A THEORETICAL PORTFOLIO SELECTION APPROACH FOR INSURING PROPERTY AND LIABILITY LINES

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The recent financial literature contains numerous applications of portfolio selection that generally attempt to develop optimal diversification strategies and (perhaps inappropriately)[1] to gauge investment performance. Most of these efforts, however, have been limited to common stock portfolios mainly because equity price movements provide a convenient input to investment models that measure risk by variability of return. The purpose of this paper is to provide a novel application of portfolio selection outside of the investment area. More specifically, it aims at providing an initial report on utilization of portfolio selection techniques to suggest the theoretical, optimal diversification of lines of insurance written by property and liability insurance companies.[2] These results are part of the author's attempts to establish operating criteria for commercial insurance operations.

DIVERSIFICATION AS A DESIRABLE OBJECTIVE

Diversification of investments is generally considered to be a desirable objective and is a widely observed aspect of investment behavior. With particular reference to insurance companies, the primary objective of a number of statutory, quantitative restrictions is to impose some degree of portfolio diversification among and within categories of investments.

Diversification is also closely related to the pooling or averaging aspect of insurance. One of the most obvious examples of "spreading the risk" is the geographical diversification of property coverages, such as fire and

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[1] Recent criticism of the use of portfolio selection concepts to evaluate investment performance has been expressed by Irwin Friend and Douglas Vickers, "Portfolio Selection and Investment Performance," *Journal of Finance*, XX, No. 3 (September, 1965), pp. 391-396.

[2] This paper will concentrate solely on the insurance portfolio. A logical extension of this effort would be to include investments, as well as insurance, in the model. Underwriting and investment results are obviously interrelated operating criteria and a more advanced model would take into account the insurance-investment interactions.

windstorm, to avoid an undue physical concentration of insureds. Nevertheless, the insurance literature gives little if any notice to the proper allocation of business among the lines of insurance now being offered by multiple-line companies. The theories and techniques of investment diversification are one possibility for explaining and/or prescribing property and liability insurance company operating behavior.

INVESTMENT THEORIES AND DIVERSIFICATION

Traditional economic theories contend that an investor facing alternative opportunities with certain returns or profits (riskless investments) will prefer the investment that offers the maximum return. When uncertainty is introduced, this reasoning is extended to the assumption that an investor will (or should) attempt to maximize the discounted value of expected, future returns.[3] This explanation of rational behavior under uncertainty is considered incomplete because it fails to recognize the aversion of risk which investors possess in varying degrees. Subsequently, the notion that an investor has an aversion to risk in addition to a preference for return was developed. However, the maximization of expected returns is generally preserved as an optimal criterion if anticipated returns included an allowance for risk, or if returns are capitalized at a rate that varies with the individual investment risk.[4] But this theory prescribes the placement of all available cash in the investment having the highest expected return. It offers no explanation for making more than one investment unless the cash available exceeds that single outlet with the maximum expected return. It is surprising that, in spite of the long recognized efforts of investors to diversify, not until 1952 was a theory of investor choice formally introduced that leads directly to diversification and admits it as a desirable goal.

Dr. Harry Markowitz, in a classic article,[5] rejects the maximization of expected returns criterion because it does not recognize investment diversification as a conscious or desirable objective. Markowitz formulates an investment model in which an investor's preference for expected return and aversion toward risk explain the desirability of a diversified investment

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[3] J. B. Williams, *The Theory of Investment Value* (Cambridge, Mass.: Harvard University Press, 1938), pp. 55-75.

[4] Harry Markowitz, "Portfolio Selection," *Journal of Finance*, VII, No. 1 (March, 1952), p. 77. Risk is represented by an appropriate reduction in anticipated returns or an increase in the capitalization (discount) rate. Either, or both, of these adjustments will lower the capitalized value which is the expected return.

[5] *Ibid.*, pp. 77-91.

portfolio. This model also appears to be applicable to the risk-return attributes of the insurance portfolio of a property and liability insurer.

RISK-RETURN CHARACTERISTICS OF LINES OF INSURANCE

Risk and Return Concepts. A property and liability insurance company will have its business diversified among a number of lines of insurance each of which has risk-return characteristics that can be subjected to portfolio selection analysis. Unfortunately, this fact has been largely overlooked primarily because of some confusion in the property-liability area over the concepts of risk and return. The traditional mistake has been to consider the consistently unprofitable lines of insurance as the riskiest business. But a more appropriate view of profitability is as a return concept, ex post, and an expected return concept, ex ante. In addition, a more acceptable and useful measure of risk for a line of insurance is the variability of operating results. Therefore, in the portfolio selection analysis which follows it is assumed that the expected return of a line of insurance is a function of profitability (as measured by loss and expense ratios) and risk is a function of the variability around the expected return.

Input Requirements for an Insurance Portfolio Model. The Markowitz technique has been applied almost exclusively to investment securities, particularly common stock, although, theoretically, any application is possible if expected values and measures of variations are determinable. A by-product of the emphasis on investments is that the available portfolio selection computer programs usually require the inputs for risk to take the form of price movements of securities and expected return is largely a function of capital gains and losses. Therefore, in order to apply portfolio selection techniques to lines of property and liability insurance, the configuration of available computer programs requires that the individual lines be viewed in much the same manner as a category of investment securities. In order to achieve this result, one approach is to use combined loss and expense ratios as determinants of expected return and variability of return.

