SOME FUNDAMENTALS OF INSURANCE STATISTICS

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This paper is written for actuarial students, for insurance workers in general, and for non-insurance statisticians. Reference is made to statistical text books in general for much of the elementary mathematics, otherwise the paper is complete in itself.

PART ONE: ELEMENTARY CONSIDERATIONS

CHARACTERISTICS OF INSURANCE STATISTICS

Primarily, insurance statistics have to do with the frequency with which untoward events occur in a class of insured individuals, and with variance in the severity of the effects of those events. Some fundamentals of insurance statistics are as follows: (1) Every risk belongs to a class of risks, but differs from every other risk in the class; (2) All classes of risk are capable of multiple subdivisions; (3) Everything is a time series, often a three dimensional time series: calendar time, age of person or property, time since insurance began; (4) Nearly everything is expressed in dollars and cents; (5) For many kinds of insurance, the class with large amounts insured may be too few in number to produce reliable data; (6) All results are influenced by human actions; (7) Many risks are classified on the basis of representations rather than on facts. Most insurance statistical data are based on second and third hand information; (8) Nearly all insurance statistics for a given class can be compared with statistics for a related class, or with statistics for a somewhat related coverage on what is otherwise the same class; (9) Often insurance statistics are used before claim information is fully developed; (10) Insurance classes are different from classes in the population as a whole because of actions of insureds, actions of the insurers, and because people who buy each kind of insurance are a different class from people who don't buy that kind of insurance; (11) Insurance itself produces changes in the statistics involved; (12) Insurance claim rates "per year" sometimes include grace periods, and are not precise as to the time when coverage actually commenced.

ELEMENTARY ILLUSTRATIONS

If an insurance company could just insure a cross section of the population, the need for underwriting of individual risks would be eliminated or sharply reduced because, to start with, the insured class would be the same as the population class. The need for claim settlement and administration would still be great because insurance changes the action of the people insured. Insureds do not consist of a cross section of the population and this difference between the insured population and the general population results in a difference between general statistics and insurance statistics. In insurance terms, we have self-selection (often equivalent to anti-selection), company selection, effect of insurance upon occurrence of events insured against (or the reporting of the occurrence), and the effect of the insurance upon the amount to be paid. These interrelations and differences of insurance and general statistics are illustrated by considering an actual case.

One of the most important recently developed lines of insurance is Major Medical Insurance. However, before considering the development of Major Medical Insurance in relation to statistics of insurance, we will first consider the development of a much simpler kind of insurance, the insurance against scoring a hole-in-one in a professional golf tournament.

In connection with the hole-in-one in a professional golf tournament recently, a very large prize, $50,000.00, was offered for the scoring of a hole-in-one. The $50,000.00 was the total prize regardless of the number of holes-in-one scored. The statistical application to the insuring of this risk is fairly clear cut to persons who keep up with golf and its statistics. The probable procedure involved illustrates many of the aspects of statistical applications to insurance operations.

In the first place, there is rather readily available a considerable volume of statistics on the number of holes-in-one scored by professionals in their golf tournaments. With some little research, this hole-in-one statistic could be analyzed by difficulty of hole. We would thus have the statistics for holes closely comparable to the one or more to be played for the hole-in-one prize. We would, of course, know how many times the hole or holes were to be played. We would also have a series of statistics for holes of somewhat greater and lesser difficulty as collateral statistics. We would also have a time series which would give us some clue as to whether professionals were tending to score more holes-in-one, or not.

We could also take into consideration some rather remotely related statistical data such as the number of birdies made on par three holes and the number of times when the balls came within a few inches of coming to rest in the hole either off line or too hard or too softly hit. We could also take into consideration similar results in amateur tournaments.

The foregoing would appear to exhaust the statistical data which could be used as a basis of determining the pure premium involved and we now come to the problem of estimating the difference between non-insurance statistics and insurance statistics.

In the case of the golf hole-in-one, there are three parties involved: the insurance company, the promoter, and the professional golfer. In this particular case, and by no means is this always the case, the interest of the insurance company is directly opposed to the interest of the other two parties. We can, therefore, expect that the other parties will do whatever they consider to be ethical in order to provide a hole-in-one with its big prize, and with its collateral publicity which would be considerably to the advantage of the promoter. There is very little risk to the professional golfer himself, since he was going to play in the tournament with corresponding time and expense in any event, so that the golfer has all to gain and little to lose by striving for a hole-in-one.

