is properly based on the same reasoning and procedures as determination of class midpoints in analysis of frequency distributions.[24]

The point in time to which a projection should be carried is the *effective* midpoint of the period during which the rates most likely will be in force. The great advantage of regular yearly rate adjustments in helping to determine this midpoint is clear. The advantage for this purpose of having all policies issued for the same term (or, if issued for different terms, of having the rates *guaranteed* for the same term, with interim rate adjustments permitted on longer term and continuous policies) is perhaps less clear but is no less important. Assuming regular yearly rate adjustments, and issuance of all policies for one-year terms, the midpoint of the period during which a given set of rates will be in effect is one year after the effective date of the rate filing. If the rates are guaranteed for three years and refiled every year, the midpoint of the period during which the rates will be in effect is two years after the effective date of the filing. The *effective* midpoint depends on the rate of change in the volume of business. The effective midpoint on a rising volume will be deferred past the temporal midpoint, and on a falling volume will occur prior to the temporal midpoint. Unless this rate of change is rapid, its effect will be negligible.

Failure to carry the projection an adequate distance can result in chronically inadequate rate levels if the trend in pure premiums is upward, and chronically excessive rate levels if the trend is downward.

When Data Should Be Gathered and Applied. Perhaps the chief problem faced by the time-series analyst is the difficulty in getting consistent series over a long enough period. A discussion of criteria for gathering and

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[24] See Yule, G. U., and Kendall, M. G., *An Introduction to the Theory of Statistics,* 13th ed. rev., Charles Griffin & Company, London, 1948, pp. 82–88, 91–92, and 160; also Neiswanger, W. A., *Elementary Statistical Methods.* The Macmillan Company, New York, 1943, pp. 212–225.

using data therefore applies as importantly to the design of statistical collecting plans as it does to use of the collected data.

Data should first be consistent as to form, i.e. all on an accident year, policy year, calendar year, or other single basis. They should be consistent as to timing, with all years ending on the same day and month and with no gaps or overlaps. Losses should be kept track of and actually developed to the same number of months for each year (with consistent formulas applied to estimate developments for the latest years). To minimize the degree of loss-reserving error incorporated in the data, it is preferable that they be developed to an ultimate basis. Loss adjustment expense should be uniformly included or excluded. If included, it should be on an actual and complete rather than on an estimated (formula) or partial basis for all but the latest years (those that are not fully developed).

Better results will be obtained if the adjustments are calculated, and rates adjusted, at the same time each year. The effect of rate level changes will in this manner be made uniform, and one more source of fluctuations in the data eliminated. Important simplifications in the work of adjusting data for rate level changes will also result from this precaution.

ANALYSIS OF SOME LIABILITY INSURANCE TIME SERIES

Nature of the Data. Automobile liability insurance data for one state were first analyzed. Later, data for another state and several provinces of Canada were also analyzed, with remarkably similar results. This discussion will first deal with the one-state data, and will then be generalized.

Suitably consistent parallel countrywide data were not available, so an intended test of the correlation between the state and countrywide data could not be performed. The basic data are pure premiums at $5,000/10,000 bodily injury and $5,000 property damage liability limits; paralleling the customary ratemaking method, they include loss adjustment expense. They are separated by coverage and grouped for (1) private passenger automobiles, (2) commercial automobiles, and (3) garages (Hazard 1, i.e. payroll-rated exposures). Data for the first two groups cover the period 1946 through 1964, while those for garages were available only for the period 1956 through 1962. The data available for private passenger vehicles were accumulated in three different ways and those for commercial vehicles in two different ways for various portions of the period. Overlapping data to be used for adjustments were available only for some of the breaks in continuity. It was possible to develop all the loss data consistently to 39 months.

Figure 4 shows the differences in six types of yearly accumulations of data. To make the six series of data as nearly continuous as possible, they were adjusted as well as possible to a calendar-accident year basis. The fiscal year data are centered at 31 December. Data for adjacent pairs of policy year and fiscal year data were therefore averaged to produce data centered at 30 June. The discrepancies between the results of this procedure and calendar year data tend to be greater for policy year data than form fiscal year data, as Figure 4 demonstrates. At a junction of fiscal and calendar year data, the latest fiscal year average was of necessity further averaged with the adjacent calendar year datum.

To make the data for the two coverages and three types of risks mutually comparable, the pure premiums were transformed into index numbers, based on the 1958–1961 average pure premium as 100.

In addition to correcting the basic data to a single (or more accurately adjusted) basis, accuracy could be improved by reflecting the changing distribution of exposures by class of use and driver, mileage driven, accident record, and limit of insurance; the increasing proportion of multiple car families, with lower exposure per car; and other factors. This could be done by development of a more complex type of index number. It would require considerably more computations and more refined and voluminous data than were available. However, it is easy to mistake the relative importance of such an adjustment in data that average a single characteristic of the whole insured population.

As pointed out in these *Proceedings*[25] *with respect to class relativities,* ". . . pure premiums obtained from a consolidation of widely divergent bodies of experience must be used with great caution since they may contain distortions." In other words, when data are classified according to one rating criterion or variable (for example, class of driver and use) — and if the effects of other variables (such as distribution of risks by rating territory and by merit rating class) are not either (1) held constant or eliminated by a technique such as multiple correlation analysis, or else (2) all very highly correlated with the variable being examined — the resulting relativities of

[25] Stern, P. K., "Ratemaking Procedures for Automobile Liability Insurance," *PCAS,* LII, pp. 169–172.

the pure premiums may not show the true relativities in hazard due to the variable being examined, because the effects of other variables will also affect these pure premium relativities. On the other hand, in data for any one year, the distribution of risks by different variables and the correlation among variables are of no importance to the average over-all pure premium. No matter what the distribution and correlations may be, there is only one total of losses and one total of exposure units and, therefore, only one over-all average pure premium.

These distributions and correlations *could* have an effect, however, on the relationships among (and predictive value of) over-all average pure premiums for a series of years if the distributions and correlations materially change in a short time. But the same effect will apply to the components of the pure premium, mean claim severity and mean relative claim frequency, as well as to the whole. Also, if such changes are material, they will have to be reflected by yearly tests of and changes in all the different kinds of class relativities named above. Further, any such rapid and material changes would severely diminish the accuracy of loss ratio tests of relativities, because the ability of the prior year's rate differential complex to offset the effects of the current year's distribution would be reduced in the degree of the changes. Since either the pure premium (Canada) or its mean claim severity component (United States) is commonly used for fitting trends, since class relativities are not changed yearly, and since the loss ratio method of testing such relativities is in widespread use, it is reasonable to infer that the majority view in North America, among those who have actually studied the matter, is that changes in distributions by rating criteria are not large or rapid enough to affect materially the predictive value of trends fitted to the data used here.

Procedure Used with the Data. Trend lines were fitted to the data by well-accepted methods.[26] The economic environment was first considered. All available economic measures that are in the form of time series point to a steady inflationary trend since World War II. The discontinuities created by war conditions made it advisable to use only post-war data. One standard that must be met by any trend line fitted to the available data is therefore that it point upward to the right. The data are seen to conform to this constraint when graphed and visually examined (Figures 5 through 8).

[26] Croxton and Cowden, *op. cit.*, Chapter XV; Flaskaemper, *op. cit.*, pp. 143 ff; Riggleman and Frisbee, *op. cit.*, pp. 297 ff.

Figure 5.-Indexes of Yearly Pure Premiums (Base Period = 1958 - 1961) for Kentucky Private Passenger Automobile Bodily Injury Liability Insurance (5/10 Limits Data) and Straight Trend Line

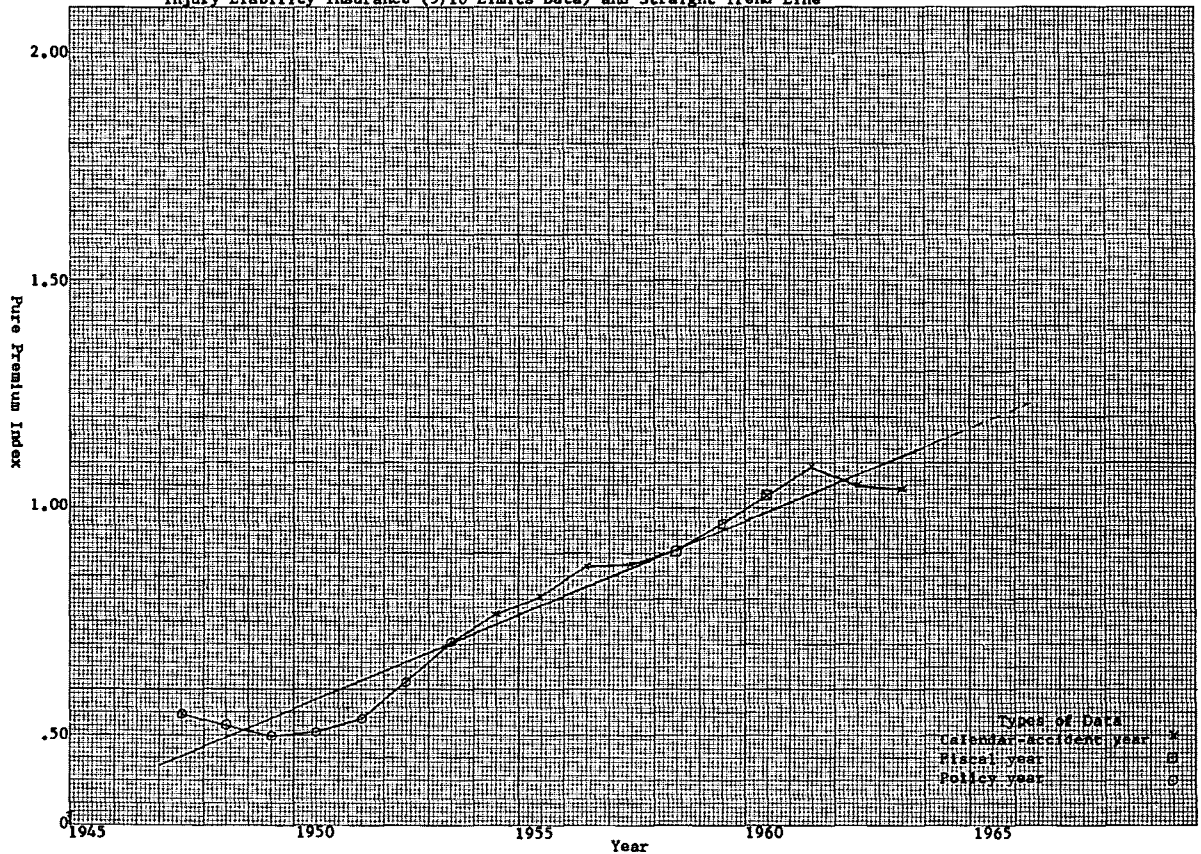


Figure 6.-Indexes of Yearly Pure Premiums (Base Period = 1958 - 1961) for Kentucky Private Passenger Automobile Property Damage Liability Insurance (5,000 Limit Data) and Straight Trend Line

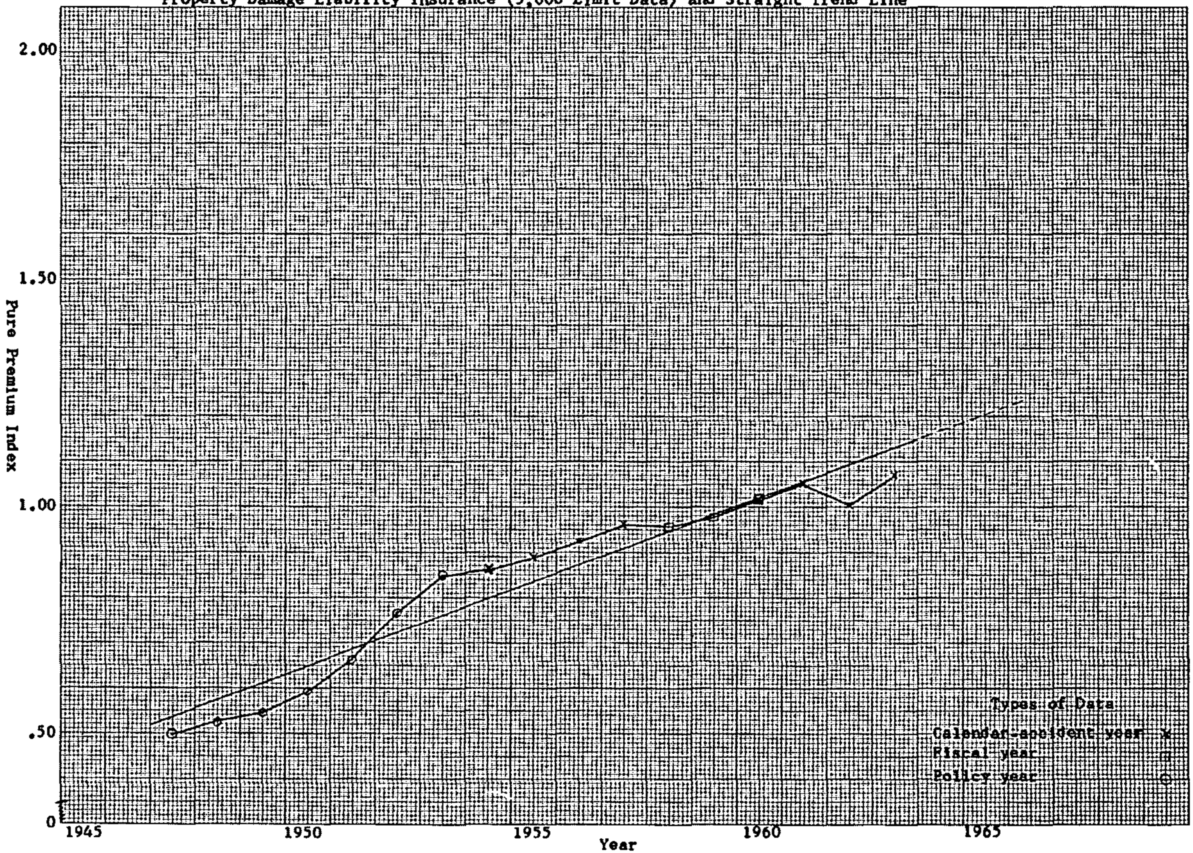


Figure 7.-Indexes of Yearly Pure Premiums (Base Period = 1958 - 1961 for Kentucky Commercial Automobile Bodily Injury Liability Insurance (5/10 Limits Data) and Straight Trend Line