Comparative Analysis of Lines of Insurance and Investment Securities. The notion of return on an investment security or a category of investments obviously differs from a concept of return on a line of insurance written by an insurer. With a security, the investor relinquishes certain assets with the hope that over time an amount in excess of the original investment will be returned. With insurance, an insurer collects a premium with the hope (at least theoretically) of disbursing less than this amount for expenses and losses.[6] In the insurance transactions the insurer does not relinquish control of any specific assets but it does legally commit a portion of its assets (or its underwriting capacity) in general as resources available for the payment of future possible claims, such claims being estimated by the insurer's reserve liabilities. Similar to investment, a sound insurance operation is based on the expectation that over time there will be a positive return on the initial asset commitment.

Another difference between investment securities and lines of insurance has to do with the potential gain or loss under each type of arrangement. The gain on an investment such as common stock may be theoretically unlimited while the potential loss is limited to the original amount invested. Alternatively, the loss on a line of insurance, particularly a line such as liability, is virtually unlimited up to the amount of the total assets of the company while the gain is limited to the amount of premium income.[7] This difference is not necessarily significant if one is interested in expected returns and the realistically possible, albeit sometimes large, variation from the expectation. Actually, the extreme gain and loss positions of the process of insuring resemble those of common stock investment when equities are sold short. In these transactions one's gain is limited but the potential loss is theoretically unlimited.

Other areas of comparison between lines of insurance and investment securities exist,[8] but these are either not germane to the basic assumptions in this paper or the differences or similarities are reflected in the risk-return measures employed.

Risk and Return Based on Combined Loss and Expense Ratios. The portfolio selection analysis in this paper assumes that the risk-return characteristics of lines of insurance are similar to investment securities and can be based on historical loss and expense ratios. More specifically, it is assumed that each line of insurance generates an annual return based on premium income, operating expenses, and insurance losses. For example, a combined annual loss and expense ratio of .95 or 95 per cent is assumed

[6] The insurer also can earn interest on funds being held for future disbursement.

[7] In a practical sense the gain will certainly be something less than premiums since expenses will be incurred in acquiring the business.

[8] For example, obvious analogies can be made between the cancellation privileges of both insurer and insured, and the marketability and callability of investment securities. Another is the relative size and variability of the expenses associated with the acquisition of insurance versus the acquisition of investments.

to be a return of 5 per cent on that line of business and, alternatively, a loss-expense ratio of 105 per cent is assumed to be a loss of 5 per cent.

In this paper the determination of risk and return on lines of insurance is made with the assumption that all historical trends and variation will continue in the near future. For this reason, the computer analysis to follow has to be considered illustrative and hypothetical; however, it is somewhat realistic since actual company data are used as a basis for the model input. Utilizing simple linear extrapolation of the most recent, historical, combined loss and expense ratios of one large company, the risk-return assumptions shown in Table I were derived and subsequently used in applying the Markowitz portfolio selection technique to property and liability insurance.

Markowitz also suggests subjective probability beliefs as alternatives to inputs based solely on a past record that may not be representative of the future.[9] He suggests a method for deriving probability beliefs by formulating expectations about the movements (for example, highest, expected, and lowest values) of some relevant index. The analyst would then state the loss-expense ratio (for example, highest, expected, and lowest ratio) for each line of insurance at each of the possible values of the index. When this procedure is carried out the individual lines are said to be "tied" to the index. With these tied estimates, correlations between individual lines of insurance can be determined indirectly from the relationships of the lines to the index. The index can be said to act as a kind of common denominator. Additional correlation factors can be introduced when two lines are more positively correlated or less positively correlated than would be indicated by their relationships with the index.[10]

Another method calls for probability beliefs for each security stated as direct estimates of return and its expected variation. With this method, correlations are determined independently for each pair of lines of insurance. This procedure, and any other input method that does not use estimates tied to an index, has the limitation of necessitating a large number of individual correlations that must be estimated or calculated.[11] Probability beliefs for individual lines of insurance are also subject to the difficulty of formulating statistically consistent estimates.

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[9] Markowitz, *Portfolio Selection, op. cit.*, pp. 26-33.

[10] *Ibid.*, p. 32.

[11] 1,225 correlations for fifty securities and 4,950 correlations for one hundred securities.

APPLICATION OF THE MARKOWITZ PORTFOLIO SELECTION THEORY TO INSURANCE PORTFOLIOS[12]

Expected Return and Risk. In order to apply the Markowitz portfolio selection theory to an insurance portfolio, it must be assumed that an insurer can formulate expectations based on the expected return and risk associated with each line of insurance. The return on each individual line is assumed to be a statistical random variable with a symmetrical probability distribution. The line of insurance can then be viewed as having an expected return which is a statistical average of the probability distribution. The expected return on a portfolio is then the weighted sum of the expected returns of lines of insurance in the portfolio; that is

$$E(R) = \sum_{i=1}^{N} a_i R_i$$

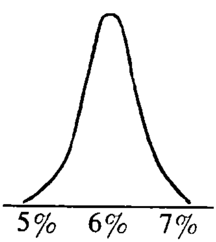
where: $E(R)$ = expected return on a portfolio.

a_i = the proportion of the total portfolio committed to the i^{th} line of insurance.

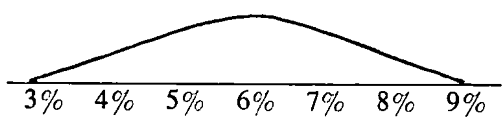
R_i = the expected return of the i^{th} line of insurance.

Therefore, the expected return on the portfolio also is considered a random variable.

The risk on each individual line is assumed to be the variance (V) or standard deviation[13] squared (σ^2) of the return described by the probability distribution for the line. For example, suppose the probability distributions of return on two hypothetical lines of insurance, A and B, can be pictured as follows:



LINE A



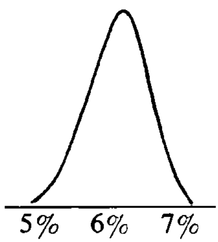
LINE B

[12] This section of this paper is purely theoretical since it ignores many practical constraints which will be taken up in subsequent sections.