The first thought in applying the original statistical basis on an uninsured event to the risk when there is insurance is that perhaps there should be some coinsurance. That is, perhaps the promoter should pay some of the prize money even though he has insurance. Coinsurance would reduce the conflict of interest between the promoter and the insurer. We then run the following gamut of thoughts with relation to the effect of the insurance upon the actions of the promoter and the golfers:

1. The golfers would practise for this hole-in-one before the tournament.
2. The golfers will be shooting directly for a hole-in-one on this particular hole rather than shooting more for the center of a green as they might do if they were interested solely in their total scores and how they finish in the tournament.
3. The promoter might easily be tempted to place the hole in the center of the green or in a flat spot, or even in a slight valley on the green, and he might well leave the hole in the same spot during the entire tournament instead of moving it as is customary. He might even leave the grass a little longer or the green a little softer than might well otherwise be the case.
4. The golfers would be somewhat inclined to pair off, or join together in greater numbers to agree to split the prize in the event that one of the group scored a hole-in-one. This is especially true because of the effect of federal income tax upon higher earnings in one year. Correspondingly, the golfers would be inclined to inform each

> other as to the effect of wind and ground conditions on the playing of the hole. They might even be a little less concerned about the avoidance of pressing their feet down into the area immediately surrounding the hole and they might be extremely careful to remove the handicap of ball marks or other irregularities in front of the hole before leaving a green.

The foregoing is merely an elementary illustration of the problems of applying non-insurance statistics to insurance operations and is not intended to indicate that the degrees of conflict between the parties involved are always the same as it is in the case of insurance against the payment of a prize of $50,000.00 for a hole-in-one.

These are factors that we, as amateur golfers and as statisticians, have thought of import. But, as a proverb of Iran states, "Only the hounds of Mazandaran can catch the wolves of Mazandaran." We should have the help of experts in our analysis of the problem; an expert golfer, an expert promoter, a greenskeeper, and, if such there be, an expert golf statistician.

As previously indicated, Major Medical Insurance and its evolution are a better indication of applications of statistics to insurance problems. One of the earliest occurrences related to Major Medical Insurance was that the medical director of a large insurance company suggested to his actuary that individuals be insured against the payment of major medical and hospital expenses on an indemnity basis rather than on a schedule basis. After a moment's reflection, the actuary asked, somewhat incredulously, if this meant that the insurance company would pay whatever the doctors decided to charge. This recognition of the conflict involved between the insurance company and one of the parties involved in Major Medical Insurance ended this attempt to initiate Major Medical Insurance.

There was a large body of incomplete statistics indirectly related to Major Medical Insurance contained in the files of insurance companies on hospital and surgical insurance even though the insurance companies did not pay the major medical expenses. Estimates could be made from the incomplete statistics. There were claim rates by cause of sickness and medical data as to the duration of such sickness. From these, and probably from non-insurance sources, Major Medical Insurance had statistics for its inception. A few years after the initiation of Major Medical Insurance, one of the large insurance companies made a study of the medical expenses of their own employees in great detail. The employees did not have Major Medical Insurance. This analysis was based on questionnaires to the employees, and the results were analyzed and reported by one of the actuaries of that company. Practically all of the problems involved in the statistical application of this experience on this particular class of individuals when they were not covered by Major Medical Insurance, but as applied to insurance operations in different parts of a changing world, were analyzed from a practical viewpoint in that report. (Thaler, *Transactions of the Society of Actuaries,* Volume II.)

The problems involved in that application to Major Medical Insurance can serve as generalizations of many of the problems of applying general statistics to insurance companies.

One of the effects of Major Medical Insurance is to provide not just the funds to pay medical and hospital bills, but to provide an actual increase in the use of medical and hospital facilities. Our hospital insurance and medical insurance, and this includes Major Medical Insurance, helps provide more and better medical and nursing facilities. This means that out of funds provided through insurance, we must be prepared to help build hospitals and improve their facilities, to help employ more nurses (this may mean paying them relatively higher scales and improving their working conditions in accordance with the law of supply and demand), and to reimburse doctors for any added work; all this in the shadow of inflation.

Insurance statisticians, in their consideration of the applications of their statistics, must take into consideration the immediate effect of the insurance and also the sociological and economic forces which, in turn, are largely capable of being understood to a degree only with the help of statistical analysis. Again, experts should help analyze the problem; doctors, hospital administrators, nurses, labor experts and, once the insurance is operating, experts in that line of insurance.

