DISCUSSIONS OF PAPERS READ AT THE NOVEMBER 1960 MEETING

ANY ROOM LEFT FOR SKIMMING THE CREAM?

BY

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DISCUSSION BY J. T. LANGE AND R. M. MUNIZ

(National Bureau of Casualty Underwriters, presented by invitation.)

Mr. Bailey is to be commended for the excellent work that he is doing, as revealed by this and other papers of his, in bringing mathematical analysis to bear on the problems of rating systems. These problems are extremely difficult, and the final analytical solutions are still to be made, but every contribution, such as Mr. Bailey's, is another step along the way. In this paper Mr. Bailey considers the problem of measuring the amount of skimmable cream to be found in the classification system for automobile liability insurance. He draws several conclusions from his analysis, his final conclusion being that the present rating system is not perfect and still has skimmable cream in it. No one will disagree about there still being *cream* in the rating system; Mr. Bailey, himself points out that perfection can only be achieved if there is a separate rate for each risk or, more precisely for each group of risks with the same accident-potential. As soon as you combine into one rate-class a group of risks with differing accident-potential, no matter how slight the difference, there will of necessity be some risks that are better than average: thus there will always be cream.

But is it skimmable? More precisely, is there so much cream that there is a sizable danger of some other rating system successfully attracting these better-than-average risks? Mr. Bailey concludes that there is, and he bases his conclusion on a comparison of the variation in the rates of the present rating system with the variation in the inherent hazard in the total population of risks.

There are a number of assumptions underlying Mr. Bailey's analysis that warrant further discussion. To begin with, he develops a figure of 1.00 for the relative variation in the hazard and finds support for this in the figure of .977 computed by M. Delaporte (Sixteenth International Congress of Actuaries, 1960, Vol. II). M. Delaporte's figure, however, is for the inherent hazard in only one particular rate-class in Paris and not for all pleasure-use cars in France as Mr. Bailey states. If .977 represents the variation in one class in one territory, the variation of the hazard for all classes in all territories, that is, for the entire population, must be considerably higher than unity. On the other hand, using the Canadian data Mr. Bailey computes a figure of .87 for the coefficient of variation of the risks in Canada. This figure of .87 is derived from a formula he developed previously, namely, $a/(a+n)$, which represents the expected claim freqency for risks accident-free for n or more years relative to the expected claim frequency for all risks. Specifically, the coefficient of variation of .87 is based on the Canadian relative claim frequency for $n = 1$, that is for one or more accident-free years. For $n = 2$, that is for two or more accident-free years, the same computation gives a coefficient of variation of .72; for $n = 3$, the coefficient of variation is .63. These varying values for the coefficient of variation computed by the same formula from the same body of data raises the question of whether the basic mathematical theory is in fact a proper model for this type of analysis. Mr. Bailey believes that these varying values are accounted for by the fact that there is an assumption in the mathematics being used that the inherent hazard for each individual risk remains unchanged from year to year. Since the .87 refers to the variation of the group already having had one accident-free year he concludes that the coefficient of variation for all risks would be higher, closer to 1.00. This hypothesis sounds reasonable and may be right. However, the computation of the coefficient of variation is based on first evaluating the two parameters, r and a, of the negative binomial distribution which has been assumed as the proper model; there is no indication in this mathematical model that these parameters would vary for the same body of data as they seem to do for the Canadian data.

All the above serves to illustrate that the value of 1.00 which Mr. Bailey uses for the coefficient of variation for risks is only an estimate. Also, it would seem that the coefficient of variation for risks would vary from one population of drivers to another to the degree that some of these populations were more homogeneous; for example, it would seem that there would be less variation among drivers in Iowa (which is largely rural) than among drivers in New York (which includes both rural and urban areas). The overall result of these considerations is to demonstrate that while unity may be used as an estimate of the coefficient of variation of the risks it is still an approximation which might be subject to considerable refinement, and it therefore should not be treated as a universal constant applicable in all situations.

