tingent lives. In dealing with benefits to survivors under a Workmen's Compensation Law where the survivors normally are the widow and children of a deceased workman, the assumption of the independence of the force of mortality among the members of the same family is open to serious question. While I did not raise this question in the paper itself, I did mention it at the original presentation of the paper in Washington last November. I do think this question of independence or dependence must be resolved before any further practical use is made of the actuarial model created in my paper.

The author of the paper under discussion is unable to resist one further comment which is in the nature of speculation. I believe that we are on the threshold of a major revision in the theoretical approach to the general subject of life contingencies.

I would like to express my appreciation to Professor Mayerson for his time and effort in presenting his review, and I would like to express the hope, which is probably common to every author, that this paper will lead to further study in this field.

THE NEGATIVE BINOMIAL APPLIED TO THE CANADIAN MERIT RATING PLAN FOR INDIVIDUAL AUTOMOBILE RISKS

BY

CHARLES C. HEWITT, JR.

Volume XLVII, Page 55

DISCUSSION BY O. D. DICKERSON

Mr. Hewitt's interesting paper carries on the discussion of automobile rating plans which consider the accident, conviction, claim and/or fault, experience of the auto and its drivers. There is a lack of general agreement whether such plans properly should be classified as individual risk rating plans or as extensions of the classification system.[1] The Canadian plan, to which Mr. Hewitt refers specifically, is designated as a "Merit rating plan"; the European plans are referred to as "no loss bonus" plans; and the bureau plan in the United States bears the hopeful appellation "Safe Driver Insurance Plan". By whatever name called and however categorized, such plans have been the subject of much current discussion and many papers.[2]

Recently the negative binomial distribution has become popular as a model to describe the theoretical distribution of accidents (convictions, claims, or

[1] *See, e.g.:* Kulp, C. A., *Casualty Insurance*, 3rd ed., New York: The Ronald Press Co., 1956, pp. 513 & 515-516; Simon, LeRoy J., "Myths and Mysteries Concerning the Actuarial Soundness of Merit Rating", paper presented to the Casualty Actuaries of Philadelphia, Sept. 7, 1960.

[2] Mr. Hewitt's footnotes cite most of these; the footnotes to this discussion cite a number of others.

accidents involving fault) by number for a single exposure unit during a finite time period. Mr. Hewitt carries on this discussion, deriving formulae for forward claim frequencies for the various classifications of the Canadian plan, and compares his theoretical frequencies with the actual experience for policy years 1957 and 1958 (developed through June 30, 1959). He implies that the fit is reasonably good but does not perform any statistical tests of goodness of fit.

It is difficult to criticize Mr. Hewitt's paper for what it contains. The mathematics seem correct and the fit looks plausible. It is for his sins of omission rather than commission that Mr. H. should be castigated, if at all. The failure to make any statistical tests of the fit has been mentioned above. While not all of the theoretical requirements for a Chi-square test may be present, it still would seem worthwhile to make the test. Not enough data are available for the reader, even if sufficiently ambitious, to do this himself. Moreover, it is not clear how many parameters were fitted, so the number of independent dimensions (degrees of freedom) is difficult to determine. Some amplification of the example would make such an additional test possible and also would clarify Mr. Hewitt's comments on his last page relative to the coefficients of variation of the five classes. Since he does not give these, nor the grand mean for all five classes combined, it is impossible to verify his conclusion or to determine the magnitude of the "smaller relative dispersion" to which he refers.

Another area of criticism is the duplication of the early part of this paper with that by Lester B. Dropkin[3] which apparently was developed almost simultaneously, but to which Hewitt refers. Hewitt's first five pages seem to contain nothing that is not in Dropkin's paper except a bit of more detailed explication. One is inclined to wonder why the two did not collaborate on a jointly authored article under the circumstances.

Several minor points may be raised. Rigor would be added by appending to formula (1.5) the qualification "when P(x) and T(m) are defined as in (1.1) and (1.2)". Finally, this discussant questions whether the "process of creating groupings based upon driving record . . . is *completely* random" (Page 56, emphasis added). How a particular driver is classified in a particular year may be completely random, but it seems that the creation of the group is not.

In terms of statistical theory, readers with slightly different backgrounds might be helped if the author would point out that the negative binomial distribution is also known as the Pascal distribution and sometimes as the Polya-Eggenberger distribution. It might also help to point out that the Pearson Type III distribution is a special case of the Gamma distribution where the origin is set at zero, and that other origins produce somewhat different results. In other words, the negative binomial described is a special case of a whole family of related distributions.

This discussant's major concern with the paper applies also to almost all

---

[3] "Actuarial Note—Automobile Merit Rating and Inverse Probabilities", CAS XLVII, p. 37. (Hewitt's footnote 4.)

the recent American literature which he has had the opportunity to read. There is apparently a total lack of familiarity with European literature on the subject on the part of most Americans.[4] This lack of communication is most unfortunate since it results in much unnecessary duplication of effort—effort which might be devoted more fruitfully to pushing forward the frontiers of knowledge.