The first statistical application to any somewhat isolated insurance problem involves a great deal of judgment. This judgment, of course, was illustrated by the incident in which the actuary of one large company refused to let doctors and hospitals say how much his company should pay on a claim and, therefore, refused to initiate Major Medical Insurance. In a sense, this judgment was based upon parallels as to what the charges might be for various kinds of services when there was no bargaining between the seller and the buyer. Often, however, we have related coverages or we can extend coverage rates for various other age brackets and may have the statistics for such related insurance.

INTANGIBLES

In any statistical process we are looking at cold figures on paper, but we should be thinking of tangible objects and of live persons. The human element underlies much of the factual material on which statistics are based. The output of one factory manager, one factory inspector, or one factory worker, forms a significant sub-class from the output of other human beings and other human organizations. Insurance is no exception to the impregnation of statistical data with the influence of humans. "Intangibles" we call some of them, but, to those humans near the grass roots they are not intangible. The workers know the influence of their own endeavors and of the endeavors of their fellow workers. To those at the grass roots the reasons for differences are not intangible, they are real. Consider one insurance case. Why did valuable property floater coverage suddenly change from unprofitable to profitable for one large company in a certain large city? The way one appraiser went about his job involved what home office underwriters may call "intangible," but the appraiser could describe precisely how he got more premiums and better experience out of floater insurance. Conversationally – "Mr. Jones, did Mrs. Jones get this dress from Macy's basement?" Mr. Jones (the man that the neighbors are keeping up with) – "No – Saks Fifth Avenue – and the jewelry from Tiffany's, etc. etc." Result – 120% increase in appraised value; 50% drop in loss ratio; 99.44% drop in trouble adjusting claims. Statistical sub-class – property appraised by an appraiser who knew property and who knew people, and who wanted to do a good job. Similar sub-classes – agents who knew; home office underwriters, et al, who knew their business, knew people, and who set high standards of performance for themselves. These generalities of knowledge of people are partly intangible when expressed as generalities, but are tangible to those near the grass roots. The insurance statisticians cannot know all that brokers, agents, appraisers, and underwriters know about a risk. But, translating intangibles into tangibles, picking the significant good or bad subclasses from the commonly recognized class, getting down to individuals, brings statistical figures to life and to extra productivity. *The Ugly American* put it this way – "Have you been out in the boondocks?"

DEFINITIONS AND DESCRIPTIONS OF INSURANCE CLASSES

The precision of our descriptions of insurance statistical classes varies all the way from fairly precise descriptions when detailed research is involved to obviously incomplete descriptions of conglomerates in intercompany investigations. Risks have claim experience because of pertinent characteristics, not just because of the name of a class. The pertinent characteristic of a student applying for accident-medical coverage may be that he is a football player, not that he is a student. The pertinent characteristic of an armed forces lieutenant applying for automobile insurance may be that once a week he races back from the nearest town or entertainment center to his armed forces base, Cinderella-like, to beat the stroke of twelve, or the stroke of two a.m. Statistical thinking is not just thinking about classes; statistical thinking is comparative quantitive thinking about characteristics in an effort, among other things, to come closer to fundamental causes.

UNITS AND VALUES

Statisticians deal in dollars, index numbers, standard deviations, and more purely mathematical standards of comparison. These are technical units or standards of comparison. "The dollar is a unit of value." But is it now? Oh, we all know we get into financial trouble with the growing cost of automobile repairs and property and personal damages in terms of a declining dollar, but insurance is a two-edged sword. The dollar has no value except in what it will buy. Where is the value in a fire deliberately set as an alternative to bankruptcy; in unemployment "benefits" to those who are thereby encouraged to idleness or discouraged from enterprise; and how great the value in rehabilitation, in prevention, in insurance dollars when put to the improvement of human lives, in unemployment benefits that preserve dignity and give hope? Likewise, the cost of paying for the insurance needs measuring in truer values. For example, premiums or taxes to pay for benefits may reduce savings or reduce worthwhile expenditures in other directions. These are indications of true costs and of true values; true statistical measures – truer than the easily expressed and varying unit – the dollar.

THE INEXORABLE APPLICATION OF STATISTICS

Insurance statistics are inexorably translated into accounts, cash accounts and claim reserve accounts, among others, whether they are translated into technically statistical reports or not. In a truly mutual and equitable insurance operation, claims plus expenses are premiums. A bad experience would force premiums to rise very quickly if all companies were limited to just one line of insurance and had very limited amounts of capital. It takes a long time for the facts to catch up with a powerful financial or governmental complex unless it is managed by those who are strongly motivated by competition and by profits and losses, but the facts have a way of catching up with everybody in the long run.