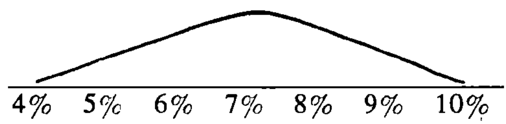
[13] It is helpful to think of the risk in terms of the standard deviation of return when one wishes to draw upon probability theory for a notion of risk. For example, if actual return is assumed to be normally distributed around expected return, then the probability of actual return being less than expected return by more than one, two, or three standard deviations is .16, .02, or .001 respectively.

Both lines have the same expected return of 6 per cent, but line A is considered the safer line because of the lesser variability of return. That is, there is more certainty that the actual return of A will be equal to the expected value. It is assumed that a rational insurer will always prefer A to B since he can expect the same return with less risk.

The situation becomes more complex if an insurer has to make a decision between lines of insurance A and C with probability distribution of return as follows:



LINE A



LINE C

Now the line with the greatest variability of return (risk) also has a higher expected return. The insurer must decide whether A or C is the more desirable line of insurance. With C there is an additional risk for a higher expected return, while with A there is less expected return with more certainty. A preference for one or the other will be a function of the objectives of the insurer and its attitude toward risk and return. That is, the final decision can only be explained by some concept of utility and the introduction of the relevant practical constraints.

The problem of portfolio selection is introduced when an insurer faces a number *(N)* of available lines of insurance that present numerous possible combinations of risk and return. It is assumed that for each individual line an insurer can formalize his beliefs about expected return and risk in the form of a probability distribution.[14] The expected return on an insurance portfolio consisting of any or all of the available lines has already been shown to be the weighted sum of their expected returns; namely

$$E(R) \sum_{i=l}^{N} a_i R_i$$

The risk of such a portfolio, however, is not simply the weighted sum of the individual variances, but it is a function of both the risk of each indi-

[14] The limitations imposed by this assumption are ignored for the time being.

vidual line and the correlation of returns between each pair of lines. This latter phenomenon is termed covariance, and the formula for covariance between two lines of insurance is

$$\sigma_{ij} = E\,[R_i - E(R_i)]\,[R_j - E(R_j)]$$

where: σ_{ij} = the covariance between the i^{th} and the j^{th} lines of insurance.
R_i = the actual return on the i^{th} line of insurance.

In this form covariance is the expected value of the deviation of the return on line i from its mean times the deviation of the return on line j from its mean. An alternative expression is the product of the standard deviation of the i^{th} line times the standard deviation of the j^{th} line times their correlation coefficient, as follows:

$$\sigma_{ij} = \rho_{ij}\,\sigma_i\,\sigma_j$$

where: ρ_{ij} = correlation coefficient for the returns on the i^{th} and j^{th} lines of insurance.
σ_i = standard deviation of return on the i^{th} line of insurance.

The variance of return or risk of a portfolio can be expressed as a weighted sum of the variances of all individual lines plus the weighed sum of the covariances for each pair of lines, as follows:

$$V(R) = \sum_{i=1}^{N} a_i^2\,V_i + 2\sum_{i=1}^{N}\sum_{j>i}^{N} a_i\,\sigma_{ij}\,a_j$$

$$V(R) = \sum_{i=1}^{N}\sum_{j=1}^{N} \sigma_{ij}\,a_i\,a_j$$

where $V(R)$ = variance of return on the entire portfolio.
a_i = proportion of the total portfolio invested in i^{th} line of insurance.
V_i = variance of return on the i^{th} line of insurance.

The correlation of returns between two lines (covariance) is a primary element in the Markowitz Portfolio Selection Theory. Insurer diversification can be viewed as a procedure for reducing aggregate risk by holding lines of insurance whose returns are not likely to vary in the same direction at the same time. The benefits of diversification are most fully realized by writing business with negative correlation, thus reducing the degree of risk for the insurance company.

The Efficient E-V Criterion. For a given set of available lines of insurance, various E-V (expected return on the portfolio and its variance) combinations are possible depending on the probability beliefs established for the individual returns, variances, and covariances. Purely for illustration, the set of all obtainable E-V combinations might be represented by the circle and the enclosed shaded area in Chart 1.

Markowitz has formulated a rule which states that in surveying all of the possible E-V combinations, one should select only the ones that are "efficient" portfolios. An efficient portfolio is a combination with the minimum variance (or standard deviation) for any given expected return and/or with the highest possible expected return for a given variance (or standard deviation). In Chart 1, the efficient portfolios are those described by the E-V combinations on the arc *ab*. All other combinations are inefficient. For example, it may be seen in Chart 1 that portfolio *c* is inefficient because a higher expected return at the same risk is possible at *d,* while the same expected return at less risk is possible at *e.* Portfolio *f* is inefficient because a higher expected return is available at *e* with the same level of risk. Portfolio *g* is inferior to *d,* because the latter offers the same expected return at less risk. Portfolio *h* is not an obtainable E-V combination given the available investments and their attributes.

It is important to note that no one efficient portfolio (in our hypothetical example those on arc *ab*) is better than any other based on return and risk considerations. Choosing between efficient E-V combinations always involves giving up some expected return for less risk, or alternately, taking more risk for an increase in expected return.

The final choice among efficient portfolios is further limited to those that meet the standards for being "acceptable" portfolios. An "acceptable" efficient portfolio is one that complies with all legal and operating policy constraints imposed on the aggregate commitment to lines of insurance. These constraints can be a minimum commitment to certain lines, a minimum overall rate of return, a maximum allowable proportion in certain lines, etc. Depending on the willingness and ability of an insurer to assume risk, it is necessary to select from among the acceptable, efficient E-V combinations the one that best meets the requirements and objectives of the insurer.

