

DISCUSSION OF PAPER PUBLISHED IN VOLUME LI

A BAYESIAN VIEW OF CREDIBILITY

ALLEN L. MAYERSON

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DISCUSSION BY CHARLES C. HEWITT, JR.

This is a perfectly delightful paper and could not have come at a more propitious time in the development of our Society's research efforts. The paper is not without flaws, but I hope that by pointing out such items I will in no way diminish my appearance of enthusiasm for Professor Mayerson's effort.

Heresy

There is another, and entirely separate reason, why Professor Mayerson's *clarification* of credibility concepts comes at an excellent moment. In listening to a sermon recently I was reminded that in the Anglican Catholic Church one of the four major obligations of the priesthood includes the stamping out of heresy. Gentlemen, I am not an Anglican priest (although my only brother is), but I say that there is heresy amongst us on the very basic issue of the meaning of credibility. One might have hoped that the recent publication and reprint issue of Laurie Longley-Cook's thorough and definitive study on credibility¹ would have dispelled what Arthur Bailey referred to as the "profound mystery" which has surrounded the basis for credibility formulas—at least among our own membership. Alas, such is not the case for we have been confronted just this Spring with an article² (in another insurance journal) which purports to explain credibility to non-actuaries, but which hopelessly confuses the term "credibility" with the statistical term "dispersion." (At one point this article refers to the terms "measures of dispersion" and "measures of credibility" as having identical meaning.³) Laurie Longley-Cook correctly points out,⁴ ". . . credibility is not a simple property of data which can be calculated by some mathematical formula as can the standard deviation or other measures of the effect of chance variation on a body of statistical data. While credibility and statistical variance are related,

¹ An Introduction to Credibility Theory—L. H. Longley-Cook—*PCAS XLIX* (1962).

² Actuarial Science and Credibility—John S. McGuinness—*CPCU Annals* (Spring 1965).

³ *Ibid.* p. 20.

⁴ An Introduction to Credibility Theory—p. 4.

the former is meaningful only against a stated or implied background of the purpose for which the data are to be used and a consideration of the value of the prior knowledge available.”

Not to belabor reference to the aforementioned article for non-actuaries, but it is appropriate to this discussion of Professor Mayerson's work to add that he does give us two mathematical formulations for credibility, applicable in specific instances, which make it abundantly clear that credibility may under certain circumstances be a function of:

- (1) sample size,
- (2) underlying hazard (mean of prior distribution), and
- (3) underlying dispersion (variance of prior distribution).

And ironically (but not surprisingly) it turns out that credibility *increases* with variance (of the prior distribution). Thus, imprecise conclusions, such as equating wide dispersion with poor credibility,⁵ can be seriously misleading to both professional and lay readers.

Synthesis

The essence of Professor Mayerson's paper is the bringing together of the late Arthur Bailey's pioneering work on credibility with the most up-to-date techniques of statistical decision theory. My advice to all interested persons, who have not already done so, is to read:

- 1) Mayerson's paper, then
- 2) *Applied Statistical Decision Theory* by Raiffa and Schlaifer—Harvard University (1961) — with emphasis on Chapter 3 (Conjugate Prior Distributions), Sections 10 and 11 of Chapter 7 (Negative Binomial and Beta-Binomial), and Chapters 9 (Bernoulli Process) and 10 (Poisson Process), and then
- 3) “Credibility Procedures” by A. L. Bailey—*PCAS XXXVII* (1950) with extremely interesting discussions and author's reply.

You should then have a superior grasp of what credibility formulas are bottomed on, and incidentally a deeper appreciation of Professor Mayerson's perception in conceiving this paper.

Perhaps a further synthesis with the works of Dropkin, et al, on merit rating in private passenger automobile can be achieved with the following illustration:

⁵ Actuarial Science and Credibility—p. 19.

Dropkin⁶ and Hewitt⁷ point out that a risk in a class with mean r/a which has c accidents in s years indicates an expected frequency of $\frac{r+c}{a+s}$. In credibility terms:

$$\frac{r+c}{a+s} = (1-Z) \frac{r}{a} + Z \frac{c}{s}$$

from which

$$Z = \frac{s}{s+a}^8$$

but $a = \frac{\text{Class Mean}}{\text{Class Variance}}$, and substituting for a in the above expression

produces Mayerson's expression for credibility in the Gamma-Poisson case.

The Neglected A. W. Whitney

My attraction to this paper prompted me to correspond with Professor Mayerson concerning the extent to which Albert W. Whitney⁹ had been a forerunner of some of the conclusions reached by (A.L.) Bailey and Mayerson. For example Whitney and his colleagues were well aware that the K in

$$Z = \frac{P}{P+K}$$

was not really a constant. Starting with the presumption that the frequency distribution of risks within a particular classification is normal, Whitney arrived at an expression¹⁰ for Z which can be reduced to:

$$Z = \frac{n}{n + \frac{P(1-P)}{\epsilon^2}}$$

n = exposure (to hazard) of a particular risk

P = indicated class hazard

ϵ^2 = variance of risks within the class

How close this comes to Mayerson can be seen by comparing it to Mayerson's Beta-Binomial derivation of Z , which can be expressed:

⁶ Automobile Merit Rating and Inverse Probabilities—*PCAS XLVII* (1960).

⁷ The Negative Binomial Applied to the Canadian Merit Rating Plan for Individual Automobile Risks—*PCAS XLVII* (1960).

