DISCUSSIONS OF PAPERS READ AT THE NOVEMBER 1960 MEETING

AUTOMOBILE MERIT RATING AND INVERSE PROBABILITIES

BY

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DISCUSSION BY D. C. WEBER

Mr. Dropkin's paper is a natural extension of his previous paper, "Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records",[1] in which he introduced the negative binomial distribution as a proper model for the distribution of risks by number of accidents. Since that introduction, several papers dealing directly or indirectly with this frequency function have appeared in C.A.S. literature. Briefly, let us examine this theoretical function to determine how it fits in the area of accident distribution.

If p is the probability of success on a single trial (called a Bernoulli trial), p remaining constant from trial to trial, and q is the probability of failure on that trial such that $p + q = 1$, then the probability of x successes in n trials is given by the binomial probability function. Although a theoretical distribution in its own right, the Poisson distribution is generally thought of as the approximation to the binomial distribution when n is large, p is small and np remains constant in the limit. The variance of the Poisson distribution is equal to its mean so that the ratio of its variance to its mean is 1. Now if we assume that a given population is homogeneous with respect to inherent accident potential, that is, there is no difference in individual risks, then the distribution of the number of accidents is due to chance and the Poisson is applicable.

The probability that the rth success will occur at the Bernoulli trial number $x + r$ is given by

$$(1) \qquad N(x;r,p) = \binom{r+x-1}{x} p^r q^x, \quad x = 0,1,2, \ldots$$

The distribution defined by (1) is called the negative binomial distribution and its moment generating function is

$$M(\theta) = p^r(1 - qe^\theta)^{-r}$$

Obtaining the proper moments by use of $M(\theta)$, we find that the mean of the negative binomial is rq/p and the variance is rq/p^2. Thus the ratio of its variance to its mean is $1/p$ or greater than 1 for $0<p<1$. Now the negative binomial remains meaningful if r is not an integer provided that $r \geqq 0$. If we let $q = 1/(1 + a)$ so that $p = a/(1 + a)$, then (1) takes on the form employed by Mr. Dropkin in his papers.

Empirical accident statistics frequently exhibit a variance greater than the mean which would lead one to suspect the validity of the assumption used

[1] CAS XLVI, p. 165.

in applying the Poisson frequency function. It is the variance greater than the mean property of the negative binomial that lends support to the use of this function in accident distributions. In using the negative binomial distribution we assume that the accident potential of the population is not homogeneous, that is, differences in individual risks exist. For each sub-group of the population, the inherent hazard is constant, but a variable accident potential exists between groups. Such an assumption is fundamental to any merit rating plan, automobile or otherwise.

A function which gives the probability that an individual will have x accidents in the next t years given that he has had c accidents in the past s years is truly an exciting notion. I believe Mr. Dropkin's paper is a remarkable contribution to the idea of merit rating. I have checked through the formulas in this rather mathematical work and have found them to be accurate. In the development, however, the author is a little sketchy on the application of inverse probability. Bayes' Rule is an extension of conditional probability and it is the latter concept that Mr. Dropkin has used in deriving the expression for $T(m|c,s)$. Assuming that clarification will not detract from the paper, the reasoning is as follows.

The probability for the occurrence of event A given that event B has occurred is given by the relationship

$$(2) \qquad P(A|B) = \frac{P(A \text{ and } B)}{P(B)} \qquad , P(B) > 0$$

Let us make the following notation definitions for clarity.

$P(m)$: Probability that an individual has accident potential, m.

$P(c,s)$: Probability that an individual has c accidents in time s.

$P(c,s|m)$: Probability that an individual has c accidents in time s given that he has accident potential, m.

$P(m|c,s)$: Probability that an individual has accident potential m given c accidents in time s.

By multiplication in formula (2) we see that

$$P(c,s \text{ and } m) = P(m) \cdot P(c,s|m)$$

But

$$P(m|c,s) = \frac{P(c,s \text{ and } m)}{P(c,s)}$$

$$= \frac{P(m) \cdot P(c,s|m)}{P(c,s)}$$

Replacing the probability expressions above by Mr. Dropkin's symbols gives us his formula (8), the crux of the entire derivation.

Someone working in the automobile merit area is more qualified to comment on the applications of the development by Mr. Dropkin, but in the interest of completeness I shall make a few observations. In his paper the writer pointed out that the general expression for risk distribution, $N(x;t|c;s)$, is of interest to rating systems which determine credits and debits on the basis of fixed experience periods. The average future claim frequency by each sub-group is given by the mean of N(x;t|c;s); namely, t(r + c)/(a + s). Hence it is possible to compare the expected claim frequency for risks having had 1 accident in the last 2 years, say, with risks having had 2 accidents in the last 4 years. In these cases, the claim frequencies are (r + 1)/(a + 2) and (r + 2)/(a + 4), respectively. Also, by comparing the expected claim frequency for a certain sub-group with the average annual claim frequency for all risks, r/a, one is able to determine debits and credits as previously noted. This procedure was demonstrated by Mr. R. A. Bailey[2] in his discussion of Mr. Dropkin's previous paper.

An important result of the paper being reviewed is the realization that any merit rating plan which recognizes only the length of time since the most recent accident is not using all of the data available. At the same time it must be remembered that the developed formulas assume that each risk does not change from one time interval to the next, which obviously is not correct for long periods of time. Hence one may conclude that the most recent accident is more significant than any prior accident, but still the prior accidents are of some value.

The change in each risk that we know occurs and referred to in the previous paragraph brings to mind another application of the formulas. By comparing the actual with the theoretical we may be able to estimate the change in individual risks which occurs with passage of time. Also from a theoretical point of view, the formulas should be helpful in estimating the effectiveness of proposed changes in merit rating plans before any experience is obtained.

A NEW APPROACH TO INFANT AND JUVENILE MORTALITY

BY

CHARLES C. HEWITT, JR.

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Author's Review of Discussion by

A. L. MAYERSON IN VOLUME XLVII

This is the first time that the writer of the article under discussion has ever had occasion to respond to a review. Frankly, I find the problem of responding more difficult than the original writing of the article itself.

The standard textbook of the Society of Actuaries on this subject is "Life Contingencies" by Professor Jordan of Williams College. In that text[1] the force of mortality is graphed from the beginning to the end of the mortality table and looks something like:

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[1] "Life Contingencies", Jordan, C. W. (1952). The Society of Actuaries, p. 16.
[2] CAS XLVII, p.p. 152-154 (Bailey's discussion).