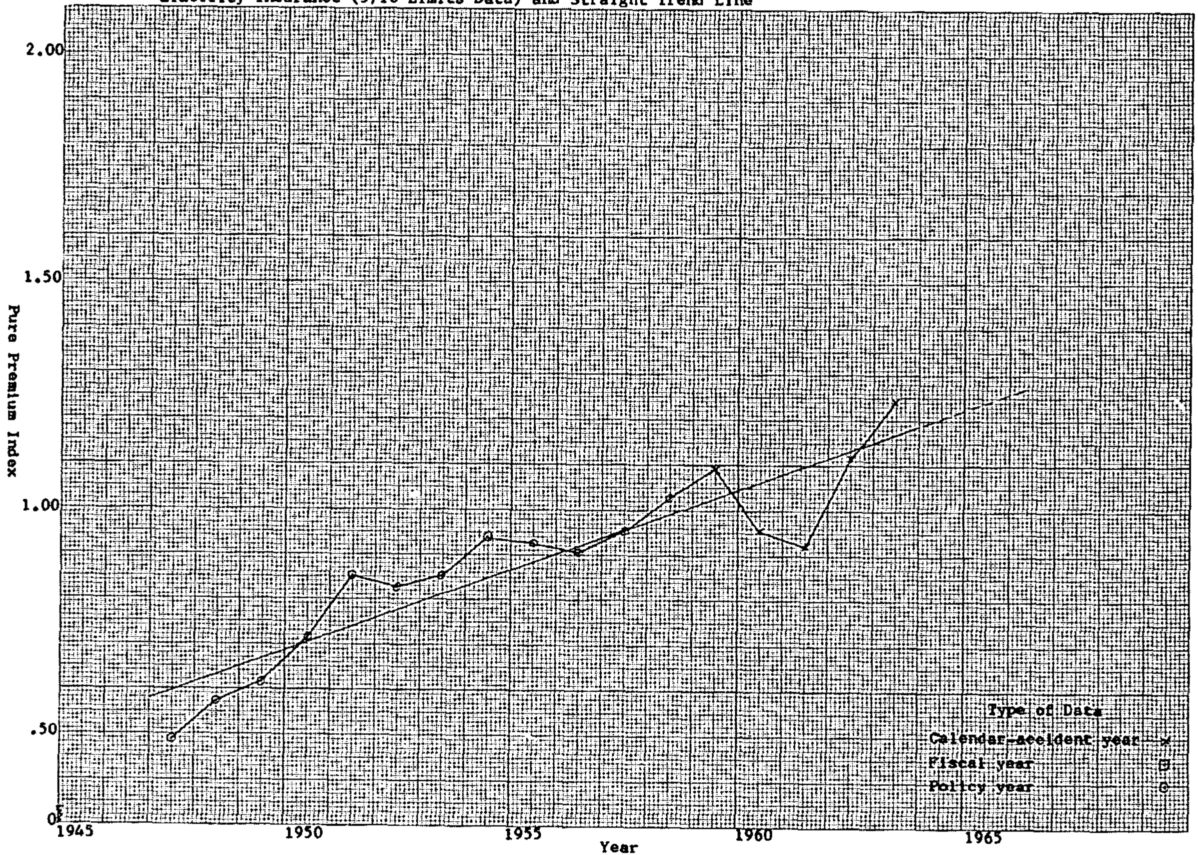
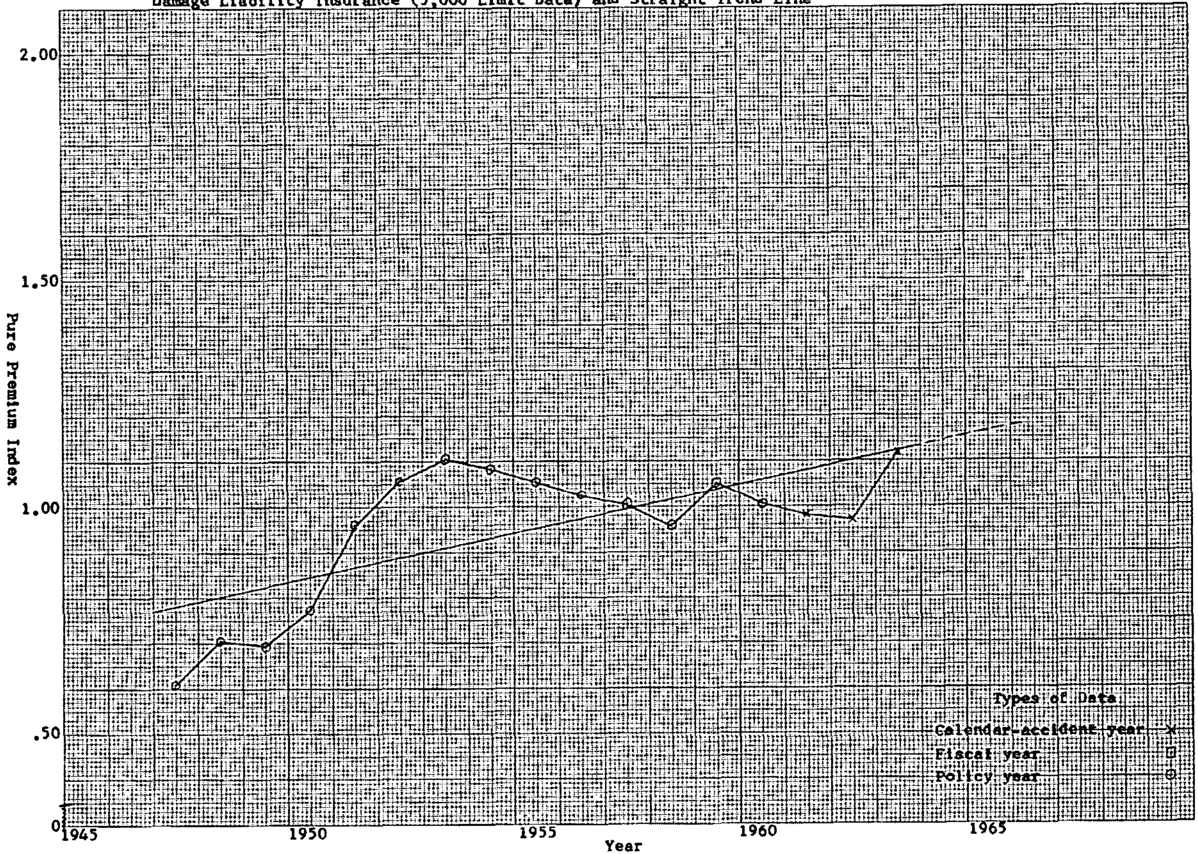


Figure 8.-Indexes of Yearly Pure Premiums (Base Period = 1958 - 1961) for Kentucky Commercial Automobile Property Damage Liability Insurance (5,000 Limit Data) and Straight Trend Line



The upward or inflationary trend is also more likely to reflect a steady rate than a diminishing or growing rate of increase over the longer term. The general economic data support this better than the insurance data available, since most of the latter are not available for long enough periods (preferably fifty or more years). The form of curve that best fits the data is therefore more likely, other things being equal, to be of second or higher degree. Most analysts have so far limited their choice of trend lines to straight lines, which do not conform to this standard. This choice is however not "wrong." A straight line has the advantage of being much easier to fit to statistical data. Possibly greater accuracy in projections is sacrificed by using a straight line, but there are also greater risks in using a less simple type.

It was hypothesized that a third degree trend might be most appropriate, based on the recent history of liability claim practices. The surge in organized activity among trial attorneys representing claimants, that began in the early 1950's, could reasonably be expected to steepen the rate of rise in pure premiums from that time on. An offsetting attempt by defense attorneys that has primarily been confined to the 1960's could be expected at least to begin to offset the results of the plaintiff-attorney activities, thereby tending to flatten the curve again. The two changes in direction of the trend line would accord with the shape of a third degree curve.

Objective criteria for selecting the most appropriate type of trend are given by one authority as follows:

(1) If the first differences are constant, use a straight line.

(2) If the second differences are constant, use a second degree curve.

(3) If the third differences are constant, use a third degree curve.

(4) If the first differences are changing by a constant percentage, use a modified exponential.

(5) If the first differences resemble a normal curve, use a logistic.

(6) If the first differences resemble a skewed frequency curve, use a Gompertz curve or a complex type of logistic.

(7) If the first differences of the logarithms are constant, use an exponential. (Fit a straight line to the logarithms.)

(8) If the second differences of the logarithms are constant, fit a second degree curve to the logarithms.

(9) If the first differences of the logarithms are changing by a constant percentage, use a Gompertz curve.

(10) If the first differences of the reciprocals are changing by a constant percentage, use a logistic curve.[27]

Examination of the first, second, and third differences gave no clear indication as to which degree of curve would fit best. This was not unanticipated, since neither the quality nor the length of the data, as explained in the foregoing section on measuring trend, is adequate fully to support fitting of a trend of higher than first degree. As a matter of interest, however, first, second, and third degree polynomial trend lines were fitted to the adjusted data for private passenger and commercial vehicles. Because of the limited data (seven years) only straight trend lines were fitted to the garage liability data. The standard error of estimate was calculated to provide the best available measure of fit.[28] Orthogonal polynomials were used to minimize computing time.[29]

Table 2 shows that a third degree curve has the best mathematical fit in three of four cases. In one of the four cases a second degree curve fits best and in two other cases second best. Of the lines fitted to the private passenger data, only the straight lines continue upward to the right, however, so they best meet all pertinent criteria. Because of the sensitivity of

Data	Standard Errors of Estimate		
	First Degree	Second Degree	Third Degree
Pvt Pass Veh BI	.051	.053	.031
Pvt Pass Veh PD	.071	.032	.033
Com-BI	.088	.075	.064
Com-PD	.117	.119	.068
Gar-BI	.160	—	—
Gar-PD	.068	—	—

Table 2. Standard Errors of Liability Insurance Data

27 Note 18.

28 *Ibid.*, p. 462.

