A NEW APPROACH TO INFANT AND JUVENILE MORTALITY

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Introduction

Considerable space is devoted in the literature of this Society to the problems of evaluating the rights of survivors under the benefit provisions of various Workmen's Compensation statutes. By the very nature of the problem of industrial deaths, survivors are, in a great majority of the cases, widows and orphans. In treating of the life expectancy of orphans, the Casualty Actuary is confronted with the use of life contingency functions developed by the Life Actuary whose primary concern is with individuals who have arrived at maturity or are very close thereto.

There is no satisfactory analytical expression for mortality with respect to infants and juveniles. Values for the various life functions at these ages are, at best, non-analytical and highly empirical.

When the problem of multiple-life contingencies is superimposed upon this situation, the difficulties encountered by the Casualty Actuary in measuring the value of orphans' benefits become considerable.

Summary

This paper offers a different approach to the problem of the analysis of mortality during the early years of life by distinguishing between the force of mortality as a group average and the individual forces of mortality of the various members of the group. It recognizes that the term "force of mortality" as now expressed in the literature on the subject of life contingencies actually means the *average* force of mortality of a group of individual lives.

A simple example is used to illustrate what happens to the group force of mortality when the individual life members of the group have varying individual forces of mortality.

The individual force of mortality is analyzed and is seen to be a combination of three components which for convenience are referred to as "obsolescence", "chance" and "incapacity to sustain life".

An *a priori* expression for the frequency distribution of the "incapacity to sustain life" is introduced. Making use of this *a priori* distribution, a new expression for group mortality rates is obtained, and the component effects of "obsolescence", "chance" and "selection" become evident.

A "complete" expression for the group force of mortality is obtained. From this expression other life functions may be derived.

A relatively crude test of the values obtained by use of the "complete" expression for group mortality shows that the general shape of an actual mortality curve may be quite well represented.

Areas for further development of the principles brought forth in this paper are indicated.

1. *Force of Selection*

It seems self-evident that expressions of the force of mortality (μ_x) and other functions such as p_x and q_x imply that each is in fact an average for the group of lives whose attained age is x. Actual values for these functions have proven reasonably satisfactory. This undoubtedly results from the circumstance that for those values of x which are most commonly used in life insurance (very roughly speaking from age 17 to age 85), there is relatively little variation of the individual values about the group average.

It might be suspected, however, that at the very beginning of the life span, there exists a considerable dispersion of the individual forces of mortality about the group average, and it is precisely for these ages that there has been the greatest difficulty in expressing the force of mortality either analytically, or with any accuracy or confidence in the values obtained.

New-born children have widely-varying chances of survival. The probability of surviving depends upon physical factors, some hereditary, some environmental, including the circumstances surrounding pregnancy and birth, which will cause wide variations in the probability of living or dying during the early days, weeks, months or even years of life. The question therefore arises as to what effect this wide dispersion in individual probabilities of survival or death will have on the group survival or mortality function. Obviously the child born with the greatest chance of survival will be more likely to live during the first few years of life; the child with a lesser chance of survival will have a greater probability of dying in infancy or early childhood. This "survival of the fittest" has a profound effect upon the group forces of survival and mortality during the early years.

The example given below illustrates, by removing all factors other than selection, the importance of selection when examining group or average mortalities during the early years.

Let us assume for purposes of illustration that in a group of 100,000 births there are 20,000 infants with a constant force of mortality such that each member has a probability of death during one year equal to 4/5ths. Similarly, another 20,000 infants are assumed to have a constant probability of dying during one year equivalent to 3/5ths; 20,000 more are assumed to have a constant one year probability of dying (q_x) of 2/5ths; another 20,000 have a value for q_x of 1/5; and the remaining 20,000 infants have such a strong probability of survival that it will be assumed that, for the purposes of this example, q_x is equal to zero.