Suppose it could in fact be determined that the relative variation in the inherent hazard is some constant, K. Can this be used as a basis for measuring the effectiveness of a rating system? Mr. Bailey says it can; he says that if the relative variation in the rates is A then the ratio A/K times 100 gives the percent effectiveness of the rating system. There are a number of things against this reasoning. First of all, the absolute variation in the rates will always be less than the absolute variation in the inherent hazard. Theoretically, the inherent hazard has no upper bound, whereas the highest rate that can be charged is limited by practical considerations. More fundamentally, the distribution of the hazard is a continuous function; there are an infinite number of values for inherent hazard. The distribution of the rates, however, is discrete; only a finite number can be established. The variance of a discrete approximation to a continuous function is necessarily less than the variance of the continuous function. In short, for a given number of rates, (and assuming equal means in order to simplify this discussion) the variation, A, is limited by an upper bound which is less than K. Call this upper bound M. Mr. Bailey gets a value for A/K equal to 0.5 and concludes that the rating system is only half as effective as it could be. But A cannot be greater than M; is not A/M a more appropriate ratio? It is possible that A/M would be substantially higher than A/K and thus give a much better picture.

Estimating A, the coefficient of variation of the rates, also presents a problem in that estimates of the coefficient of variation of the risks are based upon broad sets of data (e.g. all drivers in California, all drivers insuring in Canada) and an estimate of A should therefore be based on an equally broad sample if it is to be comparable. If an estimate of the coefficient of variation of the rates is based upon a sample population which is more homogeneous than the total population, then the resulting estimate of the coefficient of variation of the rates will be lower than if it were based upon a broader sample. While it is impossible to compare the exposure distribution Mr. Bailey uses with the actual distribution of cars in Pennsylvania to see if this distribution is typical of that of all cars in Pennsylvania, it is possible to compare it with data compiled by the National Bureau, and with data from other sources. First, it was found that in comparison with the National Bureau's distribution (based upon 1,000,000 cars) in Pennsylvania, Mr. Bailey's distribution (based on 12,000 cars) was biased with respect to territory in that a a large percentage of his total exposure is in a single territory while certain other city territories have a relatively low exposure which leads to a lower coefficient of variation than would result from a National Bureau distribution. With regard to merit rating, Mr. Bailey's distribution would appear to be atypical since over 90% of the risks fall into the lowest rated sub-group. National Bureau data from Pennsylvania indicates that 80% fall into this sub-group while data from California (where the plan has been in effect for a longer period) show closer to 60% in this sub-group. (This data is summarized in Table 1 which appears following the conclusion of this paper.) It is interesting to note that the merit rating plans produce a greater coefficient of variation in states where a large majority of drivers are insured under the plan. In Canada and Texas where all auto insurance is written under a merit rating plan, coefficients of variation of .225 and .232 respectively are produced; using the California merit rating plan with the distribution of drivers having accidents and convictions reported by the California Division of Motor Vehicles a coefficient of variation of .269 is produced. It would seem that Mr. Bailey's conclusions concerning merit rating plans of the type introduced in California are unjustified inasmuch as experience indicates that where the majority of cars are insured under the plan the coefficient of variation of the plans is several times that which Mr. Bailey estimates. It would seem that in some respects Mr. Bailey's distribution in Pennsylvania is not typical, and his total coefficient of variation is probably under-estimated.

Thus far in this discussion two questions have been raised in regard to Mr. Bailey's comparison of the coefficient of variation of the rates with that of the inherent hazard in the risks. Concerning his use of the coefficient of variation of the inherent hazard in the risks, it was pointed out that Mr. Bailey's figure is inconsistent with Mr. Delaporte's, and furthermore that, even for the best classification system that could be designed, the coefficient of variation of the rates must of necessity be less than that of the risks. His use of the coefficient of variation of the rates was questioned on the grounds that in some respects Mr. Bailey's sample appeared to be biased. While Mr. Bailey is justified in saying our present classification is not perfect, his statement that the present classification system takes care of only half of the total variation among risks is subject to question since this conclusion is based upon figures which do not appear to be wholly representative.

