DISCUSSIONS OF PAPERS PUBLISHED IN VOLUME LIV

THE MINIMUM ABSOLUTE DEVIATION TREND LINE

CHARLES F. COOK

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DISCUSSION BY DAVID P. FLYNN

Most ratemaking procedures may be described as simply the processes through which loss experience at least one year old is projected to estimate the loss experience of the same risks one or two years in the future. This time lag is the inevitable result of the constraint that the rates be reviewed on the basis of the total loss experience of the line. Even with high speed computers, it is physically impossible to record, gather, sort, and caress the enormous amounts of data involved in any reasonable length of time. It follows from this built-in lag that recognition must be given to the possible differences in claim cost levels between the two periods if we are to achieve adequate rates.

Many years ago it was justifiable to assume that the cost levels of the experience period under review would continue with little change throughout the period for which the new rates would be in effect. However, during the period following the Second World War, it became increasingly obvious that the combined inflationary effects of continuing federal deficits, the expanding money supply, and increased labor costs would make it necessary to include a factor in the formulas that would compensate for the marked increases in claim costs. It was for this purpose that the least squares trending procedure was introduced. The criterion underlying the least squares line is that this is the line for which the sum of the squares of the differences between it and the points to be fitted is as small as possible.

Up until the present time little has been done to modify the conditions which cause inflation. It is now widely held by government economists that a rate of inflation of 2% to 3% per year is necessary and desirable in an expanding economy. It is evident that as long as this attitude continues, some type of trend factor will be inevitable. In fact, we should not be too surprised to see the size of the current trend factors increase. In 1966, the last full year for which statistics are available, the cost of the average automobile bodily injury claim increased 6% compared with a previous yearly increase of about 3%. Automobile property damage claim costs are now increasing at a rate of nearly 10% per year. If the current three-year lag between the experience period and the time at which the losses are paid continues, we will need minimum trend factors of over 15% for bodily injury and over 25% for property damage.

Mr. Cook's paper has now given us an alternative method to compute trend lines that is based upon the criterion that the sum of the absolute values of the differences should be a minimum, rather than the sum of the squares of the differences. The author states that the present least squares procedure has two major drawbacks. The first, that of the excessive influence of an odd point, arises out of the basic least squares criterion. For example, a point that is four units from the line would be given a weight of sixteen, and to offset the effects of this single point would take sixteen more points one unit from the line. This is a general criticism in that it applies to any curve that is fitted using the least squares criterion.

The second criticism is that excessive weight is given to the extreme points. This unequal weighting arises out of the formula for the slope of the line which is given by $\sum x_i y_i / \sum x_i^2$. To appreciate this objection it is necessary to recall that the points have been centered about the origin and when the multiplication in the numerator is performed, the outer points count more toward the sum than the inner points. This criticism applies only to the least squares method of fitting a line and it should not be applied to the general least squares curve-fitting procedure.

The criticisms that Mr. Cook outlines are valid. The first may be met only by throwing out the odd point. The second is highly theoretical in that the author has ignored the influence of the other element in the product and it is impossible to say beforehand how the slope will change unless we know the value of these elements.

It may be interesting to note that when the least squares trend line is used to fit points that represent year-ending averages, a secondary weighting procedure is involved that to a certain extent offsets the second criticism. For instance, the automobile trend line consists of a time series of twelve paid claim cost amounts over a three-year period where each point represents the cost for a year ending in a calendar quarter. If we itemize the number of times each quarter is counted, the results are that the first, second, and third quarters of the time series are counted one, two, and three times respectively. The succeeding quarters up to the last three are each counted four times while the remaining three quarters receive counts of three, two, and one. Thus the earliest and latest quarters receive smaller counts than those in the middle and tend to offset any reverse weighting of the formula.

The minimum absolute deviation method of fitting a line will eliminate the deficiencies of the least squares method and, in addition, is many times easier to use. However, the absolute deviation procedure itself has a very serious drawback that was recognized by the author in his paper. This deficiency is not always present but only comes into effect when $Z_{k^*} = MX$. In this instance the slope of the fitted line is not unique and any slope within a given range will satisfy the basic criterion of minimizing the sum of the absolute values of the differences. Mr. Cook suggests that in this case we use the average value in the range. While this suggestion is reasonable, the deficiency in the method still remains in that we are forced to enter a judgment factor into what ideally should be a completely objective method. It should be noted that the condition $Z_{k^*} = MX$ is not highly unlikely since it occurs in fitting the latest countrywide automobile trend line both for bodily injury and for property damage.

The author describes the method that he has developed as an "algorithm of the operations analysis type" which perhaps could be stated as a method based upon a constructive proof. However, no matter what you call it, it is not an easy proof to read. In an effort to be concise, the author has left many gaps in the proof for the reader to fill in for himself, making it difficult for the casual reader to follow. Those with the spare time will find the exercise rewarding.

Mr. Cook has again demonstrated his unique talent for mathematics and we hope that he will come forward soon with more work in this area.

DISCUSSION BY KENNETH L. McINTOSH

This paper most certainly demonstrates, should such demonstration be necessary, that "an algorithm of the operations analysis type" need not involve complex and interminable arithmetical detail.* A word of arithmetical caution may be in order, however. Since $a_i = (y_i - \bar{y})/x_i;\ x_i \neq 0;$

* The distinction between traditional "mathematics" and "Operations Analysis" may be a matter more of semantics than of substance. *Cf., e.g.:* Newton's algorithm to obtain the roots of polynomials; also the Gauss, Gauss-Jordan, and Crout algorithms for solving simultaneous linear equations. Linear Programming is directly related to Combinatorial Analysis, and Dynamic Programming seems to have an impact upon the theory of the Calculus of Variation. Where is the line to be drawn?