INSURANCE STATISTICS IN THE REALM OF STATISTICS

Insurance statistics comprise a class of statistics which has different characteristics for different kinds of insurance, but, as a broad class, insurance statistics are different from all other statistics in their characteristics and properties. On the other hand, each of the characteristics of insurance statistics is present in certain other statistics.

Insurance statisticians are plagued by: changing economic and social conditions; difficulties in projecting; anti-selection; class selection; the effect of individual agents and other workers; the difficulties of second and third hand reports rather than of direct observations; the use of class labels rather than of characteristics. Other statisticians, who deal with humans, face some or all of these same problems. But scientific statistical thinking has to do with quantitive comparison of characteristics (not classes) in a search for causes. With insurance statistics as an illustration of the shortcoming of much of the statistics about people, no wonder Haroun Al Raschid went about disguised inconspicuously, as a beggar if it suited his purpose, in order to obtain his own statistics about the state of mind of the people of Baghdad at first hand in the days when the *Arabian Nights* were being written.

STATISTICAL THINKING IN THE REALM OF THINKING

Most scientific thinking is comparative quantitive thinking about qualities. The transition of the observations of gravitational effects from Aristotle, to Galileo, to Newton, to Einstein, to the conquest of space, represents the advance from rough statistical thinking to more accurate statistical thinking, to the thinking of pure science, and to statistical thinking again. Aristotle observed that heavy objects fall faster than light objects and, according to his medieval followers, concluded that their speeds were inversely as their weights. Statistically, he had the objects in the wrong statistical classes. He should have divided by the characteristics of surface and shape in proportion to weight, not just by weight. Galileo disproved Aristotle by further experimentation. Newton reached a "cause," a presumably universal law, the gravitational force which attracts objects to each other. Einstein refined our thinking about one of our standards, the "straight line" pursued by a ray of light. But we still have wind tunnels and experimental flights, and weather analyses to observe "statistically" the result of the interplay of the force of gravity with other forces, for example, near the speed of sound in various densities of air. Scientists and scientific statisticians alike are searching for the "causes" of our experience, and none of us has ever reached the ultimate "cause" – What causes gravity?

If Newton and his "cause," the force of gravity, represents thinking more precise than that of statisticians, on the less precise side we have thinking based on commonly observed statistics that we do not even write down, the "judgment" of an experienced actuary or other statistician or business man, for one thing. At the extreme low end of this application of statistical "thinking" we have the emotion that makes us tend to fear anything unfamiliar, or to embrace the familiar so that our home office agency may experience the most generous underwriting and the worst claim results of any of our agencies. There are numerous other instances of a tendency to approve the commonplace. From "general reasoning" it was long thought that the common occurrence of enlargement of the heart in conjunction with a heart murmur was a good sign, as representing good compensation. Actually, the enlargement indicated the seriousness of the heart damage.

Another kind of reasoning related to statistical reasoning is reasoning from a general background and a "statistic" of "one," or from extremely rough statistics. Thus, an insurance company inspector of new electric motors found metal chips inside one motor. He insisted on complete dismantling and found many more chips in six of the seven motors involved, any chip capable of causing a major breakdown. A life insurance statistical worker noticed that a policyholder who died of suicide in the second policy year had been paying seventy-five percent of his income for life insurance. This led to an investigation of the premium-income relationship on other suicide claims, and to new underwriting rules against speculation. Accumulations of statistics often commence with a thorough knowledge of one case, and then another case, and another, before the rules for gathering statistics are set forth. Sometimes, also, incomplete observations, folk lore, point to a truth. Workers in city morgues observed long ago that seven out of eight victims of lung cancer had the thumb and index finger of their right hands heavily stained with nicotine. The other one in eight was left handed.

FACTORS OF SAFETY

Applied mathematics is based in part upon pure mathematics and in part upon many variables which are not expressed mathematically. It is much more complex than pure mathematics, so much so that the "answer" involves expert judgment. The mathematical models used in applied mathematics are approximations: actuarial tables, statistical tables, probabilities assuming random fluctuations only, the average, the range. (What is the worst that can happen?) In engineering, a factor of safety is introduced to allow for approximations to the partly known; the structure may be two or three times as strong as simple mathematical models would indicate. In insurance, except when competition or rate regulation and the urge to expand get out of hand, we have our "margin for safety," "margins for contingencies," "profit margins," or safety factors in calculating credibility. The use of these margins is epitomized by the experience of Andrew R. Davidson, the historian of the Faculty of Actuaries in Scotland. To quote, with unimportant omissions, "But actuaries have always been practical statisticians; they are statisticians but also business men and, however nice the calculations, the men of that time did not forget to apply at the end an ample margin for contingencies when a contract was to be made. In the early part of this century, I can remember a certain chagrin when my official supervisor would add 25% or 33⅓% (his two favorites) to the figures I had ascertained with infinite pains."

