DISCUSSION BY ROBERT A. BAILEY

Mr. Simon's paper is indeed thought-provoking and raises many questions for further study. His study of the relationship between size, strength and profit is thorough and is based on data painstakingly compiled in such a way as to eliminate the many shortcomings that so often characterize the data used in studies of profit by size of company.

As he mentions, big companies must have some advantage over smaller companies or else big companies would not allow themselves to get big. But he shows that the advantage from size alone is small and that other factors are more important. It is evident that small companies are able to succeed from the very fact that all big companies grew out of small companies that succeeded. We should take to heart Mr. Simon's point that rather than be so concerned with the plight of the small company we should be more concerned with the weak company, and that it is the small policyholders that need to be protected, not the small companies.

We all realize that the relationship between size and profit is weak but as yet no attempt has been made to analyze the exact nature of this relationship. I believe it would be a valuable addition to Mr. Simon's important paper to try to develop a mathematical model for the relationship between size and profit, a model which recognizes other important factors in addition to size as suggested by Mr. Simon. Such a model would enable us to determine how much correlation we might expect.

It is impossible to derive the exact nature of the relationship between size and profit because the data at hand is limited—limited by the fact that there are only 180 company groups with $10,000,000 of premium or more. However, using general reasoning together with the data in Mr. Simon's paper, I would like to propose a formula and then compare the coefficients of correlation derived from the actual data by Mr. Simon with the expected values derived from the formula.

Let us assume that the profit ratio is the net result of three elements

1. a factor reflecting the type of company or the type of management
2. a factor proportional to the premium size and
3. a factor reflecting the purely random variations in the loss (and even expense) ratios.

Let us express these three elements as follows:

$$P = A + BS + U_s$$

where P is the profit ratio expressed as a percent

S is the size in millions

U_s is a random variable, mean $= 0$, variance $= C/\sqrt{S}$ for each S

A, B, C are constants for each type of company or type of management.

The variance of U_s would be expected to be C/S in the usual statistical applications since U_s is the ratio of the profit to the size. But a large company is not the same as the sum of several smaller companies because a larger company will accept larger risks and uses different reinsurance arrangements. Because of this, the variance of the profit ratio decreases more slowly than in proportion to 1/S. I have arbitrarily chose $1/\sqrt{S}$ because it produces reasonable results as pointed out later.

Now let us assume we take an infinitely large sample of companies all of the same type, and let us also maintain exactly the same distribution by size as in each of Mr. Simon's groups. What will the correlation coefficient be in terms of A, B and C?

$$r_{PS} = \frac{\dfrac{\sum PS}{n} - \dfrac{\sum P}{n} \quad \dfrac{\sum S}{n}}{\sqrt{\dfrac{\sum P^2}{n} - \left(\dfrac{\sum P}{n}\right)^2} \quad \sqrt{\dfrac{\sum S^2}{n} - \left(\dfrac{\sum S}{n}\right)^2}}$$

Since A, B and C are constants,

$$\frac{\sum P}{n} = A + B\frac{\sum S}{n}$$

$$\frac{\sum PS}{n} = A\frac{\sum S}{n} + B\frac{\sum S^2}{n}$$

Obtaining the variance of P by parts (the sum of the variances within each size plus the variance between the sizes), we obtain

$$\frac{\sum P^2}{n} - \left(\frac{\sum P}{n}\right)^2 = \frac{1}{n}\sum\frac{C}{\sqrt{S}} + B^2\left[\frac{\sum S^2}{n} - \left(\frac{\sum S}{n}\right)^2\right]$$

Substituting and simplifying we obtain

$$r_{PS} = \frac{\sqrt{B^2\left[\dfrac{\sum S^2}{n} - \left(\dfrac{\sum S}{n}\right)^2\right]}}{\sqrt{\dfrac{C}{n}\sum\dfrac{1}{\sqrt{S}} + B^2\left[\dfrac{\sum S^2}{n} - \left(\dfrac{\sum S}{n}\right)^2\right]}} = \frac{1}{\sqrt{1 + \dfrac{\sigma U^2}{B^2\,\sigma S^2}}}$$

It can be seen from this formula that the correlation coefficient would equal 1.000 if $C = 0$, that is, when there is no random variation in the profit ratios. To the extent that C is greater than zero, the correlation coefficient will be less than 1.000.

Now let us let $C = 79$. This gives us a standard deviation of the profit ratio of $\pm$ 5 percentage points for $S = 10$ million and $\pm$ 1.6 percentage points for $S = 1000$ million. These values are about what we would normally expect. Let us also assume that $B = .005$. This means that a 1000 million dollar company would have about 5 percentage points more profit than a 10 million dollar company.

With these assumptions we would obtain the following expected results corresponding to the actual results obtained by Mr. Simon for each of his seven groups. (Mr. Simon kindly furnished me the data necessary to calculate $\Sigma 1/\sqrt{S}$. All the other necessary data is included in his paper.)

Group	$\Sigma 1/\sqrt{S}$	Correlation Coefficient Actual	Correlation Coefficient Expected
1	7.40098	.194	.199
2	2.38759	.489	.252
3	8.44241	.108	.096
4	13.98084	.104	.107
12	9.78857	.211	.218
123	18.23098	.042	.180
1234	32.21182	.052	.154

The actual correlation coefficient for any group would vary either up or down from the theoretical expected value because of the limited number of companies in each group and the resulting lack of steadiness. Furthermore, as more and more groups are combined into one big group, the assumption that the constant A in the formula is constant becomes less valid and the actual correlation would tend to be smaller than the expected value. This is what Mr. Simon meant when he said, "There are times when a true correlation will be masked if two dissimilar groups are thrown together." The effects of this can be seen by comparing the actual and expected values for Group 123 and for Group 1234.

There are undoubtedly other formulas which would produce expected values just as close to the actual values. The formula proposed in this review is only one of many possible ones and was selected on the basis that it was simple, reasonable and consistent with the data available. A larger volume of data would be required to test how accurately the proposed formula describes the relationship between size and profit.

It is hoped, however, that the proposed formula will provide a framework within which we can further Mr. Simon's important contribution toward evaluating objectively the relationship between size, strength and profit.

DISCUSSION BY CLYDE H. GRAVES

In summarizing his study "Size, Strength and Profit" Mr. Simon stated, "Within the limits of the study, we find that no meaningful relationship exists between the premium size of a company and its profitability or between the premium size of a company and its strength as measured by the ratio of surplus to net premiums written."

I believe this statement will come as a surprise to many as I confess it did to me. I think of the Allstate, State Farm, Nationwide, Travelers, Aetna, Hartford, Liberty Mutual and Insurance Company of North America as large companies, making large profits and being towers of strength, and it comes as a shock to learn that there is no meaningful relationship between premium size and profitability, nor between premium size and strength. The shock was so great that I even calculated some coefficients of correlation myself to check on Mr. Simon's statement.

One item in the expense provisions which I felt would have a definite relationship to size was "general expense." The larger the company the smaller would be the ratio of general expense to premiums. I used the 1961 Loss and Expense Ratios published by the New York Insurance Department and calculated the correlation between "X" and "Y". With "X" representing net