Computational techniques are available that can determine the acceptable, efficient E-V portfolios associated with a given set of expected returns, variances, covariances, and constraints. The procedure is called quadratic programming, which is essentially optimizing (maximizing expected return

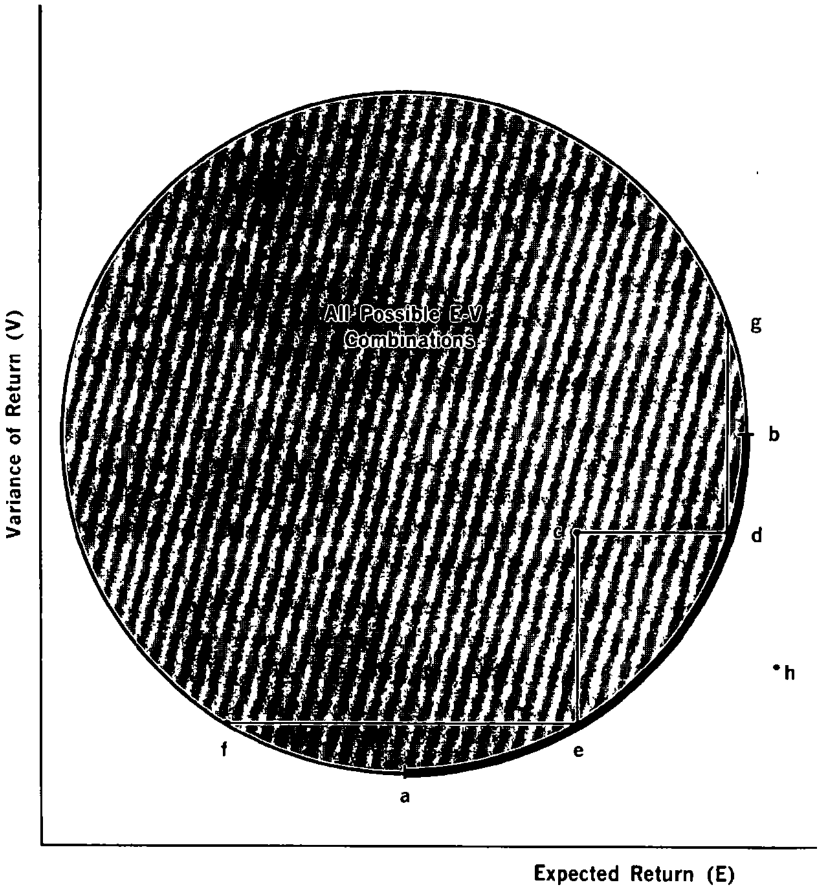


Chart 1. Graphic Representation of the Efficient (E-V) Portfolios in a Hypothetical Situation

given risk or minimizing risk given an expected return) a quadratic function subject to linear constraints. These computations are complex, but these complexities are primarily of mathematical significance. An understanding of mathematical programming procedures is not a prerequisite for using portfolio selection techniques, and the mathematical details of efficient portfolio construction are left to the original sources and will not be duplicated in this study.[15]

Constraints on Property-liability Insurance Company Portfolios. In every portfolio selection application, the combined influence of regulation, managerial policy and practical considerations places constraints on the freedom of action. Such is definitely the case in a property-liability insurance portfolio of an insurer and these constraints must be recognized before realistic results can be obtained from portfolio analysis. The crucial constraints are those aspects of the insurance business that limit the speed with which a company can move from one insurance portfolio to another. A high degree of inflexibility, at least in the short run, stems from an inability or a refusal to radically increase or decrease the percentage composition of a company's insurance business. The obstacles to such action result primarily from the maintenance of agency relationships, the insurance consumption patterns of insureds, and competition among insurers.

The computer program available to the author had the capacity for specifying the maximum percentage of a company's business that could be written in one line. Since the ability to change the insurance mix is a highly variable factor, three sets of maximum percentage constraints were selected so that, compared to the present insurance portfolio, the percentages would represent relatively "low," "average," and "high" degrees of flexibility. These assumed percentages are shown in Table 2 along with the current portfolio composition used as a point of departure.

The maximum percentage constraints are effective in limiting the increase in any one line of insurance to realistic proportions. Unfortunately,

[15] Markowitz describes the procedure called quadratic programming which computes the E-V efficient set of portfolios. See Harry Markowitz, "The Optimization of a Quadratic Function Subject to Linear Constraints," *Naval Research Logistics Quarterly, III* (March-June, 1956), pp. 111-133, and *Portfolio Selection* (New York: John Wiley and Sons, Inc., 1959) Chapter 8. For other related discussions see John Frederick Weston and William Beranek, "Programming Investment Portfolio Construction," *The Analysts Journal,* XI, No. 2 (May, 1955), pp. 51-55; A. D. Martin, Jr., "Mathematical Programming of Portfolio Selections," *Management Science,* I, No. 2 (January, 1954), pp. 152-165; William F. Sharpe, "A Simplified Model for Portfolio Analysis," *Management Science,* IX, No. 2 (January, 1963), pp. 277-293; and Philip Wolfe, "The Simplex Method for Quadratic Programming," *Econometrica,* Vol. 27 (July, 1959), pp. 382-398.

the available computer program did not allow minimum percentages to be specified. Consequently, some of the portfolios derived in the subsequent analysis suggest that certain lines of insurance should be dropped completely or reduced to very small proportions. In many cases this would be either impossible or highly undesirable. Regardless of this deficiency, minimum percentage constraints can still be recognized indirectly by considering as practical only those efficient portfolios that contain a realistic de-emphasis of certain lines of insurance. Using Markowitz's terminology, only the "acceptable" portfolios[16] of the entire "efficient" set can be considered relevant.