It is recognized, of course, that in real life the force of mortality with respect to an individual life normally varies with age. However, in order to make this illustration as simple as possible, it will be assumed that the individual lives referred to have a constant force of mortality throughout the first four years of life as represented by the various values of q_x which have been assigned. It will be recognized that where q_x remains constant, the force of mortality, μ_x, remains constant and can, in fact, be obtained by the expression:

$$\mu_x = \operatorname{colog}_e(1 - q_x) \qquad (1.1)$$

Under the assumptions made, the values of l_x and d_x for the first four years of life are set forth in Table 1.

TABLE 1

ILLUSTRATION OF THE EFFECT OF THE FORCE OF SELECTION ON GROUP MORTALITY RATES

Group Number (i)	q^i	1st Year l_0	1st Year d_0	2nd Year l_1	2nd Year d_1	3rd Year l_2	3rd Year d_2	4th Year l_3	4th Year d_3
0	0	20,000	0	20,000	0	20,000	0	20,000	0
1	$\frac{1}{5}$	20,000	4,000	16,000	3,200	12,800	2,560	10,240	2,048
2	$\frac{2}{5}$	20,000	8,000	12,000	4,800	7,200	2,880	4,320	1,728
3	$\frac{3}{5}$	20,000	12,000	8,000	4,800	3,200	1,920	1,280	768
4	$\frac{4}{5}$	20,000	16,000	4,000	3,200	800	640	160	128
Total		100,000	40,000	60,000	16,000	44,000	8,000	36,000	4,672
$\bar{q}_x=\frac{\sum_i d_x}{\sum_i l_x}$		0.400		0.267		0.182		0.130	

The resultant mortality table would look as follows:

TABLE 2

GROUP MORTALITY TABLE BASED UPON RESULTS OF TABLE 1

Age	l_x	d_x	$\bar{q}_x$	$\bar{\mu}_x$*
0	100,000	40,000	.400	.652
1	60,000	16,000	.267	.391
2	44,000	8,000	.182	.244
3	36,000	4,672	.130	.160

*See narrative for derivation of $\bar{\mu}_x$.

It is possible to represent the group force of mortality in this illustration by the following expression:

$$\bar{\mu}_x=\frac{\sum_{i=0}^{4}\mu^i l_x^i}{\sum_{i=0}^{4} l_x^i} \tag{1.2}$$

where

$$\mu_x^i=\mu^i=\mathrm{colog}_e\left(1-\frac{i}{5}\right) \tag{1.3}$$

Since

$$l_x^i = {}_xp_0^i \, l_0^i \tag{1.4}$$

and

$$ {}_xp_0^i = e^{-\int_0^x \mu^i dt} = e^{-\mu^i x} \tag{1.5}$$

$$\bar{\mu}_x = \frac{l_0^i \sum_{i=0}^{4} \mu^i e^{-\mu^i x}}{l_0^i \sum_{i=0}^{4} e^{-\mu^i x}} = \frac{\sum_{i=0}^{4} \left(1-\frac{i}{5}\right)^x \operatorname{colog}_e \left(1-\frac{i}{5}\right)}{\sum_{i=0}^{4} \left(1-\frac{i}{5}\right)^x} \tag{1.6}$$

Also from this example, it is worth while to examine the change in the frequency distribution of lives with respect to individual rates of mortality:

TABLE 3

FREQUENCY DISTRIBUTION OF LIVES (TABLE 1) BY RATE OF MORTALITY

Group Number (i)	q^i	AGE 0	1	2	3
		FREQUENCY			
0	0	.200	.333	.454	.556
1	$\frac{1}{5}$	.200	.267	.291	.284
2	$\frac{2}{5}$	.200	.200	.164	.120
3	$\frac{3}{5}$	.200	.133	.073	.036
4	$\frac{4}{5}$	.200	.067	.018	.004
Total		1.000	1.000	1.000	1.000

In examining the resulting values for rate of mortality and force of mortality, it must be re-emphasized that, with respect to the individual life members of this group, the force of mortality and the mortality rate remain constant throughout the entire period. Thus, it becomes completely evident that the group force of mortality is favorably affected by the survival of the fittest lives.