29 See Fisher, R. A., *Statistical Methods for Research Workers*, Oliver & Boyd, Edinburgh, 7th ed., pp. 148–155.

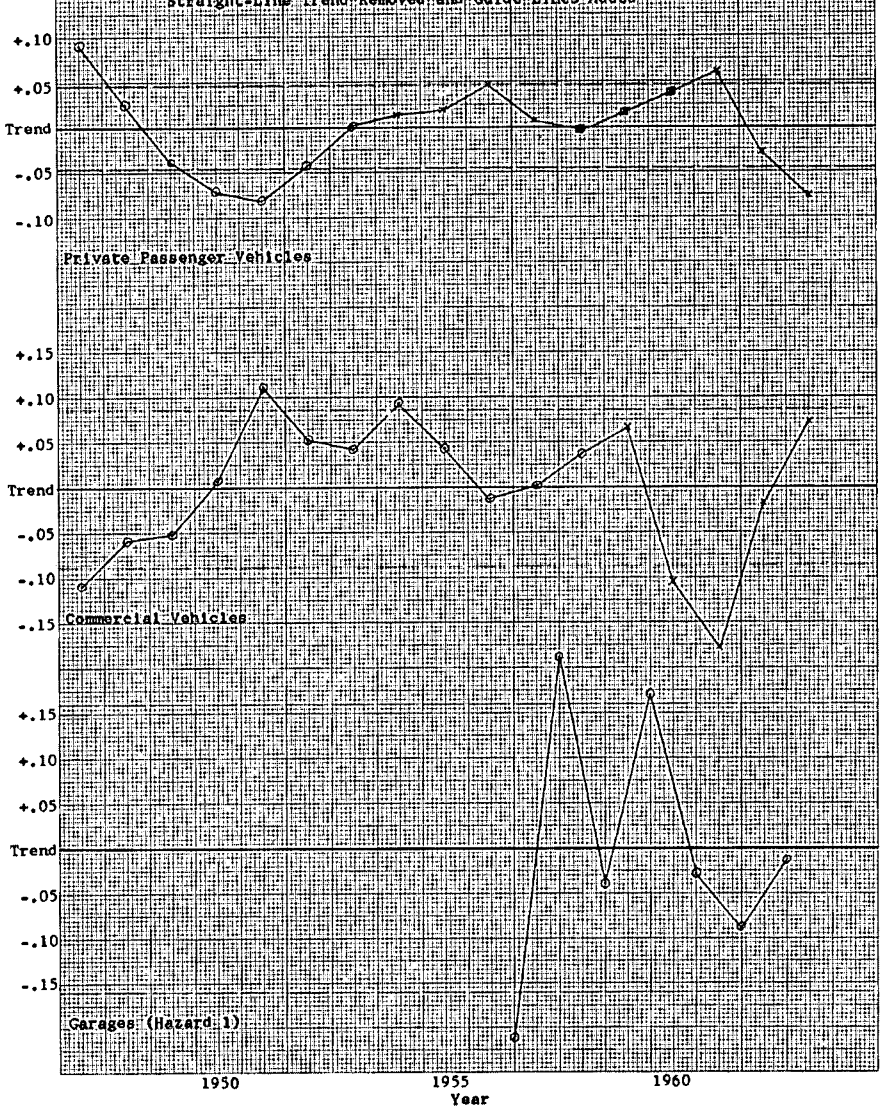
higher degree curves to small differences in data, the results obtained with them are judged to be preliminary. They do indicate however that further testing, with more refined data, is merited. Similar results with more accurately adjusted figures and (in view of their limited length) after extension of the series between endpoints that are at the same stage of a cycle would fully sustain selection of a curve of higher than first degree.

As a check on the general tendencies noted in these data, and on the capacity of the methods of trend analysis here described to handle diverse kinds of data, other data from a second state and from two provinces of Canada were examined. Although to save space only private passenger data are shown here, no difficulties with either commercial or garage data from these other areas were noted. The other state data in Figures 11 and 12 and those for two Canadian provinces, exhibited in Figures 1 and 14, all lend themselves excellently to our methods of analysis. The different legal climate in Canada gives no reason such as was mentioned for the United States for expecting a third degree trend, but the combination of bodily injury and property damage liability into a single limit package in more recent years cannot be seen to have made the Canadian pattern deviate materially from the American pattern. While the Canadian data are on the less desirable all-limits basis, this has not prevented an adequate analysis by the methods described.

A visual inspection of the data also shows a pronounced and relatively regular multi-year cyclical movement. It is easiest to see this from Figures 9, 10, and 13, from which the straight line trend has been removed. Figures 1, 5 through 8, 11, 12, and 14 show that in many years the cyclical movement causes considerably more variation in the data than does the trend movement. It is as important that this cyclical component be cared for in some orderly manner as it is similarly to care for trend.

It is seldom required or feasible to project data of as much variability as these for more than one year ahead in fields outside insurance. Insurance ratemaking may require, however, the projection of the pure premium or rate level as much as three or more years. Until the period over which the forecast or projection must be made can be reduced (by securing more recent data) to not more than, say 18 months, it therefore appears most feasible to project along or close to the trend line. However, if the point from which the projection is made departs widely from the trend line, a trend adjustment alone will produce a forecast that departs equally far from the trend line, often an improbable result. A more accurate projec-

Figure 9.- Indexes of Yearly Pure Premiums (Base Period = 1958-1961) for Kentucky Automobile Bodily Injury Liability Insurance (5/10 Limits Data) With Straight-Line Trend Removed and Guide Lines Added



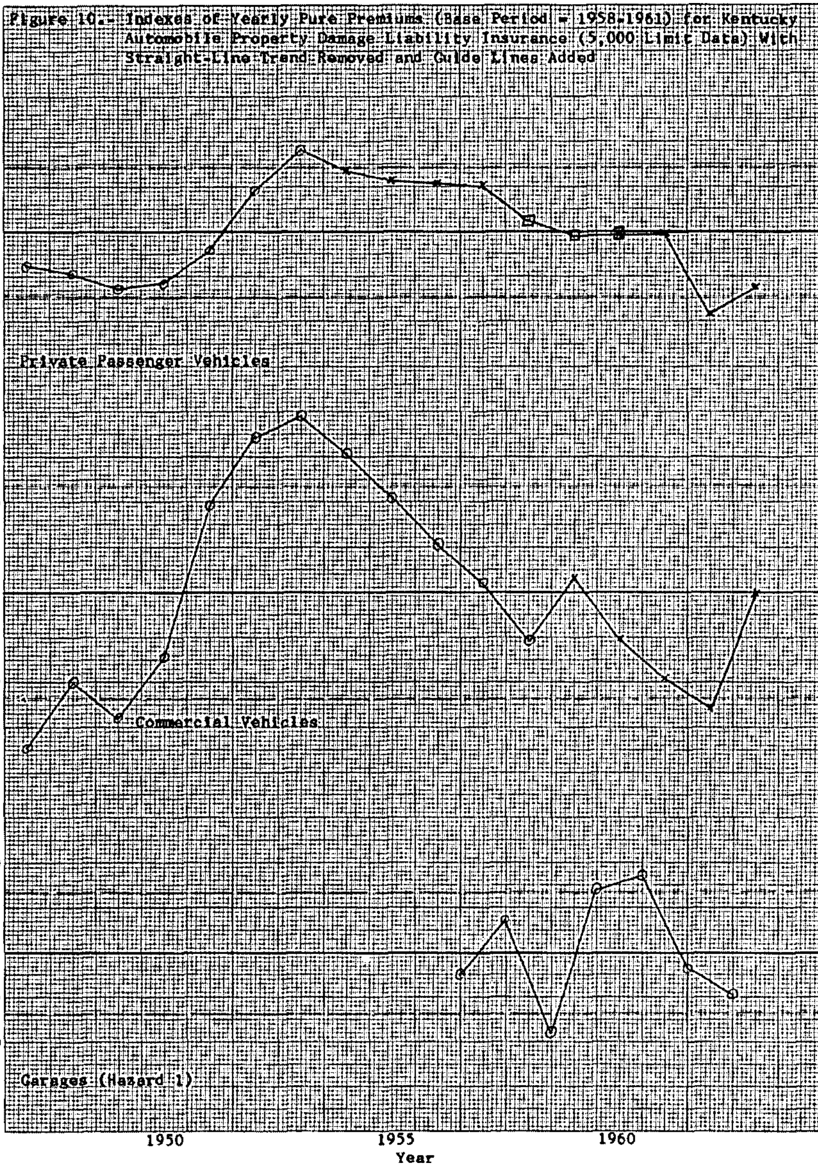


Figure 10.- Indexes of Yearly Pure Premiums (Base Period = 1958-1961) for Kentucky Automobile Property Damage Liability Insurance (5,000 Limit Data) With Straight-Line Trend Removed and Guide Lines Added
+.15
+.10
+.05
Trend
-.05
-.10
-.15
Private Passenger Vehicles
+.15
+.10
+.05
Trend
-.05
-.10
-.15
Commercial Vehicles
+.10
+.05
Trend
-.05
-.10
Garages (Hazard 1)
1950
1955
1960
Year

Figure 11.- Indexes of Yearly Pure Premiums (Base Year = 1960) for Illinois Private Passenger Automobile Bodily Injury Liability Insurance (Adjusted 10/20 Limits Data) and Straight Trend Line

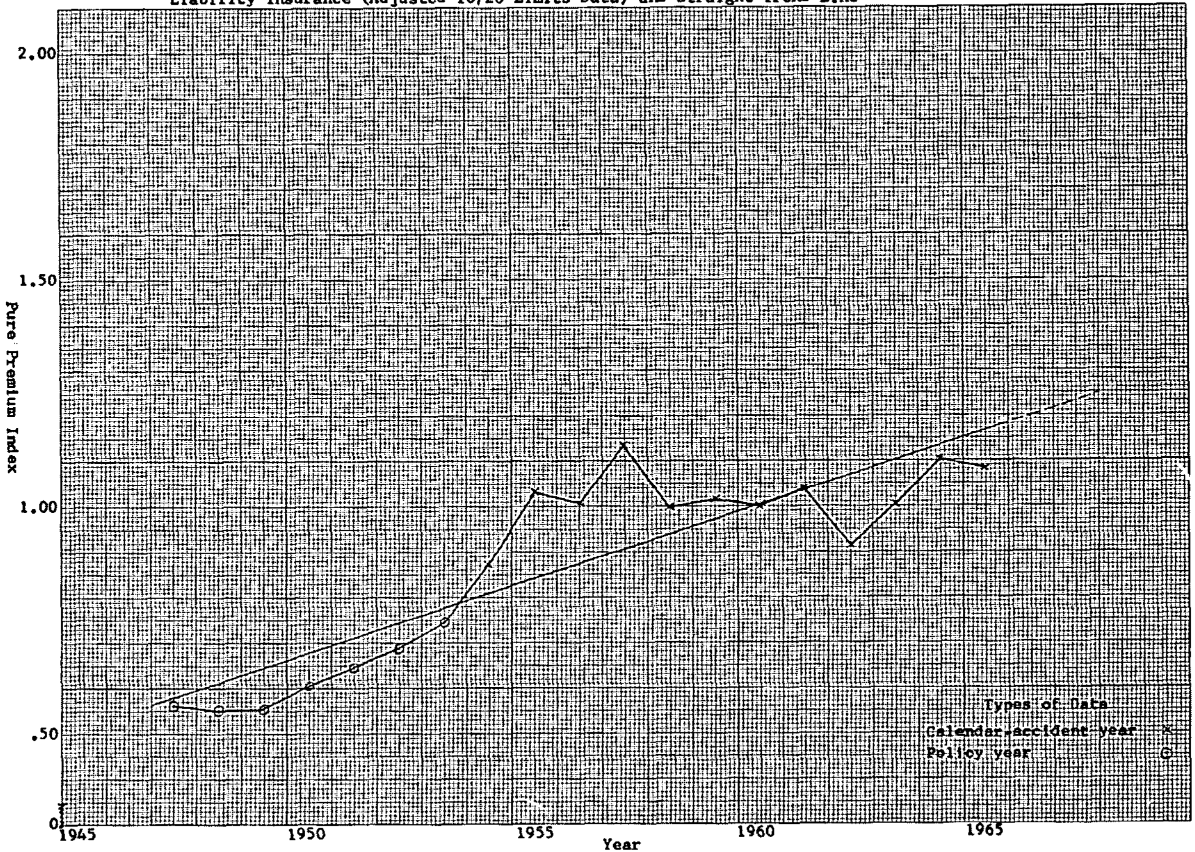


Figure 12.- Indexes of Yearly Pure Premiums (Base Year = 1960) for Illinois Private Passenger Automobile Property Damage Liability Insurance (5,000 Limit Data) and Straight Trend Line