Efficient E-V Property-Liability Insurance Portfolios. Using the input items summarized in Tables 1 and 2, the risk and return characteristics of the efficient E-V property-liability insurance portfolios produced by the I.B.M. Portfolio Selection Program (1401-FI-04X) are shown graphically in Chart 2. Tables 3, 4 and 5 contain the same expected returns and variation of returns plus the percentage compositions for representative portfolios under each of the three sets of allocation constraints. In each case, for ease of direct comparison, the attributes and composition of the current portfolio are shown along with the efficient portfolios.

Chart 2 clearly shows that the insurance portfolio of the company whose data were used will be inefficient and non-optimal on the basis of the Markowitz E-V Criterion, if the present percentage composition is maintained. This criterion suggests, for example, that a shift from point P to a point P_I directly horizontal on the efficient portfolio curve under constraint set I would produce a portfolio with the same risk (standard deviation equal to 4.80 per cent) but improve the expected return from .68 per cent to something over 1.1 per cent. Table 3 indicates that this shift would be accomplished largely by relative decreases in extended coverage, ocean marine, and auto property damage liability insurance and relative increases in fire, auto bodily injury liability (surprisingly enough), and treaty reinsurance.

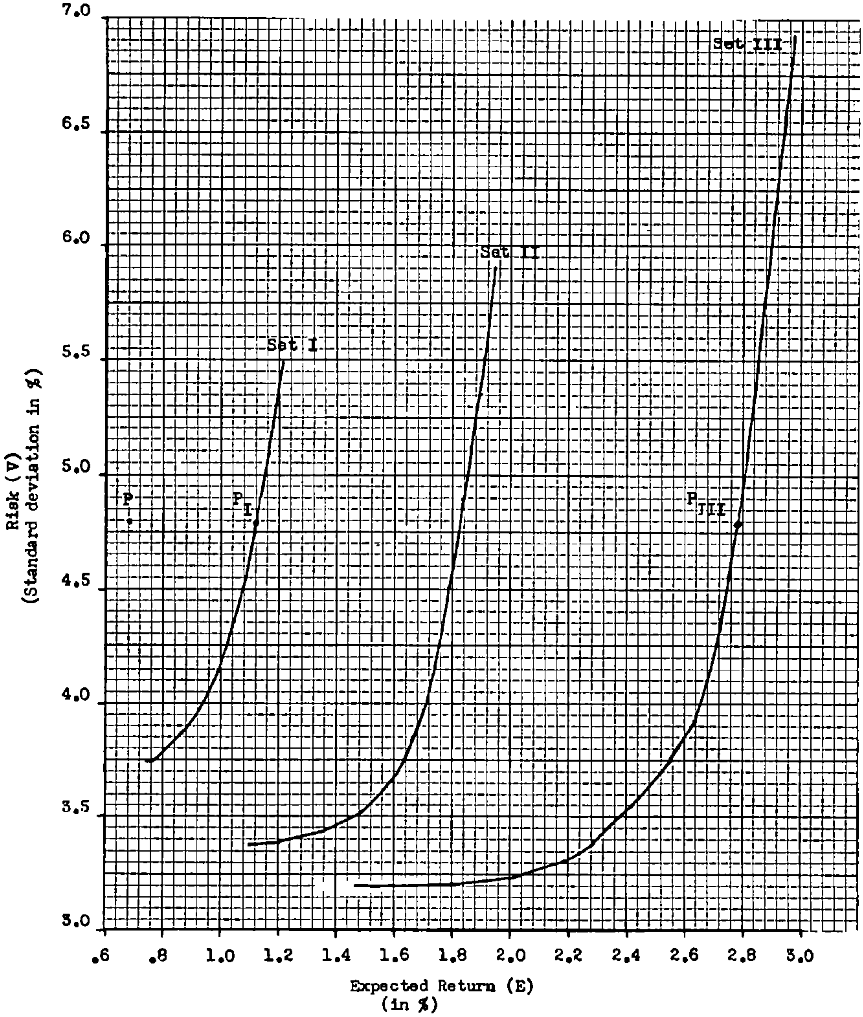
To illustrate the significance of the constraint assumption,[17] consider another horizontal shift of P to P_{III}, one of the portfolios described under the more liberal Constraint Set III. Now, under the assumptions, a portfolio

[16] That is, those portfolio that meet various legal, managerial policy, and other constraints that can't be incorporated automatically in the computer computations.

[17] A different kind of limitation on the choice of portfolios that doesn't happen to apply to the portfolios in this paper, but could in other cases, is discussed in William J. Baumol, "An Expected Gain-Confidence Limit Criterion for Portfolio Selection," *Management Science* (October, 1963), pp. 174-186.

CHART 2

Expected Return and Risk of Efficient E-V Insurance Portfolios



is theoretically attainable which has the same risk as current holdings but which increases the expected return from .68 to over 2.7 per cent. While this would represent an admirable improvement under the E-V criterion, the dramatic changes necessary for such a portfolio are evident upon observing Table 5 that fire, extended coverage, ocean marine and auto property damage liability insurance would not be sold. This latter portfolio is in all likelihood not an acceptable one.