This dramatizes the approximations involved in the use of mathematical models in applied mathematics, except that 25% or 33⅓% is not even of the same magnitude as the greatest safety factor used. Applied mathematics is important; it is complicated. No wonder, then, that Count Swedenborg in 1724 declined the chair of mathematics at the University of Uppsala on the ground that it was a mistake for mathematicians to be restricted to pure mathematics.

THE BUSINESS DECISION

Those involved in deciding how to apply statistics include members of state rate regulatory bodies, insurance company presidents, sales vice presidents and statistical experts or near experts, such as actuaries and home office underwriters, brokers or salesmen of individual cases, and, in the case of government "insurance," government executives and administrators, legislators and various classes of voters. All of these bring to the problem their own viewpoints, their own financial interests, their own desires, knowledge and background, their own bargaining abilities and deficiencies, their own emotional involvements and their own concepts of equity and acceptability. The actuary or other statistical expert who has command of the statistical information has the task of dealing with all of these people with their many diverse human qualities and with their specialized knowledge and abilities, and they, in turn, have the task of dealing with him. The task involves understanding the individuals, understanding the sales and acceptability facts, and the realization that there is no easy road to obtaining and analyzing the facts involved in the sales-acceptability and public relations aspects of applying insurance statistics. Sales and public relations aspects are not often investigated as thoroughly as are claim statistics, and we are often dependent on mere opinions of probable effects on sales, colored by very recent experiences with very few cases. Often the claim statistics are the only statistics thoroughly analyzed and recorded in writing. In deciding how to use insurance statistics, not only cold logic and judgment but human emotional reactions are involved – not just in "the decision," but in making the decision as acceptable and as effective as is practicable.

A fundamental difference in individual viewpoint is that for some their background is such, in sales for example, that one "success" in ten attempts leads to an overall success in their major activities. For a particular company decision being made, perhaps one profitable venture in three decisions will lead to an overall profit. On the other hand, some individuals will fear the prospect of even one unprofitable venture in a great number. Nor can people be wholly objective about this. The man who fears one failure is likely to speak of a one in two situation as a one in ten shot. The man who is accustomed to operate successfully with one success in ten attempts is likely to evaluate the probable effect of every likely looking product or rate reduction as a "sure thing," and to think that the power of positive thinking can keep loss ratios down. Perhaps each should continue his own way of thinking in order to offset the bias of each other.

In the application of statistical data, a major portion of the responsibility rests upon the actuary or other statistician who presents the statistical data. Often the statistician is in command of most of the hard facts: the statistics, including related statistical data, probable causes, and the human angles involved. His recommendations will loom large in the minds of others in the future and this may be especially true if the results go awry whether because of unfavorable action or because inactivity seems to be the cause of a lost opportunity to grasp affairs at an optimum time. Often the statistician has a veto or nearly so on the development of new coverages and the changing of other coverages involved in sales expansion. His positive suggestions are listened to with respect. His negative suggestions are sought, or feared. His written and spoken summaries and individual illustrations of the facts are the most important factors in the minds of many as they make their decision, especially his written reports. (When King Ahasueras could not sleep, he commanded the records to be read, and so Mordecai was remembered and honored.)

A business decision involves a weighing of the probabilities and of the stakes involved in various courses of action. The most difficult stakes to weigh are the stakes of goodwill, of respect, and of cooperation; these stakes can be exaggerated or they can be underestimated, or even evaluated in the wrong direction.

Business decisions are not made just at the apparent point of decision. Making a business decision is a developmental process. Pertinent statistics, statistical background and statistical viewpoints are involved at all the stages of developing the ideas and making the decision – the final judgment – not just in the conference room, not just in direct business discussions, but also in informal conversations under various circumstances. The character and characteristics of those making the decisions are among the important factors in decision making – and so is the knowledge that each person has of the characteristics of the others, primarily through personal contact, but including also the statistical knowledge of the characteristics of the various classes of persons to which the individuals involved belong. It is vital to know with whom we are dealing among policyholders, agents and brokers, and the makers of formal decisions.