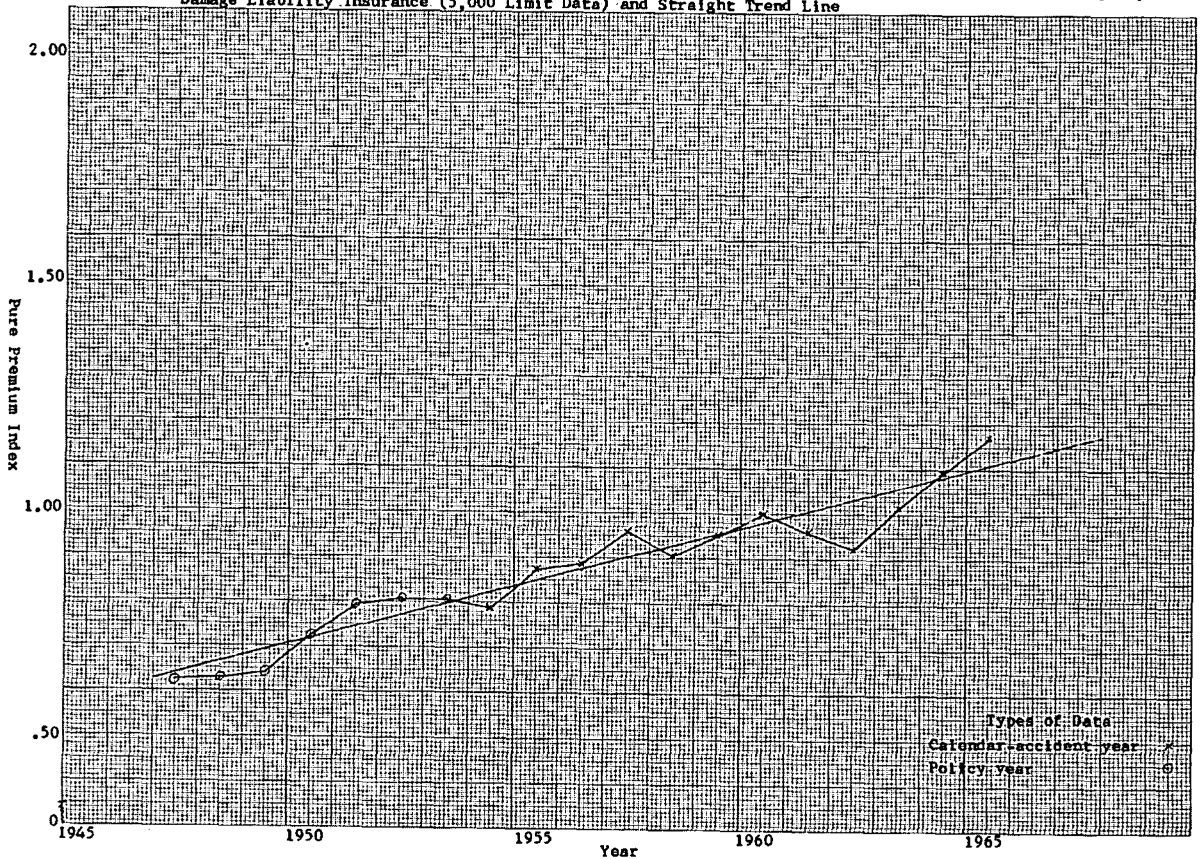


Figure 13.- Illinois Private Passenger Automobile Liability Insurance Data with Straight-Line Trend Removed and Guide Lines Added

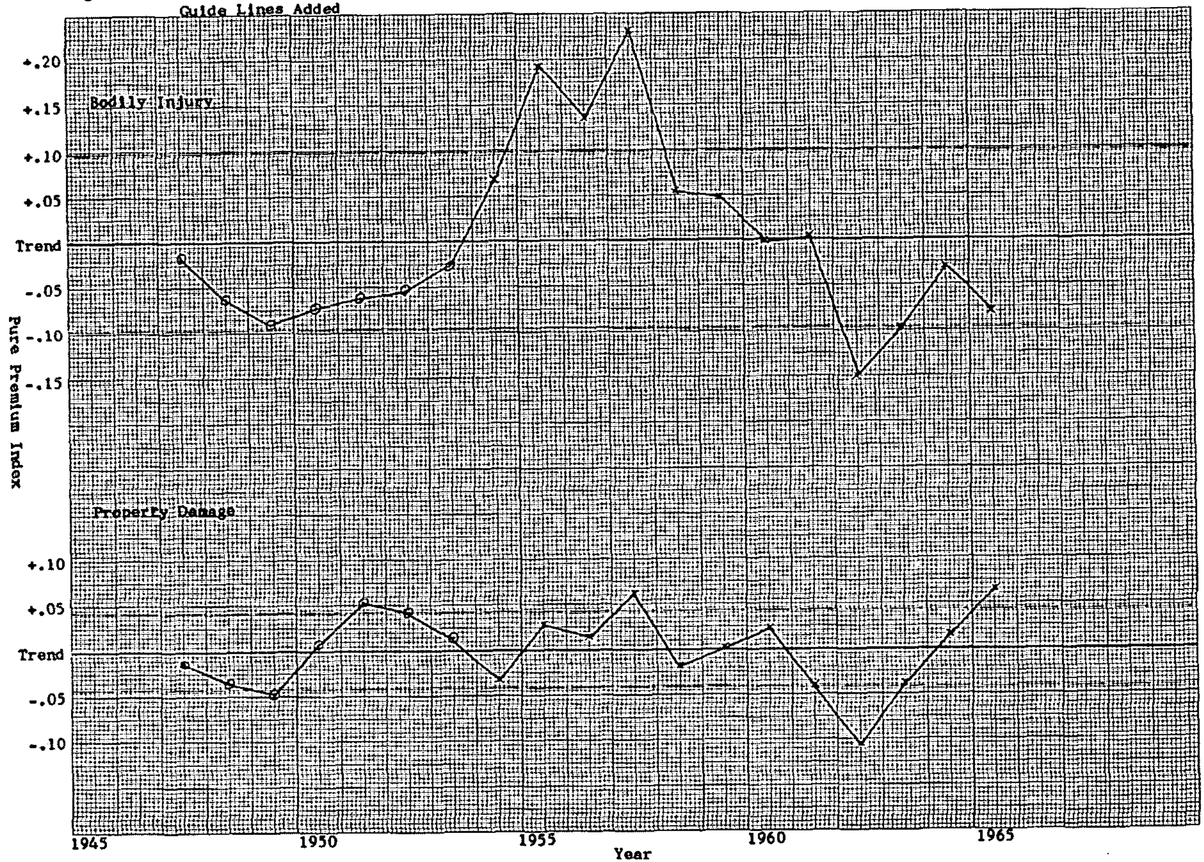
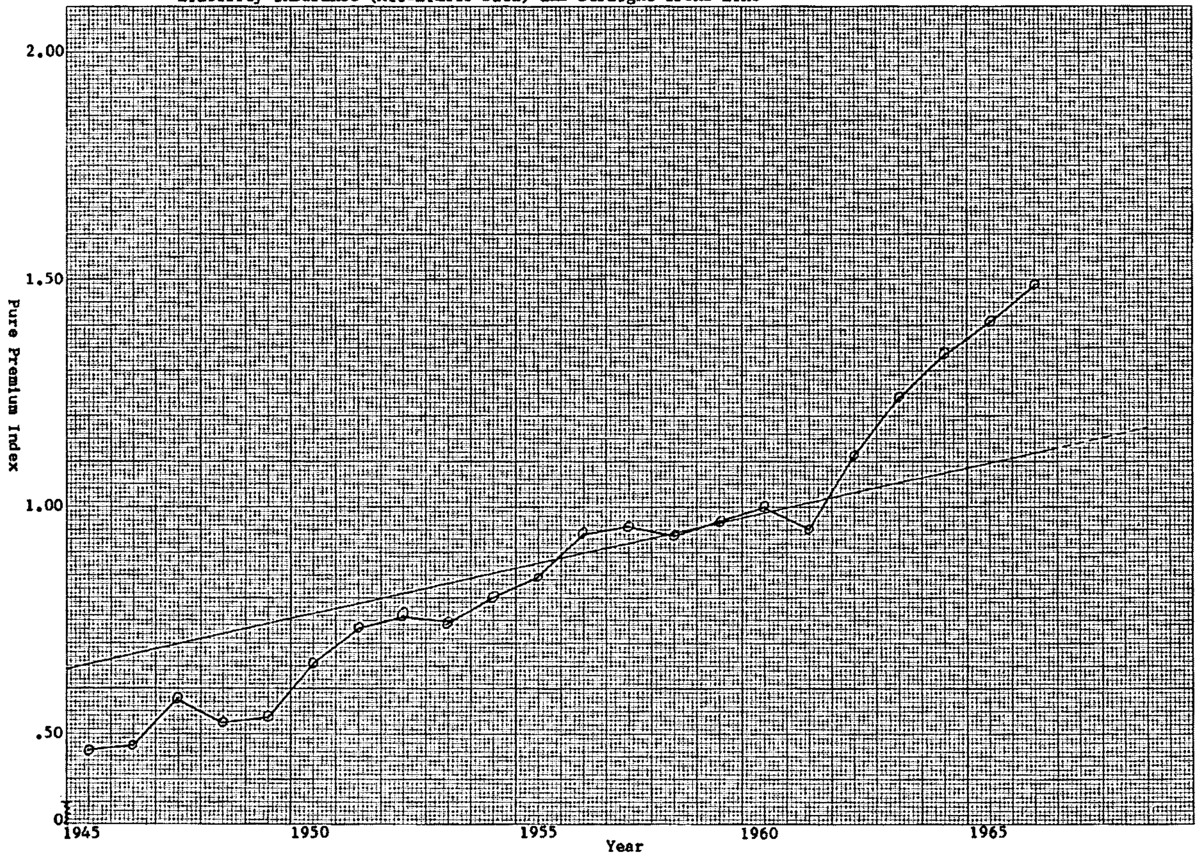


Figure 14.- Indexes of Yearly Pure Premiums (Base Year = 1960) for British Columbia Private Passenger Automobile Liability Insurance (All-Limits Data) and Straight Trend Line



tion is apt to result if an adjustment is made for the cyclical departure of the starting point from the trend line. The method previously described in the section on measuring cycles proved useful with all the data here presented. An example with the initial state data will illustrate the procedure.

The top portion of Table 3 shows the calculation of combined trend and cyclical adjustments to the six classes of liability data, which have three different midpoints in time, using the straight-trend values. The trend values were first projected forward from the last datum point, as shown in columns (2), (3), and (9). This adjustment for private passenger bodily injury liability is graphically portrayed in Figure 15. The vertical distance between the left-hand arrows is the amount of trend adjustment (.118).

The dashed guide lines in Figure 15 lie one standard error above and below the solid trend line. Since the last datum point is outside the lower guide line, the cyclical adjustment called for by the rule equals the vertical distance from the datum to the guide line. This vertical distance, between the right-hand arrows in Figure 15, is the amount of cyclical adjustment (.019). The determination of this adjustment is shown in columns (4) through (8) of Table 3.

Adding the trend and cyclical adjustments, column (10), yields the total time-series adjustment. The fully adjusted, predicted, point for 1 April 1966 would lie on the lower guide line of Figure 15, directly above the trend-adjustment arrow. Dividing the total adjustment by the trend value at the starting point yields the total time-series adjustment factor, column (11), to be applied as part of the whole rate level adjustment.

ANALYSIS OF SOME PROPERTY INSURANCE TIME SERIES

Kinds and Characteristics of Property. In contradistinction to a liability insurance loss, the size of which is relatively independent of the insured interest, the characteristics of a property insurance loss are highly correlated with those of the subject of insurance. Property insurance applies to losses to specific property, while liability insurance does not apply to losses to either specific persons or specific property. Analysis of property insurance time series must therefore take carefully into account several types of factors in addition to those considered in analyzing liability insurance time series.

One such factor is the relative uniformity or diversity of the property involved. The degree of uniformity in size, shape, and value decreases as we consider in turn automobiles, items customarily scheduled in inland marine

Table 3.—Summary Comparison of Time-Series Adjustments

(1) Type of Data	(2) Starting Point or Last Datum[a]	(3) Trend Value at Starting Point	(4) Actual Value at Starting Point	(5) Deviation of Actual Value from Trend	(6) Standard Error[b]	(7) Adjustment per Rule	(8) Amount of Cyclical Adjustment	(9) Trend Value on 1 Apr 66[c]	(10) 1 Apr 66 Trend Value + Cyc.Adj.	(11) Indicated Time-Series Adjustment Factor
Automobile Liability Insurance Data										
Pvt-BI	1 Jun 63	1.112	1.042	-.070	±.051	to guide line	.019	1.230	1.249	1.123
Pass Veh-PD	1 Jun 63	1.127	1.064	-.063	.071	½ way to trend	.032	1.233	1.265	1.122
Com-BI	1 Apr 63	1.161	1.211	.050	.088	½ way to guide	.019	1.269	1.288	1.109
Com-PD	1 Apr 63	1.115	1.082	-.033	.117	½ way to trend	.017	1.179	1.196	1.073
Gar-BI	1 Oct 62	1.113	1.080	-.033	.160	½ way to trend	.017	1.292	1.309	1.174
Gar-PD	1 Oct 62	1.063	1.022	-.041	.068	½ way to guide	-.014	1.175	1.161	1.091
Automobile Property Insurance Data										
Coll	1 Mar 64	1.058	1.097	.039	.080	½ way to guide	.021	1.053	1.074	1.015
Non-coll	1 Mar 64	1.150	1.124	-.026	.116	½ way to trend	.013	1.209	1.222	1.063
Fixed-Property Data for Extended Coverage										
Combined	1 Jul 64	1.673	2.062	.399	.732	½ way to trend	-.200	1.689	1.489	.890
[Non-ded	1 Jul 58[d]	1.622	1.952	.330	.799	½ way to trend	-.165	1.694	1.529	.943]
[Deduct	1 Jul 62[d]	1.545	1.439	-.105	.495	½ way to guide	-.195	1.847	1.652	1.069]