The rather drastic effects resulting from the "force of selection" in this example were achieved by the choice, for most individuals, of relatively high rates of mortality and by the selection of a population with a wide dispersion in the rates of mortality of the individual members. It becomes evident that as the process of selection continues the average rates of mortality of the group diminish and the dispersion of the individual mortality rates about the group average diminishes.

Thus, it may be anticipated that the effect of the "force of selection" while extremely important in the earliest years of life diminishes very rapidly and becomes of relatively minor importance during the middle and later years of the life span.

2. *Individual Mortality*

In 1825 Benjamin Gompertz[1] stated with respect to the problem of mortality, "The average exhaustion of a man's power to avoid death (is) such that at the end of equal infinitely small intervals of time he (loses) equal portions of his remaining power to oppose destruction which he had at the commencement of these intervals."

Application of this basically philosophical premise led Gompertz to the hypothesis that the individual force of mortality could be expressed as follows:

$$\mu_x = Bc^x \tag{2.1}$$

It is particularly significant in examining this concept to note that Gompertz referred to the capacity of the *individual* to resist or avoid death.

At the same time that he presented this important concept, Gompertz stated, "It is possible that death may be the consequence of two generally coexisting causes: the one, chance, without previous disposition to death or deterioration; the other, a deterioration, or increased inability to withstand destruction."

In other words, Gompertz foresaw that in addition to the contribution of deterioration to the force of mortality, there was an additional force which he attributed to chance and which was independent of the age of the individual.

The combination of these two forces was stated analytically by Makeham[2]:

$$\mu_x = A + Bc^x \tag{2.2}$$

Thus, today it is generally recognized that the two major contributions to mortality as expressed in the Makeham formula are the deterioration of the body's ability to resist death (which may be referred to as "obsolescence") and the force resulting from chance causes independent of age (which will be referred to as "chance").

Gompertz' use of the phrase "without previous disposition to death" is interesting and with respect to this paper significant. It has already been asserted here that at the time of birth each individual has his particular ability to sustain life. This ability results from heredity, environment, including the conditions surrounding birth and pregnancy, and from other causes which may be considered metaphysical in nature. Suffice it to state that the individual force of mortality may be viewed as a combination of the forces recognized by Gompertz and Makeham with a third element which recognizes the individual's predisposition to death (or incapacity to sustain life). Using this concept, it is possible to state analytically the force of mortality as follows:

[1]Gompertz, Benjamin, *On the Nature of the Function Expressive of the Law of Human Mortality*. Philosophical Transactions, Royal Society of London, 1825.

[2]Makeham, W. M.:*On the Law of Mortality, and the Construction of Annuity Tables,* Journal of the Institute of Actuaries, Volume 8, 1860.

$$\mu_x = m + A + Bc^x \tag{2.3}$$

where m will be defined as the individual predisposition to death. Thus, for an individual age zero, the force of mortality may be defined as:

$$\mu_0 = m + A + B \tag{2.3a}$$

Consideration should and ultimately must be given to whether the parameters B and c vary from individual to individual as it is assumed that the parameter m does.[3] For the purposes of this paper and in order to simplify further analysis, it will be assumed that all of the parameters except m are the same for all individuals.

Using (2.3) as the expression for the force of mortality, it can be shown that the probability of an individual age x surviving n years is given by the expression:

$$_np_x = e^{-\left[(m+A)n + \frac{Bc^x(c^n-1)}{\log_e c}\right]} \tag{2.4}$$

and thus in the special case where x equals zero, the probability of an individual surviving to age n from birth is given by the expression:

$$_np_0 = e^{-\left[(m+A)n + \frac{B(c^n-1)}{\log_e c}\right]} \tag{2.4a}$$

3. *Distribution of Individual Mortality*

It has been demonstrated by Dropkin[4] that the distribution function of the predisposition of individual drivers to automobile accidents can be very well represented by:

$$T(m) = \frac{a^r}{\Gamma(r)} m^{r-1} e^{-am} \tag{3.1}$$

This distribution function is a Pearson Type III curve with a mean equal to r/a and a variance equal to r/a^2. In this distribution function, $\Gamma(r)$ has the usual meaning:

$$\Gamma(r) = \int_0^\infty e^{-x} x^{r-1} dx$$

$$\text{(for all positive values of } r\text{)} \tag{3.2}$$

and T(m) is defined over the range from m equals 0 to m equals ∞. This distribution function meets the necessary condition that

$$\int_0^\infty T(m)\,dm = 1$$

The success with which Dropkin and others have used this distribution function suggests its use as a distribution function for m where m is assumed

[3]Individual variations in A can be considered as variations in m.

[4]Dropkin, Lester, *Some Considerations on Automobile Rating Systems Utilizing Individual Driving Records,* CAS XLVI, p. 165.

to represent the individual predisposition to death in the expression for individual mortality contained in (2.3).

The range of values for m from 0 to ∞ seems quite meaningful. It should be noted that m equals 0 does not imply that the force of mortality is zero at age zero since from (2.3a)

$$\mu_0 = A + B \qquad (m = 0)$$

On the other hand, m (an annual rate) equals ∞ carries the implication that the new-born child has such serious impairments, either physical or environmental, that it may not be expected to survive for more than an infinitely short period of time. It should be noted that the equivalence of m with ∞ is not intended to imply stillbirth.

Analysis of the distribution function T(m) indicates that the mode occurs when:

$$m = \frac{r-1}{a}$$

Since one of the conditions of the distribution function T(m) is that both r and a must be positive, it can be seen that when r is less than 1, the mode must occur at the point where m equals 0. It will subsequently be seen, when dealing with the group mortality function, that values of r are quite small. Therefore the modal point seems quite reasonable since most new-born children have little or no predisposition to death.

Reference to Table 3 will indicate that the frequency distribution of lives by individual rates of mortality will change with age. In other words, the distribution of lives with respect to the rate of mortality of each life is not the same at age 1 as at age 0, and likewise will not be the same at age 2 as at age 1, and so forth. This change in the distribution of lives with respect to the individual mortality rates results from the "force of selection" already discussed. In the original example it was assumed that the force of mortality with respect to each individual life remained constant and that the only thing that changed with age was the distribution of the number of lives with respect to each individual force of mortality. However, the expression for the individual force of mortality suggested in the previous section:

$$\mu_x = m + A + Bc^x$$

indicates that the individual force of mortality does in fact change with age. From equation (2.4a) it was seen that the probability of an individual life surviving n years after birth is represented by the expression:

$${}_np_0 = e^{-\left[(m+A)n + \frac{B(c^n-1)}{\log_e c}\right]}$$

The distribution function of the surviving individual lives with respect to m after n years may be obtained as follows:

$$T_n(m) = \frac{{}_np_0 \cdot T(m)}{\int_0^\infty {}_np_0 \cdot T(m)\,dm}$$

$$T_n(m) = \left(\frac{a+n}{a}\right)^r e^{-mn} \cdot T(m) \tag{3.3}$$

With respect to $T_n(m)$:

$$\int_0^\infty T_n(m)dm = 1$$

the mean is:

$$\int_0^\infty \left(\frac{a+n}{a}\right)^r me^{-mn} \cdot T(m)dm = \frac{r}{a+n} \tag{3.4}$$

and the variance:

$$\left[\int_0^\infty \left(\frac{a+n}{a}\right)^r m^2e^{-mn} \cdot T(m)dm\right] - \left(\frac{r}{a+n}\right)^2 = \frac{r}{(a+n)^2} \tag{3.5}$$

It can be seen from (3.4) and (3.5) that the distribution function $T_n(m)$ meets the expectation that the process of selection diminishes the average mortality rate and the dispersion of individual mortality rates about the group average.

