The average absolute difference equals

$$\frac{\sum_{ijkl} n_{ijkl} \left| r_{ijkl} - x_i - y_j - z_k - w_l \right|}{\sum_{ijkl} n_{ijkl} r_{ijkl}}$$

The chi-square is proportional to

$$\sum_{ijkl} \frac{n_{ijkl} (r_{ijkl} - x_i - y_j - z_k - w_l)^2}{x_i + y_j + z_k + w_l}$$

Setting the average difference for the ith occupancy equal to zero and solving for x_i we obtain

$$x_i = \frac{\sum_{jkl} n_{ijkl} (r_{ijkl} - y_j - z_k - w_l)}{\sum_{jkl} n_{ijkl}}$$

and similarly for y_j, z_k and w_l.

If the factors are some combination of cents and percents, or are based on some other relationship, appropriate formulas can be set up.

DISCUSSION BY JAMES R. BERQUIST

Mr. Bailey's latest paper is, indeed, a timely contribution to the proceedings of our Society. Timely, not only because it provides a method of calculating rates with minimum bias, but also because it provides ideal computer application. Without the aid of a computer the method is, in fact, impractical.

The technique presented in the paper bears careful study by every ratemaker who has the task of calculating territorial or class differentials, and what ratemaker doesn't? Mr. Bailey's technique is designed to calculate the differentials which provide the best "fit" of the data. He solves for each of the various differentials by setting what he defines as the average difference equal to zero, then, by successive approximation he arrives at the set which provides the best fit.

Mr. Bailey goes on to provide an outline of a method of testing the resultant differentials, or "estimators" for minimum bias. The advantage of this system over the systems presently in use is that the differentials so calculated will yield rates which are most nearly correct for, say, "small brick buildings" as well as small buildings in total and brick buildings in total.

It is interesting to note the similarity between this method and "Method 2" advanced by Bailey and Simon in "Two Studies in Automobile Insurance Ratemaking," *PCAS,* Vol. XLVII, which, I believe, should be read in conjunction with this paper.

The equation for x_i, for example, using "Method 2" is

$$x_i = \left[\sum_j \frac{n_{ij} r_{ij}^2}{y_j} \Big/ \sum_j n_{ij} y_j \right]^{1/2}$$

while the comparable equation advanced in this paper would be

$$x_i = \frac{\sum_j n_{ij} r_{ij}}{\sum_j n_{ij} y_j}$$

The following tables show the results of applying the "Minimum Bias Method" to the data presented in that earlier paper.

Table 1 shows the rate relativities produced by this method. Table 2, which compares to Table D on page 16 of "Two Studies in Automobile Insurance Ratemaking," shows how close the combination of the Minimum Bias relativities are to the combination of Method 2 relativities.

TABLE 1

COMPARISON OF RELATIVITIES*

		Customary Method	"Method 2"	Minimum Bias Method: First Calculation	Minimum Bias Method: Second Calculation	Minimum Bias Method: Third Calculation
	x_1	.863	.881	.872	.868	.868
	x_5	1.154	1.161	1.143	1.144	1.143
CLASS	x_3	1.313	1.309	1.288	1.290	1.290
	x_2	1.372	1.367	1.341	1.345	1.345
	x_4	2.269	2.125	2.050	2.089	2.090
	y_1	.895	.906	.918	.919	.919
MERIT	y_2	1.174	1.113	1.129	1.128	1.127
RATING	y_3	1.277	1.215	1.232	1.232	1.232
CLASS	y_4	1.610	1.462	1.486	1.481	1.481

*Source: Tables A, B and C "Two Studies in Automobile Insurance Ratemaking," *PCAS,* Vol. XLVII.

TABLE 2

RELATIVE LOSS RATIOS
Minimum Bias Method — Third Calculation*

i/j	1	2	3	4
1	.798	.979	1.069	1.286
5	1.050	1.288	1.408	1.693
3	1.186	1.454	1.589	1.910
2	1.236	1.516	1.657	1.992
4	1.921	2.355	2.575	3.095

*Compares to Table D.

A fresh numerical example would have aided considerably in understanding the paper, however, after calculating the above "simple" tables, this reviewer now realizes why the author decided against it.

Mr. Bailey is to be congratulated for his generous contributions to our *Proceedings*.

DISCUSSION BY STEPHEN S. MAKGILL

Mr. Bailey has again contributed significantly to our *Proceedings* with the ideas presented in this paper. The ratemaking technique suggested is designed to utilize to the fullest the predictability inherent in the data of each subdivision created by a multiple classification system. Mr. Bailey accomplishes this maximum utilization by producing all sets of adjustments, or relativities, simultaneously. These adjustments may be either cents or percents or a mixture of both, whichever is indicated by tests for minimum bias. Such a technique represents a significant improvement over the common practice of determining percentage relativities for the divisions of each classification, the appropriate relativity from each class then being applied one on top of another to arrive at the final adjustment for a subdivision.

The requirement of complete reliability of the data for each division of each category imposes a certain limit on the applicability of the method as presented, for it sets a substantial minimum to the volume of experience necessary. This points to the necessity of ensuring that all the rating criteria used are contributing significantly to predictability. By eliminating those that do not so contribute, the volume of experience required may be decreased appreciably. The field of meteorology particularly has made great strides in developing screening methods that might well be adapted to our needs in this area.

Mr. Bailey's iterative method of calculating a set of estimated rates that are unbiased in the aggregate seems rather unwieldy, even for computer operations. Improving these techniques offers a highly worthwhile field for further investigation.

The tests for minimum bias described appear most appropriate, and Mr.