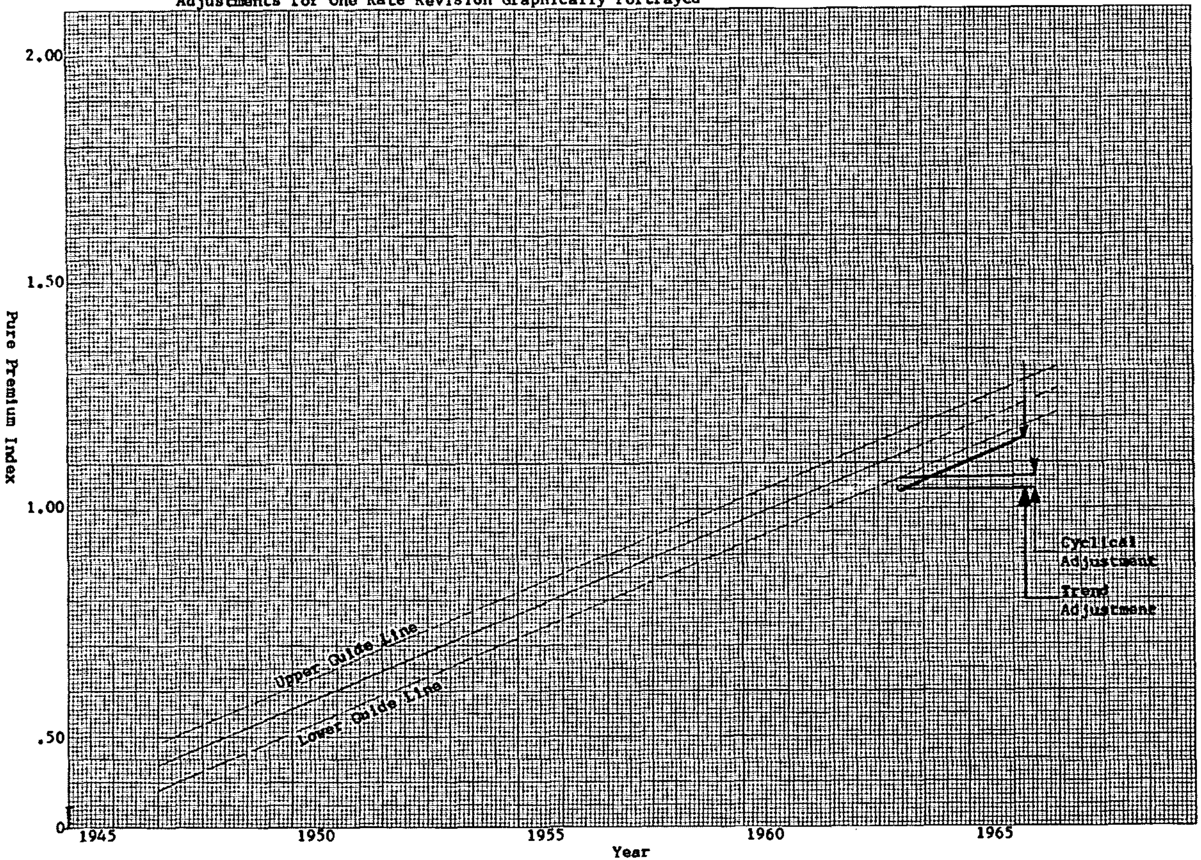
[a]This "starting point", referred to in the rule used for cyclical analysis, is either the last datum in the time series or, if two or more of the latest data are averaged (with either equal or differing weights), it is the midpoint in time of the weighted average.

[b]Distance of guide lines from trend line.

[c]1 August 1966 for extended coverage.

[d]Eleven-year average for non-deductible and five-year average for data for deductible coverage.

Figure 15.- Kentucky Private Passenger Automobile Bodily Injury Liability Data With Indicated Trend and Cyclical Adjustments for One Rate Revision Graphically Portrayed



policies (furs, jewelry, silverware, stamp and coin collections, outboard motors and boats, etc.), personal property at fixed locations, and buildings and other structures. The degree to which the amount of insurance matches the value of the property is another factor and also roughly decreases in the same order, with perhaps the first two classes reversed. A third factor is the ability of the rating system to reflect in premium charges the degree to which amount of insurance matches value or exposure to loss. Issuance of "actual value" policies, rather than policies with a stated amount of insurance against which a stated rate is applied, has introduced this third factor, which can have an appreciable effect.

Exemplifying to some extent the two extremes listed above, some automobile time series and then two fixed property time series are analyzed below.

Nature of the Automobile Data. Total limits automobile insurance data were used in order to reflect a valid cross section of the insured risks, because the sizes of property losses are closely linked to the characteristics of the insured risks (primarily value at risk). Only data for risks rated on a per vehicle basis were available. As representative of at least 85 per cent of the total exposure for one state and at least 67 per cent in another for each year, this is deemed a sufficiently large sample. There is no apparent reason to believe that addition of the data for the omitted types of risks (dealers, fleets, garage bailees' liability, single interest coverages, etc.) would materially alter the results obtained. Data for one state were first analyzed. Suitably consistent parallel countrywide data were not available, so an intended test of the correlation between the two could not be performed.

The data are grouped separately for (1) collision and (2) non-collision coverages. Collision data for $25, $50, $100, and $250 deductible options, and non-collision data for full coverage and $50 deductible comprehensive, specified individual perils, combined additional coverage, and towing were included. They cover the period 1947 through 1964 on a fiscal year basis, the first three fiscal years ending at a different time than the others. The data reflect paid rather than incurred losses, and exclude loss adjustment expense, although rates for these lines are usually based on data that include such expense. Losses from catastrophes occurring prior to 1958 are apparently included in the data, although a separate procedural allowance for such losses makes it preferable to exclude them. This feature seems materially to have affected the slant of the trend line for non-collision coverages. Since losses are on a paid basis, they are not developed to any consistent point.

The one discontinuity in the data was a three-month gap between June and September 1949. It was necessary to adjust the data for the first three years to a fiscal year ending three months later than that reported. This was done by averaging three-fourths of the prior year data with one-fourth of the later year data for 1947 and 1948. The data for 1949 were recentered by algebraically adding to the reported 1949 data one-fifth of the difference between them and the reported 1950 data.

To make mutually comparable the data for different coverages and for different deductibles, it was necessary to devise an index number for each series. Fisher's Ideal Index Number was selected.[30] It has the advantages of meeting the factor reversal and time reversal tests. This type of index number thereby eliminates bias due to a changing mix or proportionate distribution of risks by type of deductible. The index number also averages all the types of coverage according to the number of exposures for each, both in the base year (the year in which the index is 100 per cent) and in the year for which the index number is being computed. This type of index number also adjusts equitably for the absence in early years of data for some coverages and options. A sample calculation to demonstrate the procedure is shown in the Appendix.

The effect and value of the indexes can be seen from Figures 16 through 20. The patterns described in Figure 16 by the single state collision data for individual deductible forms are very similar to the patterns in the countrywide Canadian data in Figure 18. Similar patterns have been found in the data for other states and individual provinces.

Figures 16 and 18 show data for four different deductibles, as explained in the lower right-hand corner. Although the general tendencies of the four sets of data are similar, no one of them well represents the whole group. Also, the fluctuations from year to year are quite wide, caused in part by sparseness of data. Figures 17 and 19 show, respectively, how all the diverse data from Figures 16 and 18 can be combined by use of a well-designed index number. Much of the random fluctuation has been eliminated by use of the larger body of data reflected by each index. One can see by superimposing each index over its four components how it excel-

[30] See Fisher, Irving, *The Making of Index Numbers,* Houghton Mifflin Co., Boston, 3d ed., and Neiswanger, *op. cit.,* pp. 398–411.

Figure 16.- Separate Indexes of Yearly Pure Premiums (Base Year = 1960) for Kentucky Private Passenger Automobile Collision Insurance (All-Limits Data) for $25, $50, $100, and $250 Deductible Coverages

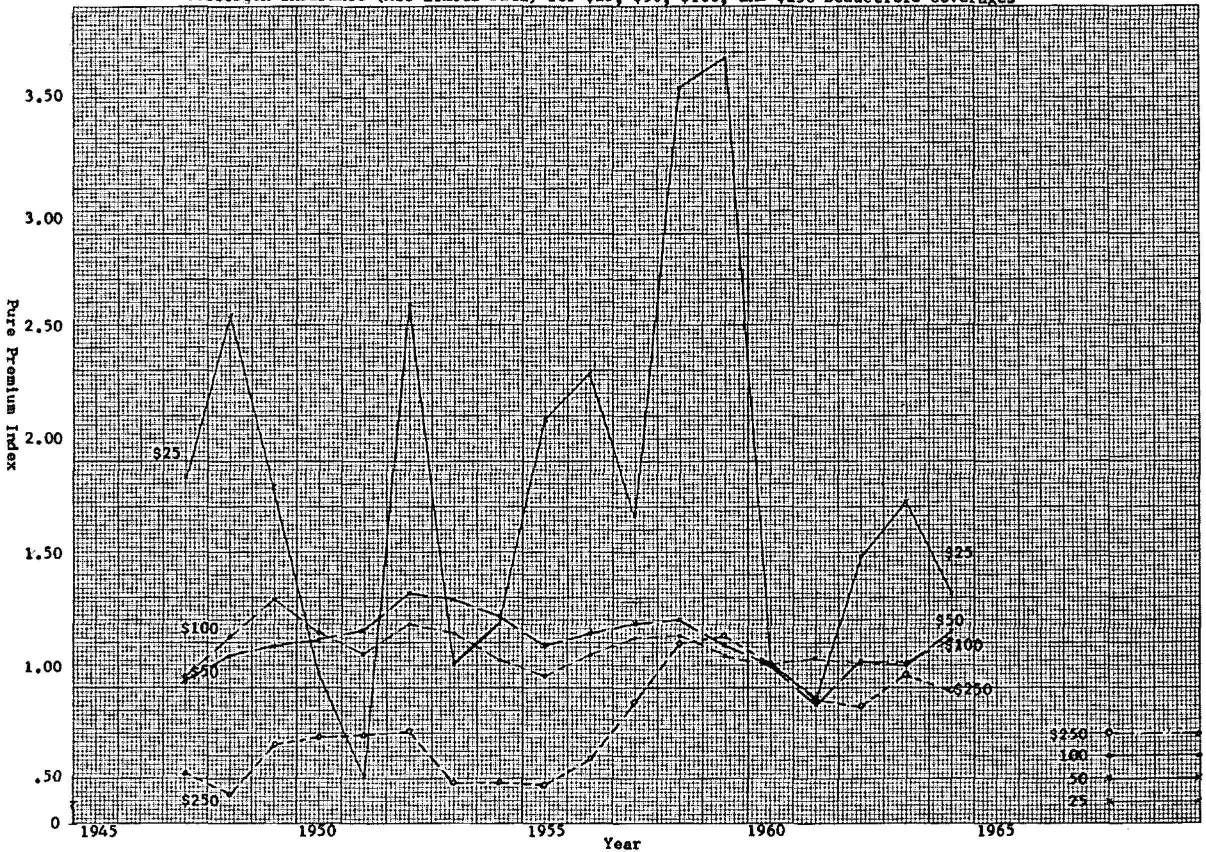


Figure 17.- Combined Index of Yearly Pure Premiums (Base Year = 1960) for Kentucky Private Passenger Automobile Collision Insurance (All-Limits Data) for $25, $50, $100, and $250 Deductible Coverages and Straight Trend Line

