AUTHOR'S REVIEW OF DISCUSSIONS

Messrs. Nelson, Cook, and Graves have each discussed my paper from a different point of view, and I will briefly review their discussions separately.

Mr. Nelson does not disagree with the conclusions concerning ratemaking in my paper, nor does he quarrel with the idea that sampling theory, and in particular ratio estimation and stratification, have implications for ratemaking. He does, however, feel that the decomposition of aggregate experience by coverage and layer is *not* stratification, as I had contended. Rather, he contends that this decomposition is desirable because it is an example of "componentwise ratio estimation," a term used by Professor Robson in a 1961 paper in the *Journal of the American Statistical Association*.[1]

I cannot fully agree with Mr. Nelson for two reasons. First, I feel that for package policies both premiums and losses may be decomposed by coverage into "mutually exclusive subpopulations," which according to his definition is stratification. Second, I do not feel that Professor Robson's "componentwise ratio estimation" is really any different from a combination of ratio estimation and stratification. I note that in his paper, Robson occasionally uses the terms "stratified" and "componentwise" interchangeably, and that Robson's example of componentwise estimation is cluster sampling with post-stratification.

Mr. Cook, like Mr. Nelson, does not in his review question the conclusions or general approach of my paper, but does have some doubts about certain details and does feel that some additional material is necessary. At the beginning of his review, Mr. Cook states that the "subdivision of experience by coverage and layer of loss is not stratification." Judging from subsequent sections of his review, he feels that subdivision by layer of loss is not stratification, while sub-division by coverage is stratification. He relies upon Cochran's[2] definition of stratification which requires that the population be subdivided into non-overlapping subpopulations whose sum is the total population. Subdivision by layer of loss and by coverage meet this criterion.

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[1] Robson, D. S. and Vithoyasai, C., "Unbiased Componentwise Ratio Estimation," *Journal of the American Statistical Association (JASA)*, Vol. LVI, p. 350.

[2] Cochran, W., *Sampling Techniques* (Second Edition), John Wiley & Sons, Inc., New York, 1963, p. 87.

Cochran adds two other desirable criteria for sampling from these strata. First, to obtain full benefit, strata sizes should be known. As will be shown, this criterion is desirable, but not necessary. Second, the sample should be drawn independently within each stratum. This condition can be fulfilled in the case of stratification by coverage, but not necessarily in the case of stratification by layer of loss. As Mr. Cook and others[3] have noted, this latter form of subdivision of experience is actuarially desirable; however, to show that its efficiency derives mathematically from stratification would require some argument beyond that given in my paper. Perhaps one might consider subdivision by layer of loss to be a form of post stratification and then attempt to show that the dependence among strata in drawing the sample is not harmful. I have not pursued that line of reasoning further because subdivision of experience by layer of loss is well established actuarially while the subdivision of package policy data by coverage is the more controversial point.

It would appear from the above definition that sub-division by coverage is a form of stratification. Mr. Cook's question is really whether, from a mathematical view, it reduces the variance. His concern is best summarized by his statement: "Unless there is advance outside knowledge of the weights Wh, stratification accomplishes exactly nothing." Mr. Cook reaches this conclusion in his discussion of the estimation of the population mean and later applies it as a criticism of subdivision by layer of loss and by coverage. He arrives at this conclusion after showing that if Wh were estimated from the sample (i.e. $Wh = nh/n$) then the sample mean under stratification is equal to the sample mean under simple random sampling. Mr. Cook neglects the fact that the reduction in variance due to stratification arises from the greater homogeneity of each stratum (as compared to the total, unstratified population). Since the procedures are unbiased, we expect identical means. However, the variance under stratification will be less, and the precision greater, than under simple random sampling provided the strata are more homogeneous than the total population. As both Mr. Cook and I have noted when the population (*not* necessarily the sample) is large, the reduction in variance due to the use of stratification is a function of the sum of squared differences of each of the strata means and the grand mean. The fact that the weights, Wh, are estimated from the sample will not alter the fact that the reduction in variance is a positive quantity greater than zero. Our state of knowledge with regard

[3] Salzmann, R., "Rating by Layer of Insurance," *PCAS* Vol. L, p. 15.

to *Wh* in ratemaking is probably no more deficient than in other experimental work, where knowledge of "true stratum sizes" is lacking, and estimates must be made from data for prior years, pre-samples, or the like. W. D. Evans, in his paper "On Stratification and Optimum Allocations" in the *Journal of the American Statistical Association,*[4] stated: "Since on the average even random stratification will not reduce precision, stratification may be employed without hesitation whenever there is even slight justification for supposing that the variable under study is related to proposed mode of stratification." I feel that Mr. Cook would agree that there is more than slight justification for supposing that the variables we study in ratemaking are related to coverage.