Kipling wrote: "I keep six honest serving-men (they taught me all I knew). Their names are What and Why and When and How and Where and Who." Insurance statisticians know the importance of "When." Everything is a time series. Our thought is ever "What of tomorrow?" and, also, "Can we beat our competition to the punch?" Rates often vary by geographic area. We know that "Where" is important. We are constantly searching for causes; Why? Why? But, pervading all the other "serving men" of What and Why and How and Where and When in insurance statistics – pervading all our decisions and all the effectiveness of our application, pervading insurance itself, all pervading in our work is the human factor – Who buys? Who sells? Who repairs? Who administers? All insurance involves human beings. Insurance is sold by human beings, sold to human beings, on products built and operated by human beings (or on human beings themselves), for statistical classes into which they are sorted by other human beings, for insurances which are administered by human beings with resulting profits and losses which are studied by human beings, including actuaries and other statisticians, and so on, all involving human beings and things controlled, in part at least, by human beings – except the weather, and we are even talking about controlling that. The result is that, for us at least, it is true that "the proper study of mankind is Man." But, that is just "study." For the end result we turn to Mr. Micawber. "Annual income twenty pounds, annual expenditure nineteen pounds six, result happiness. Annual income twenty pounds, annual expenditure twenty pounds ought and six, result misery."

CONCLUSION TO PART ONE

The science of statistics is based on similarities and on differences. Similarities lead to classes. Differences lead to sub-classes, to frequency distributions and to individuals – unique individuals, persons. Insurance statistics is in the class of human statistics and in the sub-class of business statistics. Insurance statistics of various kinds have their own distinct characteristics; each statistical study has its individual characteristics. One important characteristic of insurance statistics is change; who knows what tomorrow holds, except change? A sudden change like October 1929, or a mathematically smooth change? The application of insurance statistics to insurance operations involves the vital operations of an insurance business; sales, profit making, and the ability to provide the benefits which have been promised. So important are our statistics and our profession that actuaries, quite as a matter of course, appear before and are a part of boards of directors and governmental committees – so important, that the words "actuarially sound" have been used as part of the presidential vocabulary. Actuaries and other insurance statisticians belong to that class of individuals, purveyors of truth, of whom King Solomon wrote in The Book of Proverbs; "Seest thou a man diligent in his business? He shall stand before kings."

PART TWO: MATHEMATICAL MODELS

All applied mathematics is based upon mathematical models and upon approximations implicit in mathematical models.

In sociological and economic sciences, the approximations are very great, especially in the vast majority of cases in which the applications involve the future with all its complexities and uncertainties. For this reason, perhaps, we are often quite content with approximate but easy to apply developments of our mathematical statistical formulas.

Mathematical formulas are more acceptable than free hand or graphic methods partly because free hand and graphic methods more obviously involve personal judgment and thus are more obviously subject to slanting, whether deliberate or not. Thus, even when a supervisory statistician would trust his own judgment in free hand smoothing or in graphing, he sometimes delegates the work as a formulary application, and, when nontechnical men are in a position of responsibility, or when any conflict of interest is involved, the mathematical formula method often becomes a necessity.

All probability and all applications of statistical data are based on partial ignorance. If we knew just how a pair of dice were imperfect, were held, and thrown, and blown, and how the surface on which they bounced reacted, we could predict from tried and true engineering formulas just how the dice would fall. If we knew more about each insurance risk than we do know or even than it is at all practical to determine, we could rate each risk better, and we could build a foundation of statistics which would enable us to rate each risk still better, until, in the ultimate we could predict the actual event insured against so that savings would replace insurance as a means of mitigating the "risk," provided, of course, that our understanding and our knowledge were both built up far beyond the present ability of mankind to know, and to use knowledge.

Our mathematical formulas, per se, assume an ignorance of "major causes." The formulas deal with the assumptions of random fluctuations arising from a multitude of "minor causes" treated in our mathematics as though we did not know the causes. To the extent that we know of probable "major causes," we give less weight to the mathematics of ignorance of causes. On some small insurance classes, such as employees of one employer, we are quite likely to "know" major causes – degree of carefulness in choosing, in training, and in supervising truck drivers, and in providing them with safe vehicles, for example – and to give much weight to such knowledge, especially when it is borne out by correspondingly good or correspondingly poor experience. In a mathematical sense, we put more trust or less trust in the hypothetical claim rate which we use in our purely mathematical calculations, depending upon the extent of our knowledge of the risk. Thus the mathematical "credibility" of poor experience on a small risk often leads simply to an underwriting review of the risk which completely confirms the experience so that 100% "credibility" (in the ordinary sense) replaces 50% or 60% credibility in the mathematical sense.

Since statistical applications are founded upon probability, which is based upon partial knowledge and upon partial ignorance, we have a continual conflict between "good enough" approximations in the mathematical models and somewhat expensive improvements in the models to meet the statistical facts as we see them – "through a glass darkly."