Under the Markowitz E-V criterion, given the relevant set of constraints, the movement from point P to any point on an efficient E-V curve is considered to be an improvement recognizing that no point on a curve is ostensibly superior to another on the same curve solely on the risk-return criteria. Only two examples have been illustrated above but a large number of hypothetical portfolio adjustments are summarized in Tables 3, 4 and 5. It must be emphasized that these results were obtained from the data of only one company which in turn were affected by the author's own extrapolation and constraint assumptions.

LIMITATIONS OF THE MARKOWITZ E-V CRITERION FOR PROPERTY-LIABILITY INSURANCE COMPANIES

There are two general areas of difficulty that definitely limit the theoretical and/or practical application of the Markowitz technique to property-liability company insurance portfolios. The first pertains to the nature of the input assumptions of the model. The second is the uncertain relationship between the Markowitz E-V criterion and the objectives and behavior of non-life insurance companies.

Input Assumptions in the Markowitz Model. The programming solutions applicable to the Markowitz portfolio selection technique are complex. However, they can be performed on digital computers, and they present primarily practical problems of computer storage capacity and calculation time. The technique can be utilized without understanding the complex mathematical programming procedures. But, the preparation of a reliable and acceptable input in the form of an expected return and a variance is a major difficulty in the practical application of portfolio selection.

The formulation of probability beliefs about expected returns and variation of returns on lines of insurance is a complicated task.[18] Un-

[18] Although ostensibly this is not as difficult as the formulation of appropriate risk and return measures for non-common stock investments, particularly private placements and mortgages.

doubtedly, it is difficult for an actuary experienced in statistical methods to make reliable, consistent estimates of future return in the form necessary for the model. Naturally, the procedure is even more difficult for managers less familiar with statistical and probability concepts.

In the case of insurance uncertainties, the historical method described by Markowitz[19] can be used to calculate return, variance, and covariance, and these values then can act as guidelines for quantifying expectations. The historical method, however, is deficient to the extent that it ignores the dynamic aspects of the insurance business. For example, the relative adequacy of future rate levels may differ from that evident in the historical data.[20] This and other similar difficulties can be alleviated by introducing expectations into historical parameters by adjustments based on subjective judgment. The revised historical input still will be deficient to the extent that future developments are unforeseen or that subjective adjustments do not accurately reflect expectations in a quantified form.

Even if a reliable variance of return is available for all lines of insurance, there will still be other fundamental difficulties. Variances of return do not include many factors that are important to actual portfolio selection problems. For example, an insurer will probably be concerned with the skewness or third moment of the probability distribution of losses and expenses on a line of insurance.[21] An insurer, or an investor, might act to maximize the third moment of his probability beliefs (preference for a distribution skewed toward positive returns) since this increases the chance of a large return while decreasing the chance of a large loss.[22]

Recently, insurers seem willing to accept relatively low expected returns in many lines of business and, consequently, in the over-all underwriting operation. Considering the regulatory and actuarial difficulties in obtaining adequate rates, the insurers may (must?) be willing to live temporarily with persistent losses if such losses are relatively stable, offer no significant danger of catastrophic experience, and can be offset by investment results. Portfolios selected with conscious or unconscious recog-

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[19] Markowitz, *op. cit.*, pp. 8-26.

[20] The possible effect of the relative adequacy of rate levels at various points in time was pointed out to the author by C. A. Hachemeister of the Insurance Company of North America.

[21] For a discussion of this concern on the part of investors, see Karl Borch, "A Note on Utility and Attitudes to Risk," *Management Science* (July, 1963), p. 700; and Yale Brogen, "Discussion," *Econometrica* (July, 1951), pp. 325-326.

[22] Brozen, *op. cit.*, p. 326.

nition of skewness of results are likely to differ from portfolios consistent with the efficient E-V criterion.

Perhaps the most troublesome problem with the input to a portfolio selection model is that the assumptions of risk and return may not hold up if an attempt is actually made to acquire a prescribed portfolio. Even if a property-liability insurer could significantly alter its insurance portfolio, this action could so seriously affect loss and expense ratios as to destroy the assumptions on which the reallocation was based. For example, since return is measured as a per cent of premiums, variability of return is a function of premium volume. Thus, if the E-V criterion prescribed a reduction in a particular line, such reduction would probably increase the variability of return and this might suggest even further reductions in this line.[23] On the other hand, as business is reduced, more selective underwriting may produce a more profitable book of business, thereby increasing the expected return.

Relationship of the Markowitz E-V Criterion to Company Objectives and Behavior. Intuitively, diversification of insurance, for example, by line and geography, seems desirable for the responsible operation of a property-liability insurance business, and the insurance portfolios of the established multiple-line companies do display a great deal of diversification. The question then remains whether the Markowitz portfolio selection technique can be used to explain or to plan the diversification of non-life insurance portfolios.[24]

The structure of the insurance business is such that non-life insurance companies can attain great diversification by lines of insurance without conscious marginal risk-return decisions. A large company can be expected to establish variety in its lines of insurance simply because of the nature of the marketing channels, the sheer size of the portfolios, and the complementarity of certain lines of insurance,[25] for example, auto bodily injury liability, auto property damage liability, and auto physical damage.

[23] The possibility of this uni-directional movement in individual lines was suggested to the author by William H. Crandall of the Insurance Company of North America.

[24] For an interesting application of a portfolio selection model to the behavior of Mutual Funds, see Donald Eugene Farrar, *The Investment Decision Under Uncertainty* (Englewood Cliffs, N. J.: Prentice Hall, Inc., 1962). For a critique of Farrar's work see Irwin Friend and Douglas Vickers, *op. cit.*

[25] One study follows this line of reasoning to conclude that life insurance company investment behavior is more properly explained by the simple maximization-of-expected-return rule than by a Markowitz portfolio selection theory. See Lawrence Donald Jones, Jr., "Portfolio Objectives, External Constraints and the Post-War Investment Behavior of Life Insurance Companies" (unpublished doctoral dissertation, Department of Economics, Harvard University, 1959).