4. *Group Mortality*

Using this distribution function (3.3) with a substitution of age x for the value n, the mean value for the probability of the survival of the lives which have attained age x can be obtained as follows:

$$E({}_np_x) = {}_n\bar{p}_x = \int_0^\infty {}_np_x \cdot T_x(m)dm$$

$${}_n\bar{p}_x = \int_0^\infty e^{-\left[(m+A)n + \frac{Bc^x(c^n-1)}{\log_e c}\right]} \left(\frac{a+x}{a}\right)^r e^{-mx} \cdot T(m)dm$$

$${}_n\bar{p}_x = \left(\frac{a+x}{a+x+n}\right)^r e^{-\left[An + \frac{Bc^x(c^n-1)}{\log_e c}\right]} \tag{4.1}$$

and of course from this expression a value for the mean rate of mortality can be obtained:

$${}_n\bar{q}_x = 1 - {}_n\bar{p}_x = 1 - \left(\frac{a+x}{a+x+n}\right)^r e^{-\left[An + \frac{Bc^x(c^n-1)}{\log_e c}\right]} \tag{4.2}$$

It is important to distinguish between the probabilities of survival or death for the individual life at any particular age x which are represented by the expressions ${}_np_x$ and ${}_nq_x$, and the group or average probabilities of survival or death which have just been represented by the expressions ${}_n\bar{p}_x$ and ${}_n\bar{q}_x$. It is the latter expressions which actually correspond to the expressions currently in use in life contingencies.

Similarly, it is possible to derive an expression for the average (or group) force of mortality at any age x as follows:

$$E(\mu_x) = \bar{\mu}_x = \int_0^\infty \mu_x \cdot T_x(m)dm$$

$$\bar{\mu}_x = \int_0^\infty (m + A + Bc^x) \left(\frac{a+x}{a}\right)^r e^{-mx} \cdot T(m)dm$$

$$\bar{\mu}_x = A + Bc^x + \frac{r}{a+x} \qquad (4.3)$$

It is this expression for the average (or group) force of mortality which corresponds to the meaning of the term "force of mortality" currently in use in all studies on the subject of life contingencies. Analysis of this new expression shows that the average force of mortality is actually made up of three component forces:

(1) The Makeham component which has been referred to herein as "chance",

(2) The Gompertz component which has been referred to herein as "obsolescence",

(3) A new component which shows the effect on the group force of mortality of the elimination of those lives which are least fit to survive. This new component has been given the name the "force of selection".

In the expression for group force of mortality (4.3) the first two components referred to are independent of the distribution of lives with respect to the individual mortality rate of each life. The Makeham component is independent of age. With respect to the third component, the "force of selection," the value of r will subsequently be seen to be quite small. Thus the expression:

$$\frac{r}{a+x}$$

approaches zero as the age x increases, and the expression for group mortality $\bar{\mu}_x$ approaches the Makeham expression for the force of mortality. As was anticipated, the "force of selection" is of greatest importance and effect at the very earliest ages of life, after which its importance and effect diminishes quite rapidly and approaches zero with increase in age. Thus, it will be seen that the present expressions for the force of mortality and the present mortality tables derived in reliance thereon may be expected to be quite reasonable for ages beyond x equals 15 to 20.

The importance of the new expression for average or group mortality, however, is that it gives a complete mortality function for all ages in a relatively simple form. Furthermore, this new expression for the complete force of mortality meets the condition that at some age (x_1) there is a minimum force of mortality.

$$\left.\frac{d\bar{\mu}_x}{dx}\right|_{x_1}=0 \qquad \text{results in}$$

$$Bc^{x_1}\log_e c=\frac{r}{(a+x_1)^2} \tag{4.4}$$

(Age x_1 normally occurs during the juvenile years.)