Insurance statistical mathematics is based, in technical terminology, upon frequency and upon variance. Thus we are interested in the frequency with which automobile claims occur and in variance in the size of those claims. In nearly all cases a priori considerations lead us to a numerical probability to be tested as a hypothetical basis for frequency. For variance in severity of claims or occurrence, our bases are a distribution curve which a priori (a priori, and partly on the basis of our general knowledge as well as upon the mathematics of small causes) we would usually expect to be reasonably smooth, but which may have almost any general shape. The third dimension is time. We are always cognizant of the fact that time changes things; and time changes people. In addition, multiple characteristics make multiple sub-classes possible.

Insurance statistical mathematics is based primarily upon the theory *that any insurance statistical class is a sample of a universe.* When the entire earthly class corresponding to the specific insurance class is not large enough to be treated mathematically as a universe, the concept is of enlargement to such a universe by increasing the length of homogeneous time so that our experience hypothetically could enlarge itself to the dimensions of a universe. Secondarily, our sampling theories are for a cross section sample of an insured class, which is a sample of a limited class, the composition of which is changed as each item of the cross section sample is removed from the portion remaining to be sampled.

ELEMENTARY MATHEMATICAL STATISTICS

Elementary mathematical statistics is developed from pure probability theory in a number of text books. The student should choose a text which appeals to his method of thinking; the step by step inductive approach – or the deductive explanation from the formulas of permutations and combinations, whichever suits his fancy. For insurance statistical mathematics, a mechanical model of many sided symmetrical toy logs is superior to the usual model of coins or of six sided dice (cubes). Thus a log with 1000 sides (and two ends) can represent probabilities per thousand. One side can be labelled to show claims with a frequency of one in a thousand for, say, claims of \$100.00. For claims of \$150.00, or thereabouts, the frequency may be two in a thousand – represented by two sides of a log. The following insurance example presupposes a familiarity with the elementary mathematics.

CROSS SECTION OF EXPOSURES WITH ALL OF CLAIMS – POISSON-HYPERGEOMETRIC

Sampling of exposures deserves special attention. Suppose our attention is focussed on a sub-class – "sub-class A" in an automobile insurance experience. This could happen in several ways. For example:

1. Someone reviewing the distribution of claims against his statistical background may think claims in sub-class A are relatively frequent; or,
2. Certain sub-class A claims may arouse suspicion because of the nature of an accident causing a claim, or because of the method of presenting a claim.

If the claims are apparently frequent, random fluctuation may be a plausible explanation – some sub-class is likely to turn up with an unusual number of claims because of random fluctuation; it may easily just happen to be sub-class A. The whole procedure involves searching for causes and the use of statistical judgment. If sub-class A individual claims looked suspicious, we are already on the trail of a possible cause. In either event, the presumed degree of reliability of the original classification, together with commonplace mathematical models, form a basis for statistical judgment, when and if we know the number of exposed, as well as the number of claims, in sub-class A. Suppose, however, that obtaining the pertinent data on all the exposures is expensive, and that we use a sampling process as illustrated by the following imaginary example.

Suppose that an examination of the claims of sub-class A, which is included in an entire class of young male drivers, gives us cause for suspicion, e.g., the sub-class of non-graduates of high school or grade school who are between 20 and 25 years old and whose automobiles are at least four years old. Suppose we now engage in some research which is expensive enough to limit the number of exposed which we analyze.

As a first step, we distribute the claims and find that there are 15 claims on the sub-class under suspicion. Our information is now as follows, including the date for the "Entire Class."

	Exposure	Claims	Rate
Entire class – young male drivers	3,000	180	0.06
Sub-class A	Unknown	15	Unknown

As the next step, both because we may find something new in the process and also so that we can keep others informed who will enter into any decisions involved, we take a 1% sample of exposures, i.e. 30 of the 3000. The result is as follows:

	Exposure	Claims	Rate
Entire class	3,000	180	0.06
Sub-class A	Unknown	15	Unknown
Sub-class A in 1% sample of 30 from 3000	2	not pertinent	
100 $\times$ 1% sample	200	not pertinent	

A rough idea of the loss rate in the sub-group under suspicion is 15 divided by 200, or 0.075. Additional samples are taken which produce the following in the sub-class, including the first sample — 2 — 0 — 3 — 2 — 1 — 0 — 1 — 0 — 1 — 2 for a total of twelve in a ten percent sample. We now decide to stop, and have the following information:

	Exposure	Claims	Rate
Entire class	3,000	180	0.06
Sub-class A	Unknown	15	Unknown
Sub-class A 10% sample of 300 from 3000	12	not pertinent	
Total in sub-class A	120 (est.)	15 (Tot.)	0.125 (est.)

