CREDIBILITY OF 10/20 EXPERIENCE AS COMPARED WITH 5/10 EXPERIENCE

BY

LEWIS H. ROBERTS

Summary

Because of the admission of larger amounts on individual claims into the losses, experience subject to limits of $10,000 per claim and $20,000 per accident will be subject to more fluctuations arising from the relatively infrequent large claims than experience subject to $5,000/10,000 limits. On the basis of a study of New York State private passenger car experience it is estimated that somewhat over 40%, possibly 50%, more claims are needed for experience under the higher limits to have the same actuarial credibility as it would have under the lower limits.

In the conduct of this study, account was taken of the frequency distributions of claims by size and of accidents by number of claims. For the purpose of dealing with the subject of credibility in an analytical way, it was necessary to investigate the mathematical basis for credibility factors and to derive formulas for the coefficient of variation, or relative sampling error, of losses experienced.

In essence, both of these formulas are an extension of the Poisson theory, under which the variance of the number of independent random events occurring is equal to the expected number of events. The need for extension arises from the unequal weight that must be given to different events (due to variation between claims) and the only partial independence of events (due to multiple-claim accidents).

Technical Aspects

Because existing literature does not contain formulas for the coefficient of variation (C.V.) of losses as a function of claims it was necessary to derive such a formula. The formula showed that certain parameters or "constant" statistics descriptive of the distribution of accidents by number of claims were needed. It was possible to make only upward-biased estimates of these parameters from available experience, hence calculations were made both with the computed values of these parameters and with their theoretical minimum possible values in order to indicate the range of possible error in estimates of the C.V. of losses.

Because the C.V. of losses depends not only on the number of claims but on the C.V. of individual claims it was necessary to estimate the latter under 5/10 and 10/20 limits. Available data did not permit reflection of the change in accident limit, with the result that calculations somewhat over-estimate the relative credibility of 10/20 experience. The over-estimate arises because the distribution used (New York State Private Passenger B.I., Stock and Mutual, acc. yr.

1956) reflects an underlying accident limit of $20,000 or more both for a $5,000 claim limit and a $10,000 claim limit. Data necessary to adjust for the change in accident limit are not available. Because the reported distribution went only as high as $5,000 it was necessary to extrapolate to $10,000. Special methods were devised for this purpose to give protection against the danger of gross error inherent in extrapolation.

DETAILS OF THE STUDY

Nature of the Problem

Some Fundamental Considerations

In essence what we require is a comparison of the extent to which chance fluctuations, such as the occurrence of unusually large claims or accidents, reduce the statistical reliability of losses when the limits on individual claims and accidents are substantially increased. This comparison must then be interpreted in terms of the effect such a reduction in statistical reliability should have on the relative weight to be given to indications of experience.

This problem, the statement of which is so simple, involves a number of subtle theoretical considerations as well as the technical complications that may be expected when solutions are attempted to problems for which necessary data are incomplete or useable only in a very indirect way.

We shall implicitly define credibility through the following postulates:

> Let t be the ratio of the observed value of a characteristic (e.g., loss ratio, pure premium, etc. indicated by a body of experience) to the theoretical long-run average or expected value of that characteristic. Let P_1 be the probability that $|t-1|>k$ for one body of experience, while P_2 is the corresponding probability for a second body of experience:
>
> *Postulate I:*
>
> The credibility of the first body of experience is greater than, equal to, or less than that of the second as P_1 is respectively less than, equal to or greater than P_2 for all values of k.
>
> *Postulate II:*
>
> The relative credibilities of the two bodies of experience are indeterminate in the absence of further information when the equality or inequality between P_1 and P_2 depends on k.

For fairly large volumes of experience the mathematical derivation of credibility rests upon Postulate I. For small volumes of experience, this postulate suffices to show that 10/20 losses for a given volume have lesser credibility than 5/10 losses. Measurement of the degree to which it is less, however, proves to be indeterminate. This is so because when we attempt to ascertain what greater volume of experience under 10/20 limits would have the same credibility as a given volume under 5/10 limits we find that the hypothesis of Postulate II is fulfilled. This paragraph is amplified in Exhibit A.

For large bodies of experience, that is, where the number of claims developed is sufficient that losses have a practically normal probability distribution, credibility can be calculated in terms of the coefficient of variation (C.V.) under 5/10 and 10/20 limits. This is true because in such a case probabilities have a nearly uniform correspondence with this parameter. For small volumes, however, this uniform correspondence disappears. In terms of the probability of a chance deviation exceeding a small or moderately large percentage, the drop in credibility for 10/20 vs. 5/10 experience is less than would be indicated by the normal curve for the corresponding increase in the C.V., but at the same time the probability of a really large chance deviation is disproportionately greater.

These relationships can be appreciated from consideration of the effect of a single large claim on small bodies of experience. So long as no large claims occur, losses will exhibit fluctuations under 10/20 limits not necessarily in excess of those exhibited under 5/10 limits. Yet a very large claim or accident will obviously have twice as much effect on indications with 10/20 limits as with 5/10 limits. For example, consider an expected number of claims equal to 11, which under our present table would correspond to 10% credibility. Calculations based on the Accident Year 1956 Size of Claim Data, N. Y. State Private Passenger B. I., Stock and Mutual Combined, indicate an average claim cost of $732 with policy limits of $5,000 per claim and $827 with policy limits of $10,000 per claim (accident limitation of $20,000 on more in each instance). Expected losses would then be $8,052 with 5/10 limits and $9,097 with 10/20 limits. In the first instance an additional claim equal to the claim limit would result in a formula increase in rates of 6.2%, while in the second instance the formula increase would be 11.0% using 10% as the credibility in both cases. The formula increases with an additional accident equal to the accident limit would of course be much greater.

