arranged in such a fashion that the preponderance of risks do not cluster at either the top or the bottom limits. Moreover, there must be a relative consistency in the charge for similar hazards from risk to risk.

The Fire Insurance Industry has been operating this type of risk rating plan for many years with obvious success. Probably few of the schedule raters would recognize the association of their work with the McIntosh equations but it is believed that both are reducible to a common philosophy. Mr. McIntosh states in his paper that the fire insurance underwriters do not explicitly rate risks according to his mathematical model. Moreover, he observes that it would be impossible for them to do so at the present time. Basically Mr. McIntosh is saying that the limits which the rater must observe in scheduling his debits and credits are akin to a problem in linear algebra.

The reviewer does not believe that Mr. McIntosh offers his paper as an irrefutable proof of the mathematical basis for fire rates. Actually the value of fire schedule rating has been substantiated in many ways. Of prime importance, each assured is guaranteed that his rate takes into consideration the hazard presented by his own property as measured according to the impartial evaluation of the bureau engineer. The merit of the schedule rating approach is also proved by the fact that the insurance companies have been able to render vital protection to the public and at the same time have been able to segregate sufficient funds as a guarantee that future losses will be paid despite the magnitude of the conflagration hazard.

Lest there may be some misunderstanding of the role played by the fire insurance schedule, the reviewer would probably like to have the record straight. The schedule rate in itself is not susceptible of mathematical demonstration by insurance loss statistics. There are many students of the business who sincerely believe that no such attempt should be made.

This is not to say that fire insurance rates are not responsive to actual loss experience. The overall fire rate level is determined by comparing the actual experience with the balance point loss ratio. Once the rate level has been set for fire insurance within the State, the local rating bureau reviews the National Board tabulations of classified experience to see which types of business should be subject to rate modifications. In a way the schedule might be viewed as the means of distributing the State rate level as modified for the classification experience to the individual risk.

The Society is indebted to Mr. McIntosh for writing a paper on advanced mathematical techniques which have had wide application in many fields other than insurance. It may well be that the Society's "Special Committee on the Mathematical Theory of Risk" will indicate areas of possible application which will encourage other members to undertake writing valuable papers in this potentially important field.

AUTHOR'S REVIEW OF DISCUSSION

KENNETH L. McINTOSH

Mr. Dropkin's emphasis on the particular method of transforming the initial equations is a little bewildering, granted that in the immediate instance the method he suggests would have been a better choice than that which was

used.[1] Not only does slide rule accuracy seem sufficient for present purposes since final results here are rounded to two figures, for a number of reasons the retention of more than two significant figures in any final rate is mathematically meaningless though sometimes necessary for practical reasons.[2] In any event, transformation of the initial equations is merely an unavoidable chore, preliminary to the main development. The term "conventional methods" was intended to embrace any and all of the several techniques to be found in standard texts, leaving open the specific choice in any specific case.

The critical problem here is to develop certain significant properties, not of the initial system of linear equations, but of the system of inequalities to which these equations are converted following transformation once the condition of non-negative charges has been imposed. The easiest (if not the only) way to develop these properties is by means of certain concepts and techniques associated with certain areas of set theory, which are relatively simple of application (at least as simple of application as is the negative binomial) but which are almost impossible to explain in ordinary algebraic terms. Actually, the fire schedule problem is non-linear, though here it has been reduced arbitrarily to linear form for illustrative purposes. In the general case it can always be reduced to linear form in practice, but only at the cost of introducing an unwieldy number of additional parameters not required for the non-linear solution.

Mr. Hurley's practical maximum and minimum stop rates, as well as the practical distribution of rates within the rate range of the class, carry beyond the intended scope of the presentation. Since these points have been raised, however, the stop rates merely furnish additional equations with mathematical properties identical to those of the rate level equations and with identical parameters. These can be and should be grouped with the rate level equations to form a combined system initially, and the development then proceeds exactly as with the rate level equations alone. The practically determined rate distribution will also give additional equations, though in any given case it may be expedient to treat these separately rather than to incorporate them into the initial rate level—stop rate system.

The development as presented assumes a pre-determined set of target class rate levels obtained by unspecified methods from unspecified data. From these rate levels, limits to schedule charges are derived mathematically. If the "assumption" of class rate levels is discarded and these are derived mathematically from loss experience, the charge limits then would seem to become functions of that same loss experience. That limits to the schedule rate are functions of any existing limits to schedule charges is obvious, and the mathematical chain from loss experience through rate level through schedule charge to schedule rate seems complete. Practical considerations preventing rigorous application hardly seem to impair the theoretical rigor of the mathematical development.

To keep the record straight, however, let it be emphasized that the theory

[1] The method of Gauss was used. See M. G. Salvadori and M. L. Baron, *Numerical Methods in Engineering.* Prentice-Hall, Inc. (1952). P. 17; or V. N. Fadeeva, *Computational Methods of Linear Algebra.* Dover Paperback #S424. (1959). P. 65.

[2] The author knows of one insurance manager who must retain nine decimal places in all rate calculations solely in order to reconcile certain premium accounting balances which he must prepare.

here presented, even if extended to include derivation of the class rate levels from loss experience, is *not* proposed as a substitute for judgment in the fire rating process. It is hardly an original observation to state that so long as insurance loss remains a random variable and certain practical requirements must be met, judgment cannot be eliminated from any rating process. If the theory proposed has any practical value it will be first as a guide to judgment, second as a possible method of eliminating a certain amount of false starts and lost motion in the preparation or major revision of a rating schedule once it has been decided exactly what specific requirements that particular schedule must meet. Which, in final essence, is all that this or any other rating "theory" ever can accomplish.

OBSERVATIONS ON THE LATEST REPORTED STOCK INSURANCE COMPANY EXPENSES FOR 1960

BY

FRANK HARWAYNE

Volume XLVIII, Page 109

DISCUSSION BY SEYMOUR E. SMITH

I found Mr. Harwayne's paper quite interesting in that, while aware of the fact that there are wide variations in actual expenses by individual company, this is the first time that I have seen, in exhibit form, average expenses by premium volume and also the range of lowest and highest. To a certain extent I suspect that the difference in expense ratios between the large and small volume writers may be affected by the percentage of large risks to volume. However, this distortion should not affect the basic differences shown in Mr. Harwayne's exhibits.

Without in any way detracting from the importance of expense requirements as such, my own feeling is that expense ratios are merely one facet in the overall picture of profitability. Individual company management decisions as to policy can have quite an effect on expense ratios. For example, in certain lines of insurance a company may decide that improved profitability could result from higher levels of expenditure for such individual functions as underwriting, claim, engineering and inspection, etc. Out of curiosity, from the same source used by Mr. Harwayne, an exhibit has been prepared showing underwriting profits for a number of lines by size of company. The company size is based upon the premium volume for the individual line. There is apparently a definite relationship between the higher expense ratios noted by Mr. Harwayne for the smaller sized companies and the profitability for these same companies, as will be noted in the exhibit. The average underwriting gain for the smaller sized companies is considerably below the average for all stock companies combined. However, when considering companies, averages can be very misleading. As will be noted from the figures for the high and low profit company in each group, some small companies manage to conduct a very profitable operation and some large companies have sustained substantial underwriting losses.