⁸ R. A. Bailey—Discussion of "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records"—*PCAS XLVII*, p. 155 (1960).

⁹ The Theory of Experience Rating—A. W. Whitney—*PCAS IV* (1918).

¹⁰ *Ibid.* p. 288—Equation (23).

$$Z = \frac{n}{(n-1) + \frac{m_H(1-m_H)}{\sigma_H^2}}$$

n = (as above)

m_H = mean of prior distribution (assumed hazard)

σ_H^2 = variance of prior distribution.

How then did our predecessors get trapped into an invariant "K"? Whitney's remarks are revealing.¹¹

"We now come to the most difficult question of all, the determination of ϵ^2 . It is obviously impossible as a practical matter to determine ϵ^2 statistically in each case."

Further along:

"Mr. [Winfield] Greene made the suggestion that . . . the second term of the denominator be taken as constant."

Whitney, in defense of a constant K , says "This brings us to the question of whether it is desirable in actual practice to admit the varying credibility of the class-experience and hence of the manual rate. We know that the manual rates for some classifications are more reliable than for others *and yet it is doubtful whether it is expedient in practice to recognize this fact. . . .*"

In his later work Arthur Bailey acknowledged this earlier effort in a passing reference to Greene's practical approximation of Whitney's "more complicated formula."¹²

Whitney's introductory non-mathematical remarks are so pertinent to a clear understanding of the foundation of credibility in experience rating that portions of them must be re-quoted:

(1) *Risk-exposure*¹³

"It is evident in the first place that the weight of the risk-experience will depend upon the risk-exposure. Other things being equal, the experience of that risk which has the larger exposure will be entitled to the larger degree of consideration. In the case

¹¹ Ibid. p. 287.

¹² Sampling Theory in Casualty Insurance—A. L. Bailey—*PCAS XXIX* (1942)—p. 72.

¹³ The Theory of Experience Rating—p. 275.

of a very large risk the rate may with safety be based almost wholly upon its own experience; in the case of a small risk very little credence can be given to risk-experience and the rate must be based almost wholly upon the experience of the class."

(2) *Hazard*¹⁴

"Essentially the same relationship holds true in the case of the hazard; the larger the hazard, the larger will be the number of accidents, the exposure remaining the same, and therefore the more trustworthy the average."

(3) *Degree of concentration within class*¹⁵

"Now it is evident intuitively that if the risks are concentrated within the class, that is, if the standard deviation is small, a risk-experience that departs from the average of the class can be more easily accounted for as due to chance than as due to an inherent difference in the degree of hazard. On the other hand, if the standard deviation is large, that is if the risks are diverse, it is inherently likely that a risk-experience that departs from the average is to be accounted for by a real difference in the hazard."

(4) *Credibility of manual rate*¹⁶

"Another element that in theory may be taken account of is the varying credibility of the manual rate. The manual rate is established upon experience which in a majority of classifications is insufficient and which in many cases has been supplemented by judgment. It is evident that, other things being equal, the higher the credibility of the manual rate, the greater its weight in establishing the balance between class-experience and risk-experience. If, on the other hand, the manual rate is established upon insufficient experience, we shall be inclined to give greater relative credence to the risk-experience."

Kinds Of Credibility

If I had to choose my major criticism (in the unfavorable sense), it would be that the author leaves the implication that his approach is equally

¹⁴ Ibid.

¹⁵ Ibid. p. 276.

¹⁶ Ibid.

applicable in any credibility situation. But in *practical* casualty actuarial work there are at least two significantly different applications of credibility:

- (1) Class pure premium selection,
- (2) Experience rating.

In the former situation Mayerson's general approach is directly applicable, and, in fact, provides a quite satisfactory solution to the important question, "What credibility should be assigned to the underlying pure premium?" I think sub-consciously we have known for years that $(I - Z)$ is *not* a totally honest answer, if Z is solely a function of the volume of the current experience.

But Mayerson's general approach is not so easily applied to experience rating, if we agree with Whitney that the degree of concentration of risks within a class is pertinent. For there is now a three-way credibility problem – what credibility should be assigned to:

- (1) Current risk experience,
- (2) Previous risk experience, and
- (3) Current manual rate.

Frankly no one else has suggested a theoretical answer for this very real problem, but it should be clearly understood that Mayerson's work does not come to grips with this issue, either.

Minor Technical Criticisms

There are several minor mathematical items that can be criticized:

- (1) Meanings attached to symbols do not always remain constant (or clear). E.g., the capital letter H is used alternately to represent an *hypothesis*, a *point*, a *random variable* and a *parameter*. This failure to be more precise is confusing to the reader and may upon occasion have confused the author.
- (2) In the discussion of conjugate distributions the statement that m (assumed mean) can be a pure premium, a claim frequency, an average claim cost, or some other actuarial function is too loose. How, for example, do we choose a Beta-distribution for average claim cost or pure premium when the variable must lie within the range *zero to one*?

- (3) The statement in the last paragraph of "Choosing Prior Probabilities" that n varies directly with m is imprecise for

$$n = \frac{m(1-m)}{\sigma^2} - 3$$

The author must therefore qualify his statement by adding "for $m < 1/2$ ".

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This is one of the most significant papers presented to this Society in many years and, happily, should produce much controversy and further thought in this important area. European actuaries have outstripped us in the classical "theory of risk". Professor Mayerson has distilled the essence of American achievement in the areas of credibility and the Bayesian approach. We may well be proud of what our Society has done and hopeful for what it promises to do in these areas.