efficacy of the solution) less than precipitate rushes toward what will turn out to be a mirage.

Sometimes, too, there just ain't no way to get from here to there.

A FINAL WORD

Although I have indicated my disaffection for the example and for the method as it is presently constituted, it still seems to me that the basic approach, as a way of thinking, has a certain appeal. It may indeed be a foundation for an approach which will work. Professor Ferrari is to be commended for presenting his idea despite, I am sure, his knowledge that his example was subject to much criticism. If we focus upon that central concept, we will have extracted the kernel which I feel sure the author has wished to impart.

DISCUSSION BY ROBERT A. RENNIE*

Professor Ferrari's paper sets forth an interesting application of the Markowitz investment model to the problems of portfolio diversification among a number of lines of property-liability insurance. Apart from certain theoretical difficulties noted below, the paper makes several practical contributions. It helps to eliminate the confusion in property-liability insurance over the concepts of risk and return. The expected return of a line is defined in terms of the future profitability of that line. Risk, on the other hand, is a function of the variability around the expected return. Certainly, insurers have tended in the past to concentrate more on precise measures of return than on exact measures of risk.

The paper also shows, at least by inference, how significant the optimal diversification of lines of insurance can be to operating results and to the risk borne by a property-liability insurer. Too often in the past, management has permitted its relative product mix to follow the course of least resistance as dictated by its marketing demands.

At the theoretical level, Professor Ferrari faced a dilemma. His analysis assumed that historical risk-return trends would continue in the near future. The data in his example were based on a linear extrapolation of the recent combined loss and expense ratios of a large company.

The justification for using combined loss and expense ratios and variances over some past period is, of course, that past performance is believed to be

* Mr. Rennie, who is Vice President — Planning, Finance, and Systems of the Nationwide Insurance Company, was a guest reviewer of Professor Ferrari's paper.

indicative of the future. Such a hypothesis may be true of common stocks, but it is certainly questionable as applied to the property-liability insurance lines, particularly in the case of automobile insurance. Professor Ferrari is aware of these difficulties, and suggests that they may be alleviated by introducing expectations based on subjective judgment into the historical parameters.

I suspect, however, that the nature of these difficulties in property-liability insurance is almost fatal to any simple application of the Markowitz model in this area. The original model assumed that common stock returns and variances are independently distributed over time. In its application to insurance, even if modified historical data were used, there is a clear danger that the analysis will not take into account the tendency of the insurance rating mechanism to adjust over time to the past trends and fluctuations in pure premiums and expenses. Thus, if rates and return have been too low in the past, there is likely to be a more concerted effort to secure adequate rates in the future. There is evidence, for example, that automobile insurance has generated alternating cycles of underwriting gain and loss in the past.

Under these circumstances, the immediate past has little linear relevance to our problem. The insurer is primarily interested in the future return for the various lines of insurance. Professor Ferrari sensed this problem when he stated that the revised historical input would still be deficient "to the extent that future developments are unforeseen or that subjective adjustments do not accurately reflect expectations in a quantified form."

A second theoretical issue is raised by the assumption that the expected return and risk on each line of insurance are single valued, regardless of the proportion of the total portfolio committed to that line of insurance. The return on each line is assumed to be a statistical random variable with a symmetrical probability distribution, and the expected return is a statistical average of that distribution.

This assumption of a single-valued expected return may be valid for common stocks, but it must be questioned when applied to lines of insurance. An institutional investor can change the proportions of securities held in his portfolio at relatively uniform prices. Thus, the expected return for a particular stock will remain the same after the reallocation of his assets. However, an insurer cannot change the proportions of the total portfolio committed to specific lines of insurance and expect either the expected returns or the variances to remain the same after he has changed the relative proportions. In the case of auto insurance, for example, if an insurer consciously reduces the percentage of auto premiums in his portfolio, he will

undoubtedly seek to eliminate the marginal risks, thereby increasing his expected return and reducing his variance (risk) for that line.

Thus, if a portfolio selection model is to be developed for property-liability insurance, it must be more complex than the Markowitz model. The expected return for a line of insurance is not single-valued, but is a function of the proportion of the total portfolio committed to that line of insurance and the rate of growth of the total portfolio. Likewise, the variance of return of each line of insurance is not single-valued, but a function of the same variables.

A simple example will illustrate this point. Assume that an insurer has one-half of its portfolio in auto insurance, one quarter in homeowners, and one-quarter in commercial fire. All three lines have an expected return of 3 per cent. The insurer becomes concerned about the future risk in auto insurance, and decides to reduce his auto portfolio to 35 per cent. Homeowners is raised to 35 per cent and commercial fire to 30 per cent.

Under the Ferrari-Markowitz model, the expected return on the portfolio would remain unchanged because the expected return on each line of insurance is single-valued, and not related to the proportion of that line in the total portfolio. In fact, however, the expected returns on auto insurance would undoubtedly rise, and those on homeowners and commercial fire would probably fall if premiums were increased in those lines to maintain total premium writings at a constant level. If the expected return on auto rose to 3.5 per cent, and the return on homeowners and commercial fire both fell to 2.8 per cent, the expected return on the total portfolio would rise above 3 per cent. A similar example relating to the variance (risk) could be cited. Both indicate that the assumption of uniform parameter values for risk and return applying to all portfolio proportions oversimplifies the property-liability insurance model.

Finally, I would suggest that any further work on the Ferrari-Markowitz model might also attempt to incorporate the investment portfolio of an insurer within the model as a means of generalizing its application. Certainly, Mr. Ferrari has written an ingenious and interesting paper, and it merits further exploration and analysis by casualty actuaries.

DISCUSSION BY MATTHEW RODERMUND

Professor Ferrari's paper is scholarly, well-written, interesting, and, not least, courageous. The author is welcomed to the Society as an Associate at the November meeting, but his paper was presented to the Society in May by