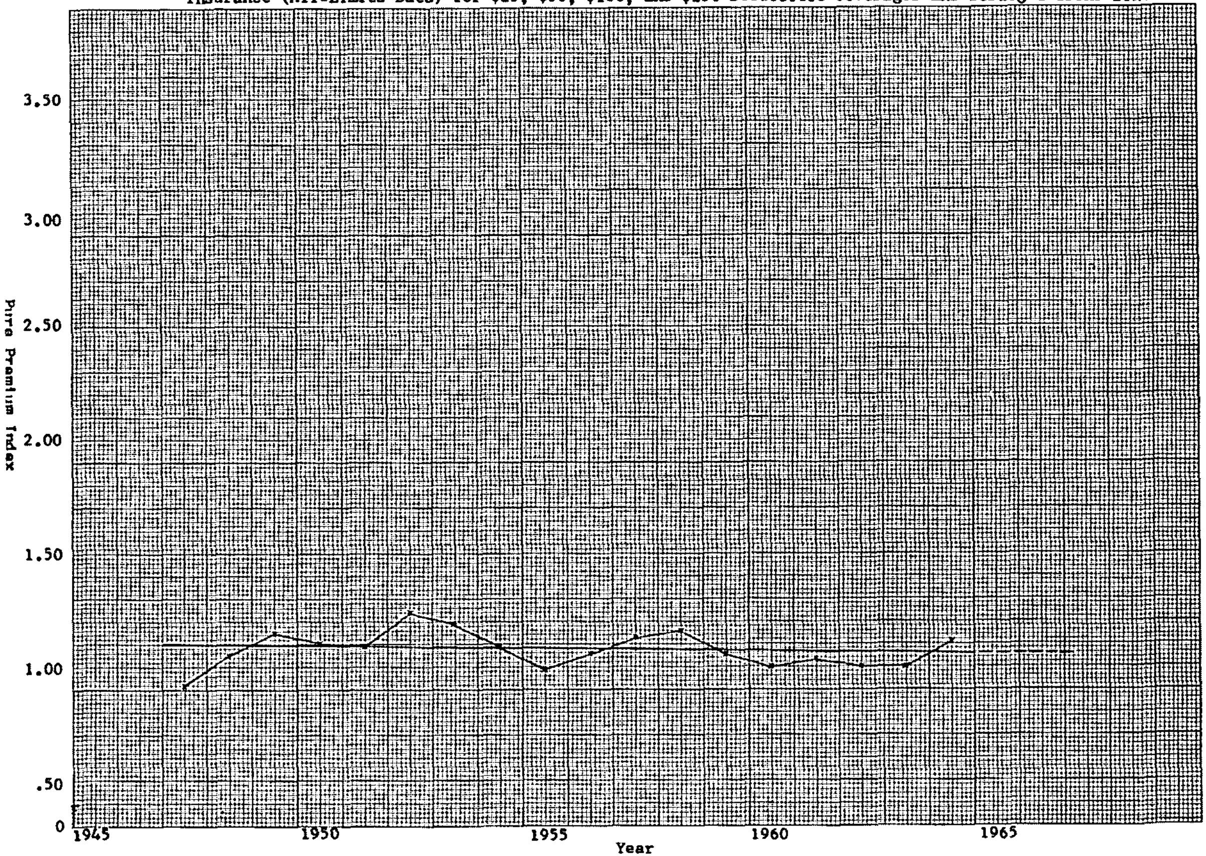


Figure 18.- Separate Indexes of Yearly Pure Premiums (Base Year = 1960) for Countrywide Canadian Private Passenger Automobile Collision Insurance (All-Limits Data) for $25, $50, $100, and $250 Deductible Coverages

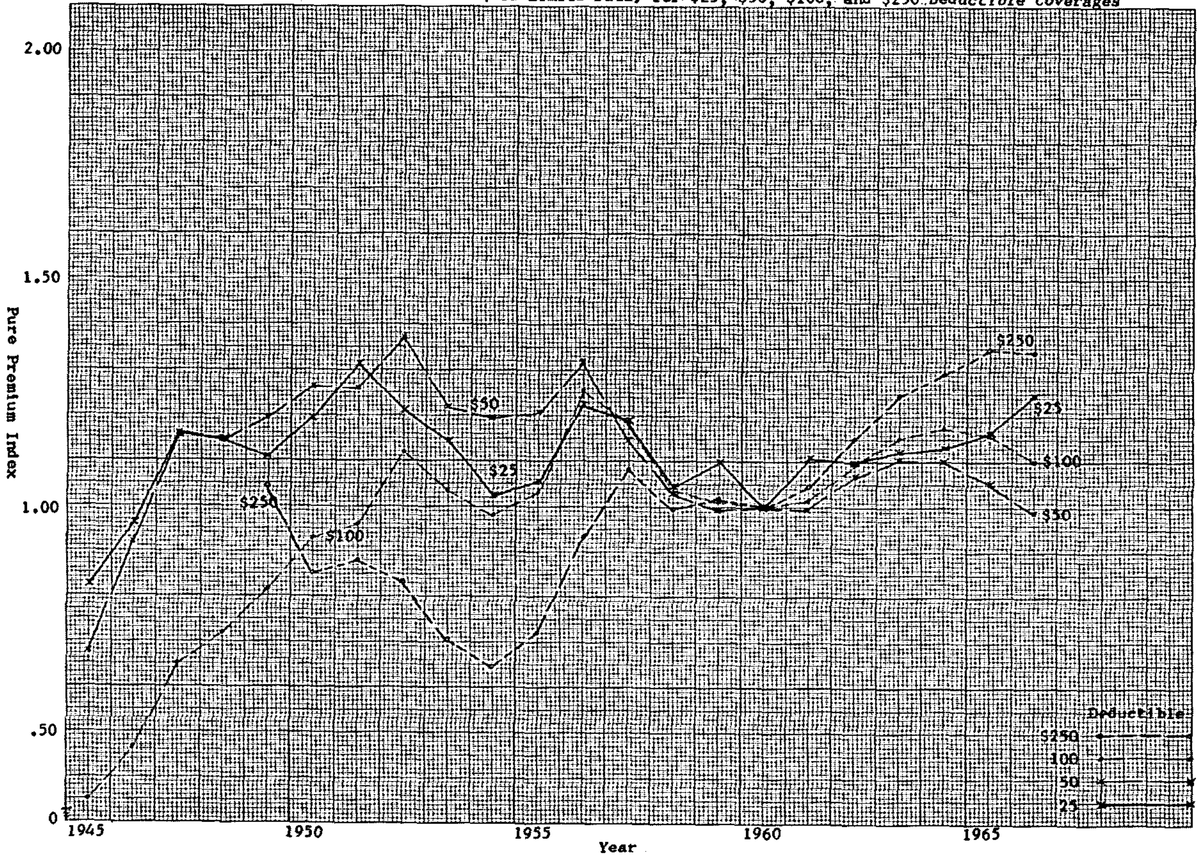


Figure 19.-Combined Index of Yearly Pure Premiums (Base Year = 1960) for Countrywide Canadian Private Passenger Automobile Collision Insurance (All-Limits Data) for $25, $50, $100, and $250 Deductible Coverage and Straight Trend Line

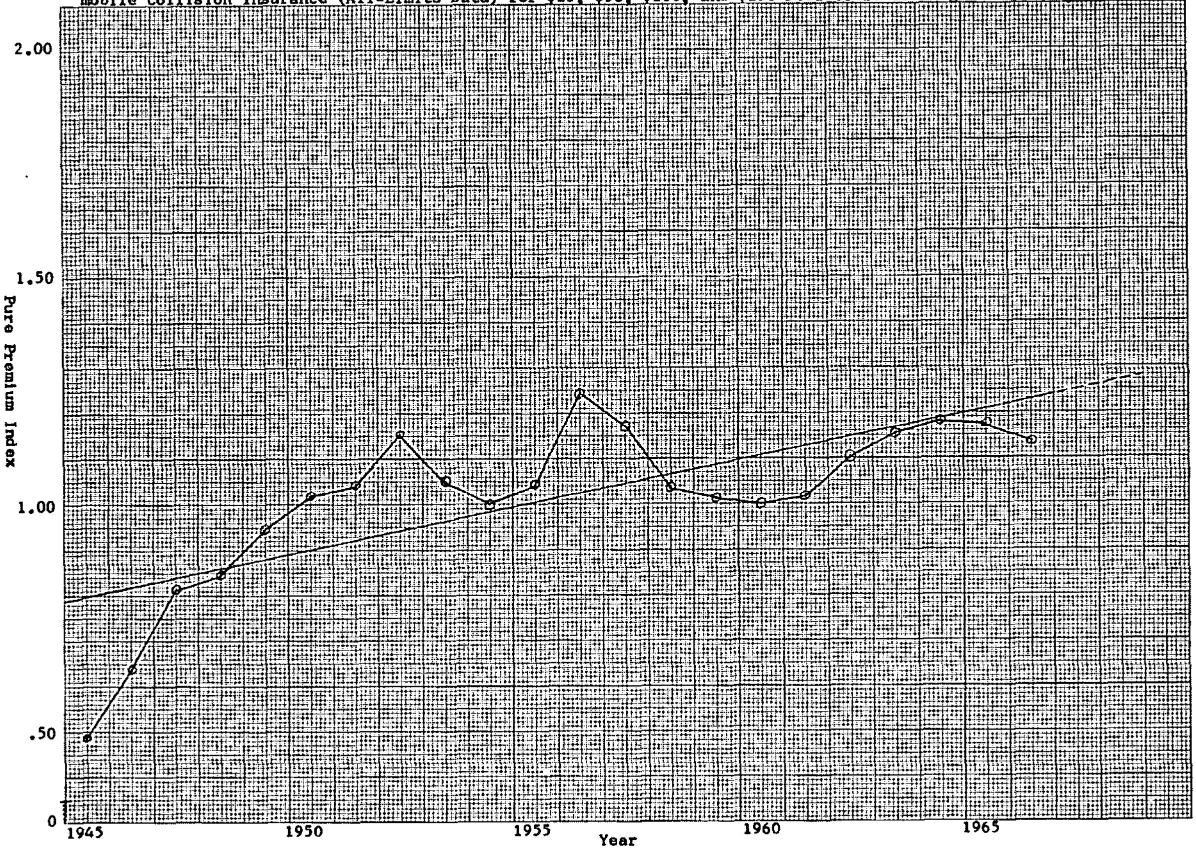
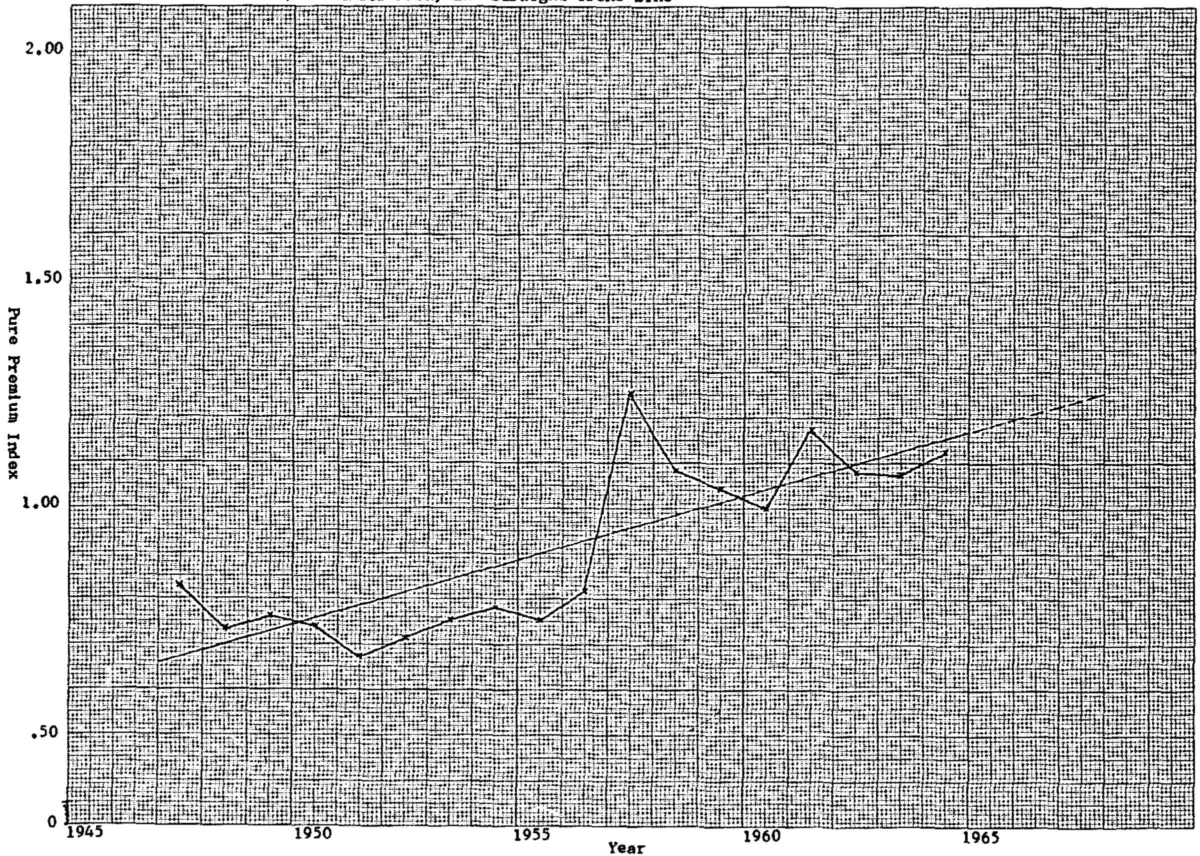


Figure 20.- Combined Index of Yearly Pure Premiums (Base Year = 1960) for Kentucky Automobile Non-Collision Property Insurance (All-Limits Data) and Straight Trend Line



lently represents the central or average tendencies of the four separate groups of data.

Accuracy of the index numbers could be improved by reflecting in addition to the mix by coverage and deductible, the changing distributions by value or price group; by use (pleasure, commercial, public, etc.); by physical type (passenger cars, trucks, buses, etc.); by rating territory; by status with respect to special charges and credits for driver experience and training, two-car families, etc.; and by all similar factors reflected in the rating system. A suitably designed stratified random sample would permit accuracy without review of 100 per cent of the data. But it is again important to point out that, since we are dealing here with a population-wide average of an individual characteristic and not with a classification or breakdown by one of several partially correlated rating criteria, the results of our procedure are of quite acceptable accuracy. The point is simply that they could be made *more* accurate.