Mr. Bailey's approach, that of looking at the total rating structure, is both interesting and enlightening, but suppose Mr. Bailey's conclusion that there is still cream in the rating structure is accepted. Is this cream really skimmable? The total cream for a rating system must be some sort of sum of the cream in each class. Now this might result from a small amount of cream in every class, which for practical purposes is unavoidable, or on the other hand, this cream may be concentrated in one or two classes from which it is easily skimmed. The coefficient of variation of the rating system as a whole seems to be too all-inclusive a measure to be used to determine whether any of the cream is skimmable. A low coefficient of variation says nothing about the individual classes themselves, whether all of the classes have skimmable cream or just one or two. Neither does this type of measure indicate what can be done to improve the rating system, if improvement is indicated. It would seem, therefore, that the proper way of judging the effectiveness of the rating structure would be to study the individual classes to see if any of these are so ineffective that there is still considerable room to skim off the cream. M. Delaporte, in the article already referred to, suggests that the difference between the mean value of a class and the modal value of that class indicates how different the rate charged the typical risk is from the rate indicated by its inherent hazard. Since the rate for any class is based upon the mean value, if the modal value is significantly lower than the mean, then the typical risk is paying a rate higher than is indicated by his inherent hazard, and he is cream that may be skimmed. This approach might be used to provide a more definite answer to the question, how much room is left to skim off the cream?

TABLE 1

Merit Rating Distributions and the Resulting Coefficients of Variation

PENNSYLVANIA

Merit Rating Code	Mr. Bailey's Exposure Distribution	National Bureau Exposure Distribution	Relativity
9	91.3%	83.6%	85
1	7.6	4.9	95
2	.8	9.3	100
3	.2	.7	120
4	.01	1.1	140
5	.04	.2	170
6	.03	.2	200
Mean Relativity	86.032	88.14	
Standard Deviation	4.315	9.93	
Coefficient of Variation	.050	.113	

Mr. Bailey's exposure distribution results from a merit rating plan which is slightly different from the National Bureau plan in Pennsylvania in that his plan uses the experience period and point system which the National Bureau used in California; therefore, in some respects National Bureau data from California provides a better basis of comparison.

CALIFORNIA

Exposure Distribution

Merit Rating Code	National Bureau Actual	National Bureau Adjusted$^{\phi}$	Dept. of Motor Vehicles*	Relativity
9	68.1%	61%	54.1%	80
1	18.5	22	22.2	90
2	8.7	10	10.8	100
3	2.7	4	5.5	120
4	1.1	1.5	3.0	140
5	.5	1	1.6	170
6	.4	.5	2.8	200
Mean Relativity	86	88.2	93	
Standard Deviation	14	16.3	25	
Coefficient of Variation	.163	.185	.269	

ϕ Adjusted to take into account changes as a result of checking policies with the driver records of the Dept. of Motor Vehicles.

* Based on a driver record study conducted by California Dept. of Motor Vehicles.

DISCUSSION BY L. J. SIMON

Mr. Bailey's paper introduces two advances in actuarial theory that make it another milestone in progress. The first stride forward is in the concept of the coefficient of variation of the rates as a method of measuring the overall effectiveness of a rating plan. This concept is destined to revolutionize our thinking with respect to classification systems because now at last we have the key to comparing two different systems of classification and also a measure which will show us how much increase in precision we will get by superimposing a new rating criterion upon the existing system.

The second advance in actuarial theory made in the paper is almost lost to the reader because it is passed over so quickly. This is the method used for determining the coefficient of variation of the *risks*. Being able to do this from risk distributions, such as the California Driver Record Study, is quite good, but being able to do it from the loss ratio of risks who were claim free the preceding year (which leads to the credibility measure, a value for the parameter "a", a value for "r", and hence to the coefficient of variation of the risks) is of major impact. This same method can be applied under many, many circumstances to determine the coefficient of variation of the risks. This,