(15) McIntosh, Kenneth L., *The Rationale of the Fire Schedule – Part I, Theory.* The C.P.C.U. Annals. Vol. 13, No. 1. (1960). p. 1.

(16) ——— *– Part II, Application. The C.P.C.U. Annals.* Vol. 13, No. 2. (1960). p. 117.

(17) Müller, W. Max, *Egyptian Mythology.* Marshall Jones Co. (1923).

(18) Pruitt, Dudley M., *St. Vitus's Dance* (Presidential Address to the C.A.S.). *P.C.A.S.* XLVI (1959). p. 149.

(19) Riegel, R. & Miller, J. S., *Insurance Principles and Practices* (4th Ed.) Prentice-Hall, Inc. (1959).

(20) Taylor, Angus E., *Introduction to Functional Analysis.* John Wiley & Sons, Inc. (1958).

DISCUSSION BY LESTER B. DROPKIN

For several years now, writers and reviewers of papers presented to this Society have stressed the desirability, and indeed, the inevitability of utilizing theory, methods, techniques and procedures derived from what may be broadly referred to as the field of Finite Mathematics. During these same years the Society has also seen an increasing number of papers dealing with the ratemaking problems of the fire actuary. Recalling Mr. McIntosh's earlier work, it is not unexpected that he would again bring these two lines together in the paper now under review.

In his current paper, "A Mathematical Approach to Fire Protection Classification Rates," Mr. McIntosh deals with the problem of determining a set of rates such that they will, in the language of the paper, simultaneously fulfill the conditions of "feasibility" and "operational constraint." These two terms, although coming from the language of linear programming, represent simple and familiar concepts. The feasibility property will be readily recognized as that old friend: a rate structure in balance by part and in total. The question of operational constraints may similarly be recognized as coming within considerations of rate relativity, albeit the rate relativities here are not specifically given. Rather, each of the rate relativities is fixed only to the extent of having given lower and upper limits, such limits being predetermined by judgment or other outside factors. It is, of course, the simultaneous existence of the feasibility and constraint conditions that make the problem a real and interesting one.

The definition of the problem and the treatment of its solution (including therein those cases where no solution is possible) proceeds via linear algebra and matrix theory. Although this mathematical area may be somewhat unfamiliar to many of us, it is fortunate that much of the terminology is natural, intuitive and extremely suggestive, and that much of the theory has direct geometric analogues.

It is clear, at least to this reviewer, that the presentation of this paper will have several most desirable consequences: First, there are those areas, both within and without the fire field, whose structure is such as to directly parallel the problem treated by Mr. McIntosh. Here the methods of the paper can be lifted bodily and immediately applied with a minimal amount of alteration. In this connection, let us specifically note how well the very detailed and comprehensive illustrative examples have been prepared. The advantage of having many concrete illustrations to follow while working through the theoretical material is obvious.

Second, there is the undoubted stimulus to a wider study of the theory, principles and applications of those many mathematical areas which may be said to come within or be related to the scope of finite mathematics. This, not only for the specific purpose of following the particular mathematics of Mr. McIntosh's paper, but also for the purpose of developing that wider background which will increasingly become more and more necessary if we are to more completely fulfill our actuarial responsibilities. It will, perhaps, be of more than passing interest to note the rising sentiment for modifying the syllabus of the actuarial examinations in this regard.

Third, there will be those generalizations and extensions of the paper's methodology and theory to a wider class of problems. The mathematical discipline of linear algebra, as is well known, has served to unify many formerly separate branches of the mathematics tree, thereby, for example, providing the theoretical base for such applications as linear programming. There would seem to be no reason why the attack on many actuarial problems could not now derive substantial advantage from just such an alternative viewpoint.

The reader, on first coming to the paper, is quite likely to feel himself overwhelmed. The paper is long; the notation and terminology is unfortunately not a familiar one in our *Proceedings;* there are many pages of mathematical symbols unrelieved by normal linguistic intercourse; and the author's style of exposition is at times too much akin to those stream-of-consciousness writers whose elliptical simplicity is sometimes bafflling. These, however, are really unimportant and passing details, for the paper is a fine piece of actuarial work. For the reader willing to give the paper the serious consideration it deserves, there will be ample repayment for any expenditure of time and effort.

It would be neither possible nor desirable to attempt to summarize or paraphrase Mr. McIntosh's paper in the short space of this review, although it may be of some interest to single out some of the most important facets. Before doing so, however, mention should also be made of one approach to the reading of the paper which I found to be quite helpful, viz., a free use of the method of general reasoning. Many of us will recall that we were first introduced to this method in connection with our study of interest and annuities, a subject which suffers no lack of multitudinous symbols.

Assuming then that it will not detract from the paper itself, where the whole theoretical construct that comprises Method II is given, I should like to point out what appears to me to be the one equation which can be identified as going to the heart of the matter, viz., equation A-27 of the Appendix. It will be recalled that this equation defines a transformation, F*, from the ratio vectors to the coefficient vectors. Basically, Method II then results from the fact that this transformation is one-to-one, and in particular that the extreme points of the ratio vectors map into the extreme points of the coefficient vectors under the transformation, together with the basic fact that the points of a convex set can be expressed as a linear convex combination of extreme points, all coupled with the properties of the parameter vectors.

Reference was made to Method II as a "whole theoretical construct." This review would hardly be complete without also mentioning the satisfaction to be derived from a consideration of the manner in which the several different aspects of the paper are brought together into an interlocking harmonious unity.

In reviewing an earlier paper of Mr. McIntosh, I said, ". . . I am sure that this Society will be looking forward to future papers in which he will carry forward the ideas and conceptions of the present notable contribution." This, in full measure, he has done and will no doubt continue to do.