Procedure Used with the Automobile Data. The economic environment was first considered. Most available economic measures that are in the form of time series point to a steady inflationary trend since World War II. During the first six years of this decade the price trend of new automobiles had flattened, but the mandatory addition of seat belts and anti-pollution devices has recently been reflected in new car prices and the prices of used cars and parts have continued to rise. Although there is a great likelihood that any time series reflecting property insurance losses will show a gradual increase, there are several factors which must be taken into account. Any one of these may offset wholly or partially an inflationary tendency in the others. For example, a gradual shift to higher deductibles for collision insurance may closely parallel the shift to higher valued automobiles, resulting in a stable pure premium for any given deductible amount. Although a greater number of vehicles on the roads may lead one to expect a higher relative frequency of accidents, this may be wholly or partly offset by an increase in two-car families and a consequent drop in the average number of miles each car is driven per year. Such factors as these are less likely to affect non-collision coverages, which relate to perils much more nearly outside the control of the vehicle owner.

The same outside economic factors affecting the shape of the trend, that were described in connection with liability coverages, apply equally to the physical damage coverages. A visual inspection of the data used here also shows the cycles observed in the liability data. It is very easy to see this from Figures 16 through 20, which show that in most years for both types of coverage the cyclical movement causes considerably more variation in the data than does the trend movement.

First through fourth degree trend lines were fitted to the two sets of single state data. The same criteria and considerations relating to choice of a curve shape that were discussed in connection with the liability data also apply here. Table 4 shows that the third degree curve has a better mathematical fit in both cases, particularly so for non-collision coverages. No theoretical grounds are apparent here, however (as they were for the liability data), for expecting a better fit by a curve of higher than second degree although they may, of course, exist. The facts that the excess portions of catastrophe data have been removed since 1958, and that there was a 1957 catastrophe, could have affected the relative size of the standard errors of the non-collision data. (Better data are, of course, unavailable.) These results may therefore also be judged to be preliminary. Data uniformly including or (better) excluding catastrophe results, preferably on an incurred loss basis, and reflecting in a controlled manner the variables mentioned above would support a firmer conclusion on the most appropriate shape of curve.

	Standard Errors of Estimate			
Data	First Degree	Second Degree	Third Degree	Fourth Degree
Coll	.080	.076	.064	.066
Non-coll	.116	.117	.097	.100

Table 4. Standard Errors of Automobile Property Insurance Data

Accordingly, the straight line trends were used as the best available practical alternative. Table 3 shows the calculation of adjustments to the two classes of data, based on these straight trend-line values. The procedure is the same as that used to produce the liability adjustments. Despite the difficulties with form and quality of data, the methods being described can be seen to produce most satisfactory results.

Nature of the Fixed Property Data. Extended coverage data for dwellings in one state are used to illustrate an application of time-series analysis to rates for fixed location properties and to loss ratio data. Even were consistent countrywide data available, windstorm conditions vary so markedly from area to area that it would make doubtful sense to use such data. Total limits data were available, separated by deductible status. The data for buildings are not kept separate from those for contents, however, even though different rates are used for the two classes of property. Pure premium data have not been available for fixed location properties since 1943, so loss ratios had to be used for analysis. Premiums and losses for calendar years 1947 through 1964 were secured. Since earned premiums and incurred losses were not reported prior to 1953, estimated earned premiums (brought uniformly to the 1 January 1965 rate level) were constructed for the early period by assuming that all policies were written for three years. This assumption was based on a review of annual statements for several years and several companies.

There being at the time covered by the data no formal countrywide arrangement for separating catastrophe loss data for ratemaking purposes in extended coverage insurance, the data fully reflect all such losses. That the procedure here used is able satisfactorily to overcome this difficulty is evidence of its usefulness and very general applicability. Absence of accident year data may tend slightly to understate the severity of catastrophic events, due to deferred loss settlements. Since the deferred losses are added to later data, however, the result is a not wholly undesirable smoothing. No evidence either of change or stability in the average ratio of insurance to value was available. The relatively steady turnover rate among existing dwellings and addition of new ones support the assumption of reasonable stability in this ratio.

Procedure Used with the Fixed Property Data. The economic considerations were similar to those for automobiles. Because extended coverage is ratably priced, and in the absence of any evidence of a decreasing ratio of insurance to value, however, a relatively flat trend could reasonably be expected.

The cycles noted in extended coverage results (see Figures 21 and 22) are by far the sharpest among the three sets of data. Cycles account for the vast bulk of the variation in the extended coverage series. Separate catastrophe data and remainder data (equivalents, respectively, of excess limits and standard limits data in liability insurance) were not available to overcome this difficulty. The relative sparsity of data in relation to this large cyclical amplitude therefore made it of little avail to fit other than straight trend lines. It also indicated the desirability, parallel to that demonstrated for automobile collision (Figures 16 and 18), of combining the data by an index number into a single series. Figure 23 shows the result.

Figure 21.- Indexes of Yearly Loss Ratios (Base Year = 1960) for Michigan Dwelling Non-Deductible Extended Coverage Insurance (All-Limits Data) and Straight Trend Lines (Fitted to 1949 - 1964 and 1947 - 1964 Data)

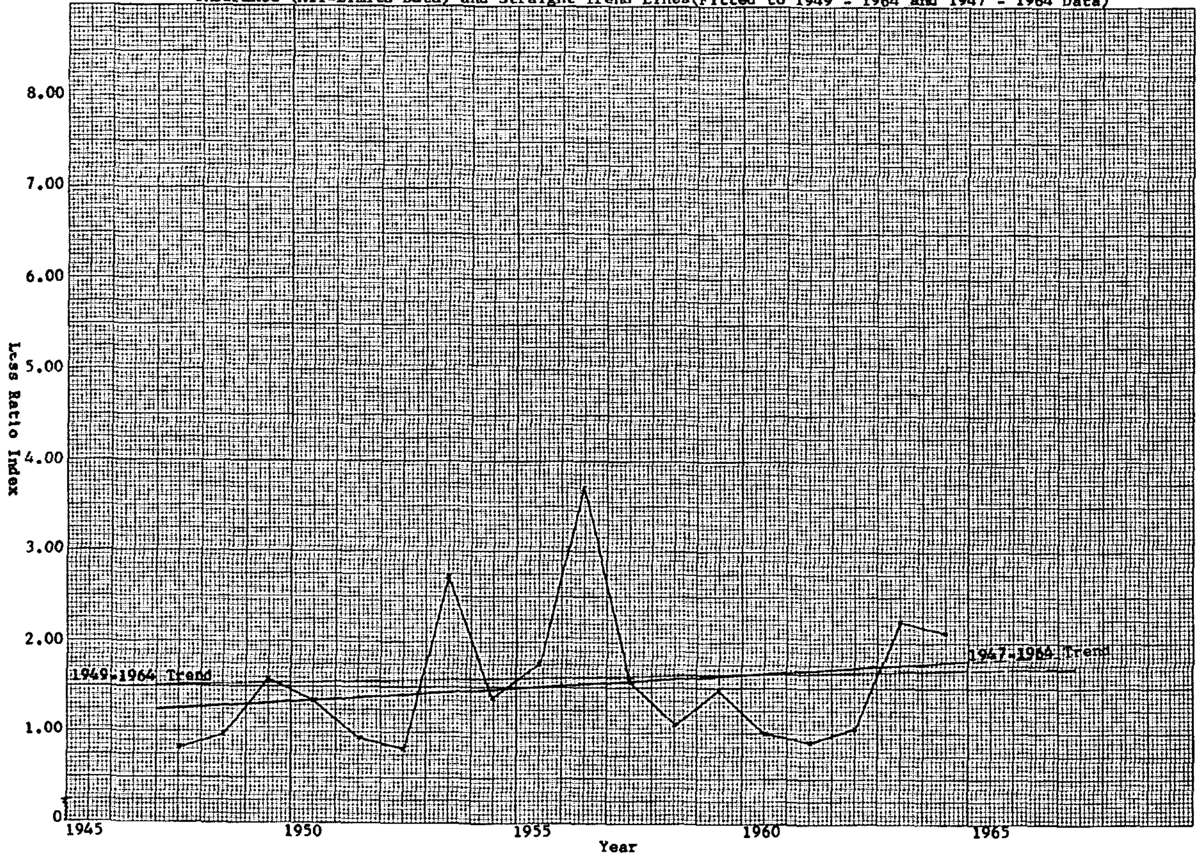


Figure 22.- Indexes of Yearly Loss Ratios (Base Year = 1960) for Michigan Dwelling Deductible Extended Coverage Insurance (All-Limits Data) and Straight Trend Line (Fitted to 1958 - 1964 Data)

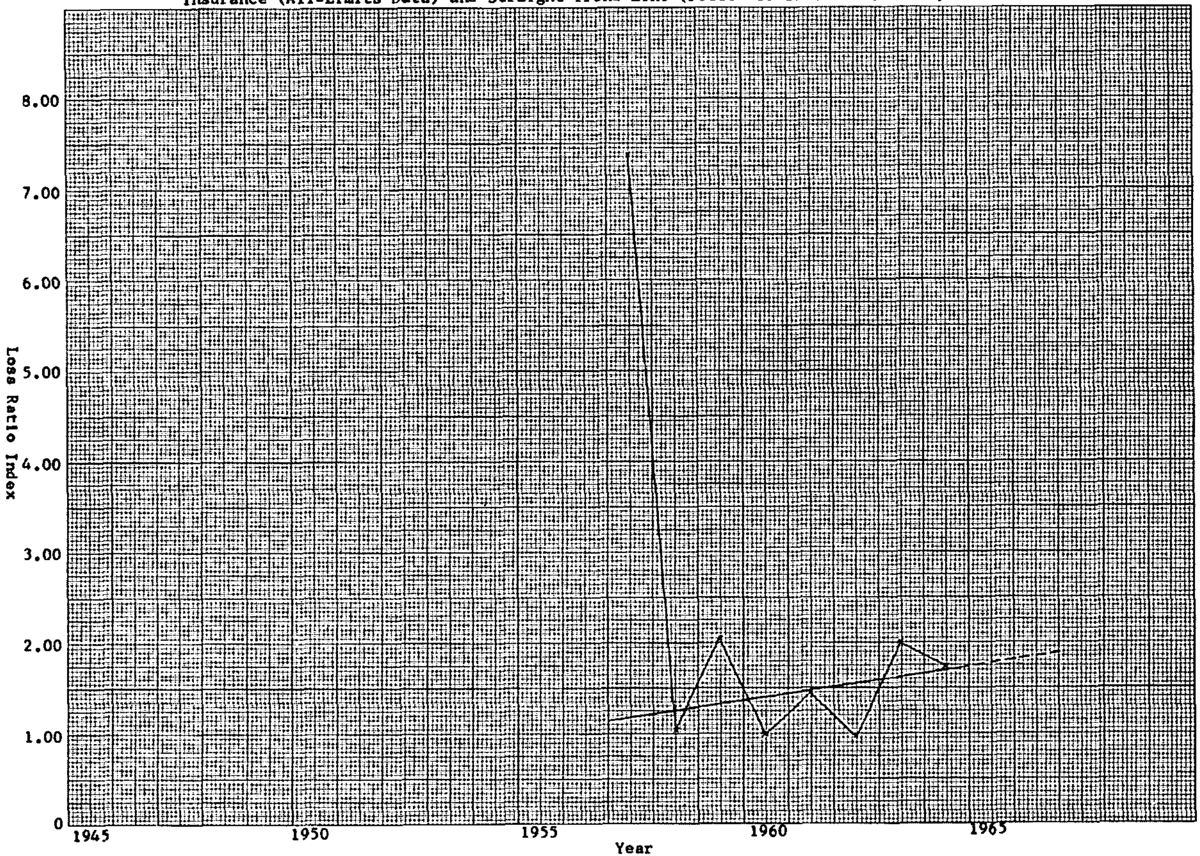
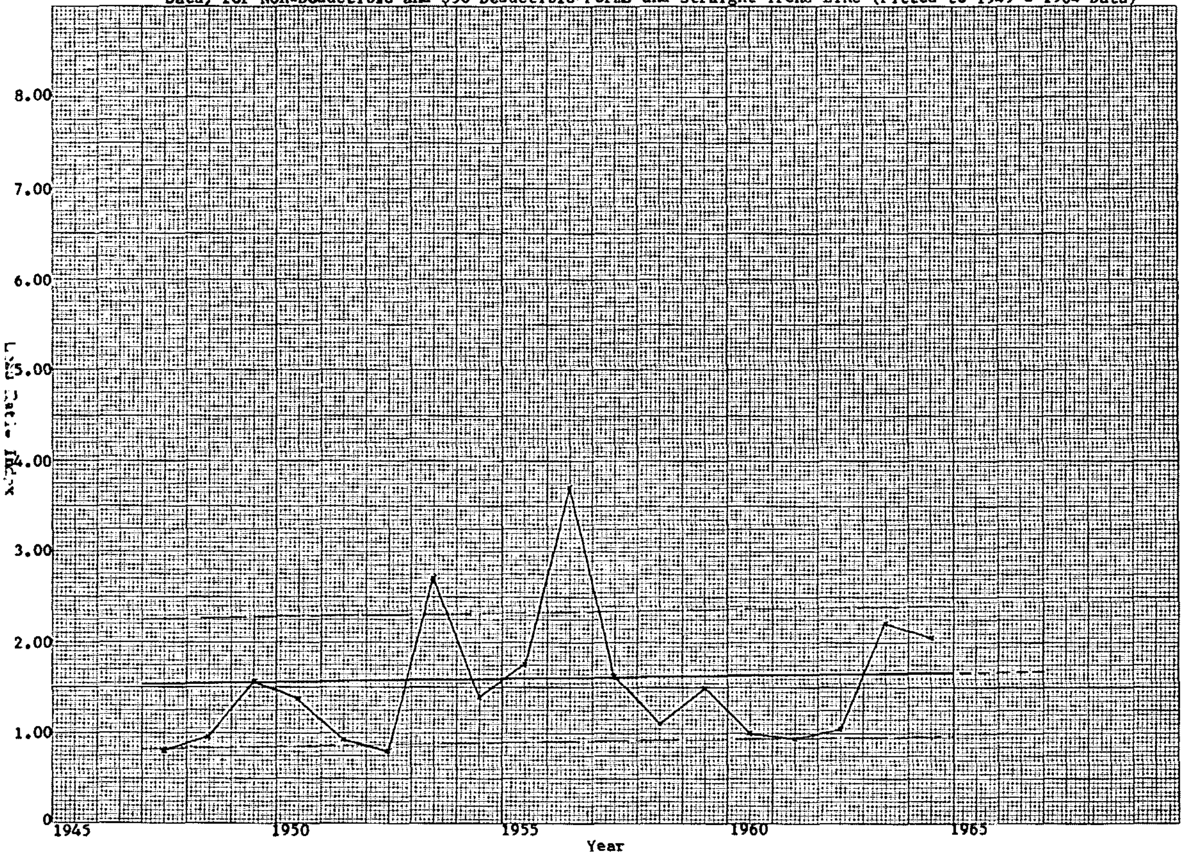