Several remedies suggest themselves immediately, First, as many members of this Society as possible should join the ASTIN Section of the Permanent Committee for International Actuarial Congresses, thus obtaining the ASTIN literature and the opportunity to participate. Second, it would be extremely valuable for this Society to establish a committee to abstract summaries of the more significant foreign actuarial literature and publish these summaries in the *Proceedings*. Perhaps it might be possible to have some of the best translated and distributed to the members in processed form. Similarly, the Society might invite occasional papers by foreign actuaries. Perhaps one noted foreign authority might be invited as a guest speaker to each meeting. It is probable that financial support for travel expenses could be obtained without undue difficulty.

Illustrative of this lack of communication is the fact that neither Mr. Hewitt nor any of the other writers on this subject has even mentioned the papers presented at the ASTIN Colloquium at La Baule, France, 11 and 12 June, 1959. The topic of the colloquium was "No claim discount in insurance, with particular reference to Motor business." Ten papers were presented and discussed. The discussion was well summarized by Ammeter[5] and in the *ASTIN Bulletin*[6]. Particularly appropriate to the subject of Hewitt's paper are the papers by Bischel[7], Delaporte[8], and Thyrion[9]. None of these articles is precisely parallel to Mr. Hewitt's but each develops the negative binomial as a suggested model for automobile insurance.

Unfortunately, there seems to be a startling paucity of data on both sides of the Atlantic. It is to be hoped that future experience will be tabulated and reported in such a way as to make possible further building and testing of models. Data on the distribution of losses by amount also would be valuable. It well may be that this would lead to a model involving a double

---

[4] Admittedly, this discussant shared this unfamiliarity until rather recently.

[5] Ammeter, Hans, "Die Rückvergütung bei schadenfreiem Verlauf in der Motorfahrzeugversicherung", *Mitteilungen der Vereingung schweizerischer Versicherungsmathematiker*, Heft 2, 1959, p. 3.

[6] Vol. I, Part III, pp. 92-105.

[7] Bischel, F., Une Methode pour Calculer une Ristourne Adéquate pour Années sans Sinestres, *The ASTIN Bulletin, Vol. I, Part II* (pp. 106-122).

[8] Delaporte, Pierre, "Quelques Problémes de Stastique Mathématique posés par l'Assurance Automobile et le Bonus pour non Sinistre", *Bulletin Trimestriel de l'Instutut des Actuaires Francais*, No. 227, pp. 87-102 (Juin, 1959).

[9] Thyrion, P., "Contribution a l'Etude du Bonus pour non Sinistre en Assurance Automobile", *The ASTIN Bulletin*, Vol. I, Part II, pp. 142-162.

compound distribution. Perhaps the Poisson Pascal distribution might serve as a useful first approximation.

Despite these comments Mr. Hewitt's paper is interesting and informative. This discussant learned a good bit from it and it does seem to be the first paper to develop formulae for forward claim probabilities by classes. It is a worthy contribution to the growing body of literature on the subject.

AUTHOR'S REVIEW OF DISCUSSION

CHARLES C. HEWITT, JR.

The author would like to express his appreciation to Professor Dickerson for his kind treatment of the paper under discussion.

He is in complete agreement with most of the items which Professor Dickerson refers to as "sins of omission." Although pleading guilty on virtually all counts of the indictment, I do wish to indicate certain extenuating circumstances:

(1) As freely indicated, my paper is built on the foundation of earlier work in this field by Lester Dropkin. As Professor Dickerson correctly infers, Mr. Dropkin and I did arrive at our conclusions with respect to forward claim frequencies of stochastic groups independently, and I suspect almost simultaneously.
Therefore, there is some duplication of Dropkin's work in my paper. Professor Dickerson will be glad to know that Mr. Dropkin and the writer got together on the matter of symbols and when these two papers are printed in the Proceedings, there will be a general agreement.

(2) Professor Dickerson suggests that the creation of groups based upon driving record is not completely random. I think that our difference on this matter is a question of semantics (he is speaking of the shell into which the group is placed, and I was speaking of the aggregation of individuals that are placed in the shell).

The author is particularly grateful to Professor Dickerson for two items in his review:

(1) He recognizes that the subject matter of this and earlier papers consists basically of actuarial models for the rating of Automobile insurance. It is to be expected that no model will exactly fit the actual data.

(2) Professor Dickerson devoted considerable space (and rightly so) to the lack of communication between European and American actuaries. He says "There is apparently a total lack of familiarity with European literature on the subject on the part of most Americans." I would like to add my voice to that of Professor Dickerson in urging this