It would appear that subdivision by coverage not only satisfies the criteria for stratification but that one may expect some reduction in variance. With regard to stratification, Mr. Cook questions whether my statement of caution concerning the improvement in precision is true in general or only for small populations. Mr. Cook claims stratification can result in a decrease in precision, independent of sample size; however, Neyman[5] has proved the converse for large samples, thus contradicting Cook.

Mr. Cook also feels that the ratio of package to non-package pure premiums is not a ratio estimate in the traditional sense (since the primary and auxiliary variables are not measured on the same units of the population) unless the unit is defined to be a class of risks. This reduces the sample size and artificially increases the correlation. He notes that I gave only a large variance formula for the ratio estimate variance; a discussion of the error involved in using this approximate variance has been given by Sukhatme.[6] He further notes that my footnote 18 refers to only one, rather poor published example; perhaps footnote 18 should have read *"Ibid.,* p. 171, 175 and 179," thus including several better examples from Cochran. In addition to preparing the published examples, prior to presenting the paper I undertook several experiments comparing a combination of ratio-estimation and stratification versus simple random sampling in which I used small samples (about 70 units) where the sample unit was in fact a group (a class or territory) and obtained results like those in Cochran. While Mr. Cook's questions were valid, it would appear that the ratio of

[4] Evans, W., "On Stratification and Optimum Allocation," *JASA* Vol. XLVI, p. 95.

[5] Neyman, J., "On the Two Different Aspects of the Representative Method: the Method of Stratified Samplings and the Method of Purposive Selection," *Journal of the Royal Statistical Society* Vol. XCVII, p. 558.

[6] Sukhatme, P. V., "Contribution to the Theory of the Representative Method," *Journal of the Royal Statistical Society* — Supplement — Vol. II, pp. 253-8.

package to non-package experience may be considered a form of ratio estimation.

Addressing himself to the more practical aspects of actuarial science, Dr. Graves gives us an example, from Virginia, of present day automobile package policy ratemaking techniques (simply to ignore the package policy data) and notes that my method might be an improvement. It would seem that the action of the Mutual Bureau Actuarial Committee to use the components of the package policy in ratemaking is almost equivalent to stratification and Dr. Graves feels that some form of ratio estimation might be helpful. It is interesting to note that Mr. Nelson found the ratemaking process implied by my paper persuasive, while Mr. Cook felt we could profitably use it next month. Apparently, Messrs. Cook, Graves, and Nelson agree with my general premise that the estimates (of pure premiums) for package policies would be more precise if the package statistics were decomposed by coverage and if the ratio of package to non-package (or total) experience were used in making the estimates. Mr. Cook and Mr. Nelson find in sampling theory (and in particular stratification and ratio estimation) some justification for my conclusion, although they both find it necessary to redefine some terms and ask for some further elaboration at a few points. Perhaps by limiting the effort (for discussion purposes) to a decomposition by coverage and by properly defining sample units, Messrs. Cook, Nelson, and I might be able to view the suggested ratemaking technique as an example of componentwise (or stratified) ratio estimation. However, my reviewers have made it clear that I have drawn implications from sampling theory and have not proved corollaries from theorems in sampling theory. It is interesting to note that one might have reached the same conclusions concerning package ratemaking by drawing implications from Monte Carlo techniques. In particular, the Monte Carlo method for the numerical evaluation of a multi-dimensional integral may be improved (in the sense of increased precision and efficiency) by arbitrarily breaking up (stratifying) the ranges of integration and by using control variates and regression (ratio) methods.[7] Such an approach might have avoided some of the difficulties raised by Messrs. Cook and Nelson, but, on the other hand, would have necessitated a longer paper since Monte Carlo methods are probably less familiar to most actuaries than sampling. The criticism of my paper for a lack of mathematical rigor may be analogous to the criticism of modern painting which often lacks a clear resemblance to

[7] Hammersley, J. and Handscomb, D., *Monte Carlo Methods*, Methuen & Co. Ltd., London 1964, pp. 50-76.

nature. Paul Klee defended his abstract works by comparing them to the fantastic images one sees through microscope and then asked the question:

> Does then the artist concern himself with microscopy? History? Paleontology?
>
> Only for purposes of comparison, only in the exercise of his mobility of mind. And not to provide a scientific check on the truth of nature.
>
> Only in the sense of freedom. In a sense of freedom, which does not lead to fixed phases of development, representing exactly what nature once was.