Using the group mortality function (4.3) we can derive the group survival function:

$$\bar{s}(x)={}_x\bar{p}_0=e^{-\int_0^x \bar{\mu}_t dt}=e^{-\int_0^x \left[\frac{r}{a+t}+A+Bc^t\right]dt}$$

$$\bar{s}(x)=\left(\frac{a}{a+x}\right)^r e^{-\left[Ax+\frac{B(c^x-1)}{\log_e c}\right]} \tag{4.5}$$

This, of course, agrees with equation (4.1) when the value zero is substituted for x, and x is substituted for n.

Finally, with respect to the specific problem of evaluating benefits to orphans where the joint-life status is involved, the joint-life probability may be expressed as:

$${}_n\bar{p}_{x_1x_2\cdots x_k}={}_n\bar{p}_{x_1}\cdot{}_n\bar{p}_{x_2}\cdots\cdots{}_n\bar{p}_{x_k}$$

$${}_n\bar{p}_{x_1x_2\cdots x_k}=\left[\frac{(a+x_1)(a+x_2)\cdots\cdots(a+x_k)}{(a+x_1+n)(a+x_2+n)\cdots(a+x_k+n)}\right]^r e^{-\left[Akn+\frac{B(c^n-1)}{\log_e c}\sum_1^k c^{x_i}\right]} \tag{4.6}$$

and since values for a will be extremely small, an approximation of the joint-life probability is given by the following:

$${}_n\bar{p}_{x_1x_2\cdots x_k}\doteqdot\left[\frac{x_1\cdot x_2\cdots x_k}{(x_1+n)(x_2+n)\cdots(x_k+n)}\right]^r e^{-\left[Akn+\frac{B(c^n-1)}{\log_e c}\sum_1^k c^{x_i}\right]} \tag{4.6a}$$

or even more simply

$${}_n\bar{p}_{x_1x_2\cdots x_k}\doteqdot\left[\frac{x_1\cdot x_2\cdots x_k}{(x_1+n)(x_2+n)\cdots(x_k+n)}\right]^r({}_np'_w)^k \tag{4.6b}$$

where

$${}_np'_w=e^{-\int_0^n \mu'_{w+t}dt}$$

and

$$k\mu'_w=\mu'_{x_1}+\mu'_{x_2}+\cdots+\mu'_{x_k}$$

and

$$\mu'_x=A+Bc^x \qquad \text{[see (2.2)]}$$

5. *Test of Group Mortality Functions*

It is impossible to find an actual mortality table which would meet any rigorous program for testing the various group mortality functions derived in the previous section.

However, it is interesting to test the general form of the "complete" expression for group mortality to see whether there is an approximate fit against actual data. For this purpose comparison was made with the mortality table for U. S. White Males 1939-41, the figures for which were obtained from the Census of 1940. The comparative values for $\bar{q}_x$ are given in Table 4, and it is interesting to note that the general shape of the actual mortality curve is quite well represented by the group mortality function.

TABLE 4

COMPARISON OF MORTALITY RATES USING "COMPLETE" EXPRESSION FOR GROUP MORTALITY WITH MORTALITY RATES FOR U. S. WHITE MALES 1939-41 (AT QUINQUENNIAL AGES)

	Mortality Rates	
Age (x)	Actual (q_x)	Theoretical ($\bar{q}_x$)
0	.04812	.04812
5	.00138	.00120
10	.00099	.00100
15	.00143	.00110
20	.00211	.00138
25	.00243	.00184
30	.00279	.00257
35	.00363	.00370
40	.00513	.00540
45	.00766	.00792
50	.01155	.01170
55	.01736	.01724
60	.02548	.02566
65	.03684	.03806
70	.05454	.05628
75	.08313	.08336
80	.12472	.12132
85	.18096	.17582
90	.24893	.25112
95	.32071	.35110
100	.38934	.47631
105	.44751	.61998

There are five parameters in the general expression for group mortality, and these parameters were obtained on a relatively crude basis by setting up the following five conditions:

(1) The minimum point of the curve representing the group force of mortality (see 4.4) was assumed to occur at age 10 years and 6 months.