The hypothesis of random fluctuation in the claims, for this experience segment which produced 15 claims, is the Poisson series. To test the cross section sample result, we may also set up the hypothesis of 250 exposed in sub-class A, corresponding to 15 claims and to the related further hypothesis that the true claim rate for sub-class A is the entire class claim rate of 0.060. For this test of the exposure hypothesis we use the hypergeometric probability function for 3,000 exposed containing 250 of sub-class A, from which a 10% cross section has produced 12 in sub-class A. The two series, the Poisson and the Hypergeometric, may be viewed separately as a basis of judgment.

On the other hand, the two series may be combined by joining the terms in which the probabilities of indicated claim rates are within a range of claim rates arising from the combined hypotheses. The term in the combined series for 8 claims and 6 in the 10% cross section of exposures "indicates" an experience claim rate of $8/60$ or 0.133. This is 221% of the hypothetical claim rate of 0.060. The term in the combined series for 16 "claims" and 12 in the cross section of 3,000 also "indicates" a claim rate of 0.133, or 221%. Since $8/6$ and $16/12$ each "indicate" 221% of the hypothetical 0.060 rate these terms could be combined, and also combined with other nearby terms, "indicative" of, say, 210% to 230% of the hypothetical 0.060 rate. The resulting combined Poisson-Hypergeometric series is illustrated by the term involving 8 claims and 6 sub-class A exposures in the 10% cross section of 3,000 exposures, as follows.

$$\frac{e^{-15}\ 15^{8}}{8!}\ {}^{300}C_{6}\left(\frac{(2{,}750 \cdot \cdot \cdot 2{,}457)\ (250 \cdot \cdot \cdot 245)}{(3000 \cdot \cdot \cdot 2{,}701)}\right).$$

The problem is of the general class, fluctuation in ratios, which has been solved by the Student t, but the normal curve component of the t is not suited for our small numbers and small claim rates. The foregoing mathematical model, while difficult to calculate, is directly suited to this double uncertainty problem arising from obtaining all claims and a cross section of exposures in an insurance experience. A corresponding double Poisson development would be a more approximate but simpler mathematical model.

The sampling and the research might be extended still further. Suppose the claimant drivers are shown a motion picture in which a driver makes a left turn across the path of another vehicle (which should slow down slightly to avoid a collision); In which the driver of the straight-ahead vehicle also fails to slow down; And a crash occurs. Now suppose that a blood pressure chart is kept of the fifteen claimants, and of twelve other drivers in the 10% sample and that 14 of the 15 show a violent upturn in blood pressure at the time when the left-turn driver starts to turn; while the twelve in the 10% exposure sample happen all to be non-claimants, and they show a similar reaction at the time when it becomes evident that the straight-ahead driver has dangerously delayed slowing down and should therefore be applying his brakes. We suspect a dangerously antagonistic attitude, a causative factor, on the part of the fourteen drivers who reacted violently to the interfering left turn.

We have these data: fourteen of fifteen claimant drivers fail the motion picture-blood pressure test. None of twelve non-claimants tested fail. The analogy is to a sample of an industrial product, with 14 of 15 indicated failures by a test of one production process and no indicated failures in 12 produced by the same test of another production process.

UNWIELDY FORMULAS AND THE PRACTICAL SOLUTIONS

Easily understood but unwieldy formulas, which are understandable mathematical models, are replaced by more tractable but less basic formulas, or by approximate formulas, in our operations. The Bayesian is replaced by a single hypothesis – or by separate consideration of two hypotheses. The complicated formula yields to the Monte Carlo technique, but not without difficulty of its own. The binomial with two parameters implying a volume of tables or an extensive calculation is replaced by the Poisson with only one parameter. The χ^2 test is applied instead of a slightly more complicated formula which could cut down on a bias in its results. The multinomial with amounts and probability is replaced by a binomial or Poisson and a separate consideration of the curve of amounts of claims. A varying homogeneity distribution which would produce several Poisson curves to be dealt with is fitted into a Type III curve to produce the single curve, the negative binomial. All of these represent simplifying the application of our mathematical models and, sometimes, complicating the underlying mathematical philosophy and, sometimes, also, an increase in the degrees of variance from the actual data, in choosing the hypothesis which we test mathematically. The data involves humans and time and changes, so that most of the approximations involved are fully justified because they are overwhelmed by the approximations inherent in our applications.

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