APPENDIX 3—PARAMETERS

Non-Participating Stock (Subscript 1)	Parameter	Mutual (Subscript 2)
1.220	a	2.223
10.0	p	21.8
.050	α	.046
11.69	β	5.25
0.94	γ	0.63
h	Compound Log-Gamma	1−h
.731	Number of risks	.269
.293	Amount of premium	.707

DISCUSSION BY JAMES R. BERQUIST

We are, indeed, indebted to Mr. Hewitt for his continual efforts to provide us with practical applications of the theoretical techniques developed by mathematical statisticians.

In this paper Mr. Hewitt suggests a model which gives a good fit for size of risk distributions. That this technique does, in fact, fit the industry data is shown in Tables I and III.

The value of the suggested model is not limited to industry statistics, however, as its most practical application for the company actuary will be in fitting the distribution of business by size of risk of his own company to the model.

For example, the table on the following page shows the differences between the actual distribution of Employers Mutuals workmen's compensation risks by size and the theoretical distribution obtained by using a compound Log-Gamma as Mr. Hewitt suggests in Appendix 2. In this case the a_2 and p_2 were determined by using the method outlined in Appendix 1. The "h's" turned out to be .861 for the distribution of business by amount of premium, and .466 for the distribution of the number of risks.

Typical of the authors of many good mathematical textbooks, Mr. Hewitt assumes a rather high degree of mathematical proficiency on the part of his readers, and leaves the reader on his own to supply some of the missing proofs.

On page 107, for example, he says the following: "if $T(x, a, p)$ repre-

COMPARISON OF THEORETICAL AND ACTUAL DISTRIBUTION BY SIZE OF RISK

Employers Mutuals of Wausau

Annual Premium Size	Number of Policies* Actual – Theoretical	Standard Premium Size* Actual – Theoretical
Under $100	-.0004	-.0047
$ 100-$ 499	+.0015	+.0056
500- 749	+.0005	+.0016
750- 999	-.0005	-.0020
Under $1,000	+.0011	+.0005
$ 1,000-$ 4,999	-.0034	-.0017
5,000- 24,999	+.0015	+.0011
25,000- 49,999	+.0011	+.0002
50,000- 99,999	-.0009	–
Under $100,000	-.0006	+.0001
$100,000-$249,000	-.0044	-.0002
$250,000 and Over	+.0049	–

*Actual and theoretical values were calculated as ratios to total number of policies or amount of premium, carried to four decimal places.

sents a distribution of amount of premium, then it is *easily** seen that $T(x, a+1, p)$ . . . is the distribution by number of risks."

This reviewer feels the paper would have been much more readable had the author reviewed for his readers some basic mathematical statistics. He could have pointed out that the basic Gamma frequency function is $x^p e^{-ax}$. The value $\frac{a^{p+1}}{\Gamma(p+1)}$, then, was obtained by integrating the frequency function $x^p e^{-ax}$ over the range 0 to ∞ and requiring this integral to be

* Reviewer's italics.

equivalent to *1.0*, the requirement of any p.d.f. Then, since the amount of premium is e^x, the frequency function for the number of risks is $x^p e^{-(a+1)x}$ upon dividing by the amount of premium. By integrating this frequency function and fulfilling the requirement that the integral equals *1.0*, we do "easily" obtain $T(x, a+1, p)$.

Mr. Hewitt's fine narrative on "fitting the data" in Appendix 1 would have been enhanced, at least for the average reader, if he had seen fit to include some of his worksheets used in obtaining the tables in the paper.

This paper is a valuable addition to our *Proceedings* despite the minor points just raised. We hope that Mr. Hewitt, and others, will continue to share their research with us.

DISCUSSION BY ROBERT L. HURLEY

While this paper, so suggestive of an austere scholarship, may seem directed to those of the avant-garde who delight in frolicking among the outer reaches of actuarial theory, Mr. Hewitt presents both a challenge and a promise to those members whose interests, like this reviewer's, may gravitate more towards the application of actuarial principles to current underwriting and rating problems.

This paper shows that the distribution by size of both the workmen's compensation standard premium and the number of policies* may be fairly described by a Log Gamma equation. It also suggests that certain workmen's compensation expenses may vary by size of risk according to a similar pattern. There is the intimation (which particularly interests this reviewer) that loss distributions may follow the same law, using the latter term in its least restrictive sense.

A quick check on Mr. Hewitt's findings by premium size (c.f. Table I) reveals a close fit of the actual to theoretical values, according to the Pearson Chi-Square or even the possibly more critical Kolmogorov-Smirnov test. While references were afforded the reader on the Gamma function, the author was understandably more interested in the potential significance of his findings to actuarial theory than in detailing the mathematics, some of which is available in the standard literature. This "Hoc age" (up and at it) approach which is not infrequently so characteristic of the scholar can be oftentimes bewildering and even exasperating to the less specialized reader.

* As given in Exhibit I of the National Council on Compensation Insurance's Report of the Special Committee to Study Expenses by Size of Risk.