(2) The value of the group force of mortality at this particular age was taken as .001.

(3) The probability of survival of the first year of life was equated to the corresponding probability in the mortality table.

(4) The probability of surviving to age 69 (approximately the median point of the survival function) was equated to the corresponding value in the mortality table.

(5) The probability of surviving an additional 25 years after age 69 was equated to the corresponding value in the mortality table.

It will be recognized that there are better methods of obtaining the parameters, particularly when greater accuracy is desired at the lower ages.

There are a number of reasons why a perfect fit of the data would be impossible:

(1) Values of various parameters used in the group mortality functions are changing from year to year. For example, the lives age 40 at the time of the Census would not have had the same values for the various parameters in the group mortality functions during the first ten years of life as the lives at age 10 in 1940, and so forth.

(2) There are undoubtedly varying individual life values for the parameters B and c. To the extent that it was assumed (in deriving the group mortality functions) that these two parameters are the same for all lives in the group, an accurate fit of the data would be impossible.

(3) The actual method of preparation of the tables[5] involved adaptations and projections for lives age 5 and under, and for lives above age 94, and, therefore, at these ages the presumption that the mortality rates shown in the original tables are more satisfactory than the group mortality rates obtained on the basis of the functions derived in this paper is weakened.

Values used for the parameters are:

$$A = 1.5194 \times 10^{-4}$$
$$B = 1.9722 \times 10^{-4}$$
$$c = 1.08388$$
$$r = 4.0802 \times 10^{-3}$$
$$a = 6.1500 \times 10^{-6}$$

[5]Greville, T. N. E., *United States Life Tables and Actuarial Tables 1939-1941,* 1946.

For T_0 (m) the mean becomes:

$$\frac{r}{a} \doteq 663$$

the variance:

$$\frac{r}{a^2} \doteq 108 \times 10^6$$

and the standard deviation:

$$\frac{\sqrt{r}}{a} \doteq 104 \times 10^2$$

μ_0 has a range from 3.4916×10^{-4} to ∞, with a majority of cases at and near the lower value.

None of these values at age zero should be considered accurate. However, they do illustrate the tremendous dispersion of the initial "incapacity to sustain life" with respect to individual lives. With such tremendous dispersion the group (or average) force of mortality at birth is seen to have little significance. Actually, the most significant value at birth is the minimum value of μ_0 (the mode). This is the force of mortality with respect to healthy infants, and it is these children who can best be expected to survive the early days, weeks and years of life.

6. *Suggested Further Development*

Although the primary purpose of this paper has been to provide a more adequate analytical approach to infant and juvenile mortality, it will be recognized that the concepts of "individual ability to sustain life" and "force of selection" have application at all ages. The further development of these concepts seems to hold more reward for the Life Actuary.

For example, the "individual ability to sustain life" (or its converse, the "predisposition to death") surely does not remain constant for a particular individual throughout life. An "incubator baby" or Siamese twin who survives the early period of infancy certainly does not retain the same "predisposition to death" at age 1 or age 2, that he had at the moment of birth. Even mature individuals will experience physical or environmental changes that increase or diminish the individual ability to sustain life. An asthmatic or tubercular patient can improve his chances of survival by removal to a dry or arid climate. An overweight person can recover normal life expectancy by reducing his weight to normal or slightly below. The reversal of these situations should result in decreased life expectancy.

The use of "select" mortality tables implies recognition of the "force of selection". The rapidity with which "select" mortality rates approach "ultimate" mortality rates is an inverse measure of the effectiveness of the selection. The degree with which this selection is exercised in the underwriting of life insurance is a measure of the recognition that there is dispersion in the individual rates of mortality about the group average.

Further research should develop at least the following:

(1) The manner in which individual "predisposition to death" varies with age,

(2) The extent to which there may be variation of the Gompertz parameters (B and c) among individual lives,

(3) Analytical expressions for, and measures of the effectiveness of selection (underwriting) by life insurers.