The application of the Markowitz technique to a financial institution such as a property-liability insurance company is bound to present difficulties, since the model is probably most appropriate for some theoretical individual who has a definite amount of assets to commit for a given time duration. In this sense the model is applicable to static situations and only those in which the prescribed actions will not alter the general market and hence the input assumptions.

The operation of a non-life insurance business is obviously not static, and the continuous marketing activity in a changing environment and the sensitivity of the Markowitz model would probably suggest continual and impractical reallocation of lines of insurance. In addition to the use of constraints, one suggestion for reducing the reallocation problem is to introduce a cost of switching to make the model less sensitive.[26] The inability to reallocate lines of insurance without affecting the market, and the loss and expense assumptions on which the switching is based, has already been discussed.

CONCLUSIONS

The application of portfolio selection techniques to property and liability insurance companies has some interesting theoretical possibilities as well as serious practical limitations. The immediate value of such models appears to stem not so much from the output of optimal insurance portfolios but from the explicit emphasis on the definition and measurability of the crucial variables—risk and return. The analysis stresses the distinction between profitability (return) and variability (risk) and the dependency of both on the portfolio mix. Of more significance than the mechanical production of optimal portfolios is the recognition that decisions related to individual lines of insurance should be considered with regard to their effect on the entire portfolio.

[26] Gordon D. Shellard, "Panel Discussion: Operations Research," *Transactions of the Society of Actuaries* (1966, No. 1), p. D. 333.

Table 1

Assumed Expected Return and Range of Variation of Return on Lines of Insurance in a Multiple Line Insurance Company*

	Expected	High	Low
Fire	-1.64	5.96	- 9.24
Extended Coverage	-2.61	10.87	-16.09
Home Multiple Peril	-1.10	14.44	-16.64
Commercial Multiple Peril	2.57	61.69	-56.55
Ocean Marine	-4.43	6.07	-14.93
Inland Marine	1.18	8.84	- 6.48
Accident	-1.06	29.98	-32.10
Group A and H	7.93	16.99	- 1.13
Workmans Compensation	- .50	10.52	-11.52
Auto B. I. Liability	.11	10.73	-10.51
Auto P. D. Liability	-2.25	5.19	- 9.69
Auto Physical Damage	- .97	8.97	-10.91
Misc. B. I. Liability	2.49	18.97	-13.99
Misc. P. D. Liability	10.53	22.49	- 2.43
Treaty Reinsurance	-1.37	5.71	- 8.45
Fidelity	7.09	27.47	-13.29
Surety	25.61	44.47	6.75
Burglary and theft	7.58	15.84	- .68

*The range was taken as two standard deviations on either side of the expected value assuming a normal distribution.

Table 2

Maximum Percentage Constraints For A
Property-Liability Insurance Portfolio

	Present Composition (P)	Degrees of Relative Flexibility Set I (Low)	Set II (Average)	Set III (High)
Fire	8.95	10.00	12.00	15.00
Extended Coverage	2.58	3.00	4.00	6.00
Home Multiple Peril	11.58	13.00	15.00	17.00
Commercial Multiple Peril	5.51	7.00	8.00	10.00
Ocean Marine	4.73	5.00	6.00	8.00
Inland Marine	4.34	5.00	6.00	8.00
Accident	0.35	0.50	1.00	2.00
Group A. and H.	7.25	8.00	10.00	12.00
Workmen's Compensation	9.07	10.00	12.00	14.00
Auto B.II. Liability	9.60	11.00	12.00	15.00
Auto P.D. Liability	3.54	4.00	6.00	8.00
Auto Physical Damage	4.51	5.00	6.00	8.00
Misc. B. I. Liability	7.35	8.00	10.00	12.00
Misc. P. D. Liability	2.48	3.00	4.00	6.00
Treaty Reinsurance	15.08	17.00	20.00	25.00
Fidelity	1.36	1.50	2.00	4.00
Surety	0.89	1.00	1.50	2.00
Burglary and theft	0.83	1.00	1.50	2.00

Table 3

Efficient E-V Insurance Portfolios Under Constraint Set I

													Present Portfolio
Expected Return	1.21	1.20	1.15	1.10	1.05	1.00	.95	.90	.85	.80	.75		.68
Standard Deviation	5.48	5.43	5.03	4.69	4.39	4.18	4.04	3.94	3.86	3.79	3.74		4.80
	Percentage Composition of Premium Volume											Set I	
Fire	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	8.95
Extended Coverage	--	--	--	--	--	1.69	3.00	3.00	3.00	3.00	3.00	3.00	2.58
Home Multiple Peril	12.00	12.15	13.00	11.56	10.99	9.97	9.38	8.20	7.11	6.01	5.40	13.00	11.58
Commercial Multiple Peril	7.00	6.85	5.61	4.91	4.01	3.34	2.62	2.41	2.22	2.02	1.60	7.00	5.51
Ocean Marine	--	--	--	--	--	--	--	1.39	2.68	3.96	5.00	5.00	4.73
Inland Marine	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	4.34
Accident	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.50	0.35
Group A. and H.	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	7.25
Workmans Compensation	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	9.07
Auto B. I. Liability	11.00	11.00	11.00	11.00	11.00	11.00	11.00	11.00	11.00	11.00	11.00	11.00	9.60
Auto P. D. Liability	--	--	--	2.52	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	3.54
Auto Physical Damage	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	5.00	4.51
Misc. B. I. Liability	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	7.35
Misc. P. D. Liability	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	3.00	2.48
Treaty Reinsurance	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	17.00	15.08
Fidelity	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.36
Surety	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.89
Burglary and theft	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.83

