plan in Pennsylvania would have a coefficient of variation of about .10 which is about twice as large as the California-type plan. Messrs. Lang and Muniz show that the coefficient of variation of the National Bureau plan in Pennsylvania is .113 which is a very close confirmation of my estimate of .10.

Messrs. Lange and Muniz said "but suppose Mr. Bailey's conclusion that there is still cream in the rating structure is accepted. Is this cream really skimmable?" Such a question reminds me of the farmer who locked the barn door after the horse was stolen. We do not need to resort to theory to find out whether there is cream and whether it is skimmable. All we have to do is look at the underwriting results of some of the independents. The rating refinements introduced recently have raised the coefficient of variation of the total rate structure only a small amount as shown in my paper, thus still leaving cream for those who know how to skim it.

A NEW APPROACH TO INFANT AND JUVENILE MORTALITY

BY

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DISCUSSION BY A. L. MAYERSON

Mr. Hewitt's paper attempts to derive an analytic expression suitable for evaluating mortality at infant ages. Noting that the Gompertz and Makeham laws, often used by life actuaries to fit mortality data (the 1941 CSO table was Makehamized at ages 15 to 95 while a Gompertz graduation was fitted to the 1937 Standard Annuity Table), are not applicable at juvenile ages, he derives formulas which may be useful in valuing orphans' benefits, especially where multiple lives are involved.

The rationale used in obtaining the formulas is to split the force of mortality operating at age x into three component parts: (1) the portion attributable to chance causes, independent of age, (2) the portion which depends upon the "obsolescence" or deterioration of the body's ability to resist death, and (3) an element which recognizes the individual's inherent predisposition to death. Mr. Hewitt then expresses the individual force of mortality μ_x as $A + Bc^x + m$ where m measures the 3rd or "inherent predisposition" factor and is a random variable with its own distribution function. A, B and c are the usual Makeham constants and measure the "chance" and "obsolescence" components of mortality. He assumes that μ_x has a Pearson Type III distribution function and, by manipulating this distribution function, determines the average force of mortality for a group of individuals, the function used in life insurance mortality studies, as $\bar{\mu}_x = A + Bc^x + \dfrac{r}{a + x}$ (r and a are the two parameters of the Pearson Type III curve). The third term, which he calls the "force of selection", is intended to measure the individual's inherent capacity to survive.

Mr. Hewitt's paper is an interesting approach to the problem of deriving an analytic formula to represent mortality rates. Its underlying rationale, namely, the conception of the force of mortality as an average of widely varying individual rates, resembles that used in the paper entitled "A Theory of Mortality Classes" by Louis Levinson which appeared in the Transactions of the Society of Actuaries Vol. XI (1959). Mr. Levinson divides the factors influencing mortality into three types, which he classifies as those inherent in the nature of man, those due to environmental influences, and those based upon the individual's propensity to survive. Mr. Hewitt uses this approach for a quite different purpose, however.

I detect one error early in Mr. Hewitt's paper. In his first section, he states that "where q_x (the rate of mortality) remains constant, the force of mortality, μ_x remains constant" and then proceeds to calculate an interesting arithmetic example based on the formula $\text{colog}_e\ (1 - q_x) = \mu_x$. In fact, $\text{colog}_e\ (1 - q_x) = \int_0^1 \mu_{x+t}\, dt$ and there is no necessity for μ_x to remain constant over the year of age x to x + 1. Furthermore, the contrary is probably true during the year of age 0 to 1, since μ_o decreases rapidly during the first year of life. The assumption that μ_x is constant for each age x does not invalidate Mr. Hewitt's mathematics, though it does make his numerical example less realistic.

Near the end of his paper, Mr. Hewitt illustrates his formula by fitting a curve to the 1939-41 U.S. white males mortality table. Though he obtains an excellent fit to the mortality rates shown by this table at ages 5 and 10 (but not very close at ages 15 to 30), he does not demonstrate that the method provides a good fit at ages below 5, which is the range he proposed to investigate. It would also be interesting to know whether as good results would be obtained if his curve were fitted to a more recent mortality table.

Mr. Hewitt's attempt to analyze separately each of the factors influencing human mortality is an interesting and worthwhile excursion into the whys and wherefores of mortality data, and his approach may well be useful in analyzing automobile accident statistics and for other purposes. Whether his formulas will produce a more accurate valuation of orphans' benefits than the methods now used is, however, not yet proven. In particular, his formula (4.6b), which expresses a joint life probability in terms of single life probabilities, is such that the law of uniform seniority may not apply. Since the utility of Makeham's and Gompertz' laws in computing annuity values depends on the fact that not only nPxyz but also a_{xyz} can be expressed in terms of single life values or in terms of values at equal ages, I believe Mr. Hewitt should have gone a bit farther and showed that this is also true for his formula. Unless a law of uniform seniority or some similar labor-saving device can be found, it might be easier to obtain joint life annuity values by programming the job for an electronic computer than to use Mr. Hewitt's methods. Even if his analysis does not lead to easier computations, however, his analysis is original and is a worthwhile contribution to actuarial literature.