Figure 23.- Combined Index of Yearly Loss Ratios (Base Year = 1960) for Michigan Dwelling Extended Coverage (All-Limits Data) for Non-Deductible and $50 Deductible Forms and Straight Trend Line (Fitted to 1949 - 1964 Data)



Although the slope of the 1947-1964 trend in Figure 21 is not steep, it is materially influenced by the combination of the large cyclical amplitude plus the fact that the series starts near a cyclical trough and ends near a cyclical peak.[31] The first two years' data were accordingly dropped, so that start and finish would be peaks. The resulting 1949-1964 trend line in Figure 21 can clearly be seen to have a smaller slope. The adjustment reduced the slope of the trend line by a factor greater than three (the respective yearly values are .030 and .009). Since the 1949-1964 trend runs between peaks, it is biased slightly upward along its whole length. The fact that the two peaks are low ones fortunately results in a degree of bias here that can safely be ignored.

The freakishly high initial loss ratio (from sparse data) for deductible coverage would alone cause a negative slope, so it also was eliminated from the calculations that were used to produce the trend line shown in Figure 22. The separate calculations for deductible data, in view of the modest period they cover, can be considered to be mainly of academic interest.

It was not necessary to eliminate this datum in calculating the combined index in Figure 23, since the premium volume weightings solved the problem in a very neat manner. The combination of more data in the index reduced the variance below that for the non-deductible data. The calculations reflected in Table 3 are based on the usual averaging of extended coverage experience over periods of about ten years. The methods described here would permit elimination of such averaging, as well as of the arbitrary weightings that are often used with multi-year averages of data. It would be preferable to use for quantity weights in the index computations the number of $1,000 of insurance exposed per year. Second to this would be the number of risk years, which would measure only partially the size of the exposure. Lacking these, the premium volumes were used, as the best available measures of exposure. They did work satisfactorily.

These extended coverage data illustrate quite well the high desirability of having in the ratemaking process an orderly plan to separate the catastrophic losses on an objective basis (preferably stated as a ratio to volume of exposure rather than as an absolute) and to average such losses over one or more periods (from 10 to 50 years) determined by the patterns of major cycles. In this way the remaining fluctuations will be small enough to provide a reasonably stable rating base without artificial weighting. The

[31] See Croxton and Cowden, *op. cit.*, p. 408.

standard errors of estimate about the trend lines (19.4 points of loss ratio for non-deductible coverage, 12.7 points for deductible coverage and 17.8 points for the combined index) eloquently illustrate this need numerically.[32]

Application of time-series analysis and adjustments of any type to these data without some long-term averaging might be expected to result in indicated yearly swings in rates of such size and of such frequent changes in direction that most ratemakers might well consider them impractical to apply in practice. The application of the described methods to these data, which were actually used in connection with a rate filing, shows however that the methods can overcome all the difficulties usually encountered with data and produce adjustments that are both responsive and stable to a desired and measurable degree.

SUMMARY

Application of long accepted techniques of time-series analysis, as shown by actual examples from the major sectors of liability and property insurance, can be of material help in overcoming some of the increasingly difficult problems faced by ratemakers. Techniques developed by economic statisticians can produce actuarially acceptable precision in many cases where other methods fall short. This is exemplified by substitution of the measurable accuracy of the statistical control chart for arbitrary and unmeasured weighting, and by the use of factually weighted index numbers in place of using only a homogeneous fraction of the whole available data. These techniques can be used with both pure premium and loss ratio methods of ratemaking. The quality of results obtained with these as well as other methods depends largely on the quality of the data collected, and therefore on the design of the data collecting plans. Specific suggestions for improving this quality have been made in several places. The methods are equally applicable to all lines of business. Contrasting the characteristics of the various lines and the risks to which they pertain can be helpful in avoiding a proliferation of approaches in individual lines that can later cause difficulties when those lines are combined in packages. Some of these contrasts have been presented in this paper as a help toward such uniformity.

The cyclical adjustments detailed in Table 3 range from −1.2 to +2.6 per cent of the total time-series adjustment for liability insurance, from 1.1

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[32] Proposals in this direction by Fire Insurance Research and Actuarial Association are in process of being implemented.

to 2.0 per cent for automobile physical damage insurance, and are 13.4 per cent for extended coverage. From Figures 1, 5 through 8, 11, 12, 14, 17, and 19 through 23, it can be seen that many cyclical changes are even greater than these, frequently dwarfing the trend change during the same year. Particularly for extended coverage, they form in our examples a material portion of the total adjustment. But even where they are small their importance is large.

Although they usually (two-thirds of the time with the guide lines spaced as in our example) lag the actual peaks and troughs by one year, they absolutely prevent the dramatic over-reactions given by the short-term "trend" lines in common use. By examining Figures 9 and 10 it can also be seen that the areas between the upper guide lines and peaks that extend above them match very well the comparable areas between the lower guide lines and troughs that extend below them. This simply indicates that undercharges and overcharges balance out quite well under the time-series adjustment system here described. Since the guide lines keep adjustments reasonably close to the trend line, the system tends to result in a rate level that is free of subjective bias.

The system also eliminates the need for all or most of the arbitrary weighting commonly used. The cyclical adjustment procedure, once set, automatically limits the effects of large fluctuations in data. In every case only the last datum need be used. There is no need to average two or more recent years' data, perhaps with arbitrary weights to boot, or to inject arbitrary judgment into individual rate decisions. In short, all available data are used, the very latest datum is the starting point for applying adjustments, major fluctuations are dampened without destroying responsiveness to recent indications, and opportunity to inject arbitrary judgment is minimized.

Areas for Further Inquiry. The results exposed in this analysis suggest the following potentially rewarding areas for further inquiry:

a. Can the theory of runs be used to develop a useful test for the existence and characteristics of trends and cycles in insurance time series?

b. Can time-series analysis be fruitfully considered as a tool for increasing and measuring credibility? For example:

(1) Does a trend line not make it possible to use a many-year series of data as the "prior distribution," gaining greater credibility as the series lengthens, and extracting the maximum indicative information from the data in an orderly manner?

(2) Does not competent time-series analysis permit in most cases supplementing the limited geographic or numerical spread of sparse data by a spread over time with a measurable credibility or margin of error?

c. Can the multi-split concept originated in workmen's compensation ratemaking and suggested here for multiple peril ratemaking be profitably applied to credibility theory?

(1) Since there are sometimes two or more pertinent "prior distributions" or sets of data available, why cannot credibility theory logically and usefully contemplate not only the traditional two-way split but also be broadened to embrace a three-way or greater split?

(2) Can objective criteria be devised for selecting and weighting or otherwise relating the sets of prior data or prior distributions among themselves, and also with the current statistical data, in forming a "posterior distribution"?

d. Can more specific criteria be developed for selecting the distance (number of standard errors) of guide lines from the trend line?

e. Can a concise summary for curve fitters of all possible second degree, third degree, fourth degree, and possibly higher degree families of curves, including their characteristics and handy criteria for selection and fit such as Karl Pearson's Beta-1 and Beta-2, be developed in a form analogous to W. Palin Elderton's *Frequency-Curves and Correlation* (2d ed., London: C. & E. Layton, 1927)?

f. Can an objective significance test be designed that will permit a decision, based on a given degree of credibility or else on a predetermined confidence interval, on whether two sets of data should or should not be used together for ratemaking purposes? Such a test could apply to deciding whether data from adjacent states or provinces are sufficiently similar to permit combining them to increase credibility, to deciding whether the results from two different areas are sufficiently dissimilar to warrant making them separate rating territories, and to deciding whether two groups of data from different time periods are sufficiently similar to combine for ratemaking purposes.

g. Can the short-term (4 or 5 years) trended averaging now widely used for "trends" in insurance be useful in rating spread loss reinsurance, where a means of keeping up with adverse reserve developments and worsening claim severity is needed, but where no prediction of future levels is required?

APPENDIX

CALCULATION OF AUTOMOBILE PHYSICAL DAMAGE PURE PREMIUM INDEXES

The long standing problem of how to combine or average data for the same coverage but for different deductible amounts can be handily solved by use of the economic statistician's index number. This device does not permit combining apples and oranges, but it does permit combining in meaningful form the *prices* and *quantities* of apples and oranges. It permits the ratemaker to combine in meaningful fashion the prices of \$50 deductible apples and \$100 deductible oranges, deriving therefrom a greater spread or stability of experience.

The basic logic of all price indexes is to combine in a suitably ordered manner the prices and quantities of disparate items. For insurance ratemaking, the prices may be in gross (rate or other unit premium) or net (pure premium) form. The quantities are exposure units. The terms used in the following calculations may be defined as follows:

p_n = price or pure premium for year n
p_o = price or pure premium for base year
q_n = quantity or number of car years for year n
q_o = quantity or number of car years for base year
$p_n q_n$ = total losses for year n

The formula for the Fisher Ideal Price Index is

$$I_p = \sqrt{\frac{\Sigma p_n q_o}{\Sigma p_o q_o} \times \frac{\Sigma p_n q_n}{\Sigma p_o q_n}}$$

The following calculations for the fiscal 1948 non-collision and collision coverage pure premium indexes should be self explanatory.

Year to 30 September 1948	p_n = Current Pure Premium	q_n = Current Number of Car Years	p_nq_n = Current Losses	p_nq_o ($o=1960$)	p_oq_n ($o=1960$)
Non-Collision Coverages					
Full coverage comp	$ 5.41	$ 83,334	$ 450,700	$1,068,464	$ 673,339
$50 deduct comp	0	0	0	0	0
Fire and theft	4.80	8,297	39,824	19,512	24,476
Towing & road svc	.10	9,878	1,036	7,634	2,272
Fire, theft, CAC	5.96	5,537	32,987	247,745	25,858
Fire, theft, wind	4.29	3,665	15,735	5,577	9,749
	$ 4.88	$110,711	$ 540,282	$1,348,932	$ 735,694

$$I_{non\text{-}coll} = \sqrt{\frac{1,348,932}{1,825,286} \times \frac{540,282}{735,694}} = .7367$$

Collision Coverages					
$ 25 ded collision	$53.10	$ 731	$ 38,819	$ 4,513	$ 15,307
50 ded collision	32.73	43,549	1,425,456	1,635,682	1,373,971
100 ded collision	28.36	12,523	355,194	3,293,362	316,456
250 ded collision	36.16	642	23,212	134,190	54,088
	$32.08	$ 57,445	$1,842,681	$5,067,747	$1,759,822

$$I_{coll} = \sqrt{\frac{5,067,747}{4,825,322} \times \frac{1,842,681}{1,759,822}} = 1.0487$$