Table 4

Efficient E-V Insurance Portfolios Under Constraint Set II

																	Set II	Present Portfolio
Expected Return	1.95	1.90	1.85	1.80	1.75	1.70	1.65	1.60	1.55	1.50	1.45	1.40	1.35	1.30	1.25	1.20		0.68
Standard Deviation	5.98	5.55	5.05	4.63	4.28	4.00	3.82	3.68	3.59	3.54	3.50	3.47	3.44	3.42	3.40	3.39		4.80
						Percentage Compositi m of Premium Value												
Fire	--	--	--	1.05	4.30	7.89	10.11	9.28	11.44	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	8.95
Extended Coverage	--	--	--	--	--	--	--	--	--	--	0.65	1.09	1.21	1.33	1.45	1.56	4.00	2.58
Home Multiple Peril	12.26	10.15	9.26	8.15	6.25	3.00	3.00	2.21	1.25	1.30	1.32	1.26	1.13	1.00	0.87	0.74	15.00	11.58
Commercial Muliple Peril	8.00	7.14	5.93	4.80	3.88	4.41	2.30	1.86	1.40	1.33	1.26	1.22	1.19	1.16	1.14	1.11	8.00	5.51
Ocean Marine	--	--	--	--	--	--	--	--	--	--	--	0.38	1.23	2.07	2.92	3.77	6.00	4.73
Inland Marine	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	4.34
Accident	1.00	1.00	1.00	1.00	1.00	0.85	0.43	0.22	--	--	--	--	--	--	--	--	1.00	0.35
Group A. and H.	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	7.25
Workman's Compensation	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	10.90	10.69	10.46	10.19	9.89	9.59	9.30	9.00	12.00	9.07
Auto B. I. Liability	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	9.60
Auto P. D. Liability	--	--	--	--	--	--	1.30	4.99	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	3.54
Auto Physicel Damage	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	4.51
Misc. B. I. Liability	10.00	10.00	10.00	10.00	10.00	10.00	10.00	10.00	9.68	8.75	7.78	7.04	6.55	6.07	5.59	5.11	10.00	7.35
Misc. P. D. Liability	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	2.48
Treaty Reinsurance	13.74	16.71	18.82	20.00	19.56	18.84	17.88	16.45	16.11	16.93	17.54	17.84	17.80	17.77	17.73	17.70	20.00	15.08
Fidelity	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	1.36
Surety	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	0.59
Burglary and theft	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	1.50	0.83

Table 5

Efficient E-V Insurance Portfolios Under Constraint Set III

																	Set III	Present Portfolio
Expected Return	2.95	2.90	2.80	2.70	2.60	2.50	2.40	2.30	2.20	2.10	2.00	1.90	1.80	1.70	1.60	1.50		0.68
Standard Deviation	6.65	6.05	4.98	4.20	3.84	3.66	3.52	3.40	3.32	3.27	3.24	3.23	3.22	3.21	3.21	3.20		4.80
					Percentage Composition of Premium Volume													
Fire	--	--	--	--	2.67	3.64	4.66	5.68	8.23	9.08	8.81	8.73	9.04	9.44	9.84	10.16	15.00	8.95
Extended Coverage	--	--	--	--	--	--	--	--	--	0.61	0.89	1.09	1.19	1.25	1.32	1.38	6.00	2.58
Home Multiple Peril	5.82	6.79	5.23	3.67	2.05	1.49	1.23	1.00	0.72	0.52	0.44	0.37	0.35	0.34	0.32	0.31	17.00	11.58
Commercial Multiple Peril	9.41	8.06	5.65	3.24	1.56	1.28	1.17	1.08	1.01	0.95	0.92	0.91	0.91	0.91	0.91	0.91	10.00	5.51
Ocean Marine	--	--	--	--	--	--	--	--	--	0.75	2.07	3.04	3.49	3.78	4.07	4.34	8.00	4.73
Inland Marine	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	4.34
Accident	1.77	1.89	1.36	0.84	0.31	0.14	0.05	--	--	--	--	--	--	--	--	--	2.00	0.35
Group A. and H.	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	12.00	11.38	10.56	9.74	8.99	12.00	7.25
Workman's Compensation	14.00	14.00	14.00	14.00	12.09	9.95	9.06	8.39	7.81	7.35	7.15	7.03	7.02	7.04	7.07	7.09	14.00	9.07
Auto B. I. Liability	15.00	15.00	15.00	15.00	15.00	15.00	15.00	14.13	13.19	12.47	12.14	11.96	11.95	11.99	12.03	12.07	15.00	9.60
Auto P. D. Liability	--	--	--	--	--	3.86	5.80	7.81	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	3.54
Auto Physical Damage	8.00	8.00	8.00	8.00	8.00	8.00	8.00	8.00	7.88	7.74	7.67	7.65	7.69	7.74	7.79	7.85	8.00	4.51
Misc. B. I. Liability	12.00	12.00	12.00	12.00	12.00	11.07	9.35	7.87	6.11	4.83	4.28	3.91	3.76	3.69	3.61	3.55	12.00	7.35
Misc. P. D. Liability	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	6.00	5.93	6.00	2.48
Treaty Reinsurence	--	0.26	4.76	9.26	11.88	11.53	11.65	12.01	13.03	13.70	13.96	14.21	14.40	14.58	14.75	14.93	25.00	15.08
Fidelity	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	4.00	3.67	3.10	2.86	2.72	2.58	2.46	4.00	1.36
Surety	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	0.89
Burglary and theft	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	2.00	0.83