To appraise the significance of such a comparison it is worthwhile to look at probabilities. To simplify matters, we shall consider it as given that 12 claims are incurred and the first eleven produce total losses equal to the expected, then compute the probability that the twelfth claim is (a) as large as $5,000 and (b) as large as $10,000. These probabilities according to Exhibit A are the complements of .970 and .987 respectively, or .030 and .013. In view of these low probabilities, moderate errors in appraisal of credibility for small volumes of experience will so infrequently produce significant departures from theoretically proper formula rates (whatever such may be) that we are justified in treating credibilities under the hypothesis of Postulate II as equal for our purposes. This assumption will enable us to calculate credibilities in terms of the C.V. of losses for all volumes of experience.

Formula

Previous formulas for the C.V. losses were expressed in terms of the number of accidents rather than the number of claims for theoretical reasons (the Poisson assumption is considered to be valid in liability insurance for accidents but not for claims). A formula expressed in terms of claims is needed, however, because it is the number of claims, rather than of accidents, which is reported. The formula (derived in Exhibit C) is:

$$V_L^2 = \frac{V_c^2 + Em(1 + V_m^2)}{EN} \tag{1}$$

the symbols being defined as follows:

V = Coefficient of variation of its subscript variable

L = Losses

m = Number of claims per accident

N = Number of claims

C = Size of claim

E = Denotes expected value of variable following

The parameters, Em, V_m^2 and V_c^2 are constants which must be determined in advance. Values of N are reported. EN may be estimated as equal to N (the most accurate estimate where N is large enough) or may be estimated in other ways.

Description of Calculations

We do not have adequate data immediately available for calculation of Em, V_m^2 and V_c^2, but the following calculations have been made from what data there are:

Calculation of Em and V_m^2

From a distribution of accidents by size based on New York State Private Passenger B.I. experience for accident year 1957 we have computed that for accidents producing excess losses $Em = 1.7$ and $V_m^2 = 1.0$. (Exhibit F.) These values should be regarded as high estimates since accidents producing excess losses, by reason of their severity, would be expected to produce more claims and a greater variation in the number of claims, than other accidents.

Calculation of V^2_c

The value of V^2_c has been estimated from the 1957 call for Size of Claim Data, Private Passenger Cars, B.I., for New York State, Stock and Mutual Combined (Accident Year 1956). It was necessary to employ certain artifices in this calculation because the data used were not strictly what was needed. Actually, two size of claim distributions were needed, one with an underlying accident limitation of \$20,000 and showing claims at least up to \$10,000, and the other with an underlying accident limitation of \$10,000 and showing claims at least up to \$5,000. The reported data showed claims up to \$5,000 and had underlying accident limitations of \$20,000 or more.

The calculation of V^2_c for a \$5,000 limitation was straight-forward and is described in Exhibit B. This would appear to be a high estimate because the underlying accident limitations were \$20,000 or more, rather than \$10,000, and with the lower accident limitation some claims would be reduced in size. If it be assumed that claims involving excess losses by reason of the accident limitation have a larger average than all claims combined, the pro-rata scaling down of claims under an accident limitation would tend to reduce V^2_c. This seems reasonable to expect, hence we have assumed an upward bias in the calculation of V^2_L for a \$5,000 claim limit.

The calculation of V^2_c for a \$10,000 claim limit required extrapolation. Since this is a "dangerous" type of calculation, special measures were taken to protect against serious error. The value of V^2_c as a function of $\hat{c}$, the claim limit, was expressed in terms of two logarithmic transformations of $\hat{c}$ to yield two curves, one concave upward and the other concave downward. Extensions of secants constructed through the last two data-supported points (\$4,000 and \$5,000 size of claim) on these curves provided upper and lower limits to the estimate of V^2_c for a \$10,000 claim limit. These limits were 3.51 and 3.33, their mean being 3.42 before adjustment for grouping error and 3.45 after this adjustment. This calculation appears on Exhibit D.

An independent calculation was made by finding a transformation of the claim-size variable that would bring the cumulative distribution of claims into agreement with the integral of the Normal Curve at \$3,000, \$4,000 and \$5,000. The distribution of claims from \$5,000 to \$10,000 and the proportion of claims that would be limited to \$10,000 was then calculated from the Normal Curve. This calculation yielded a value of 3.47 for V^2_c with a \$10,000 limit. This value (rounded to 3.5) was used in subsequent calculations since it is the most precise determination made and falls within the previously established upper and lower limits. (Exhibits B and E.)

Calculation of V_L^2

Values of V_L^2 were calculated by means of Eq. (1) using the values of 1.7 for Em, 1.0 for V_m^2, 2.2 for V_c^2 with a $5,000 limit and 3.5 for V_c^2 with a $10,000 limit. Because of the upward bias in the values used for Em and V_m^2, calculations were also made setting Em equal to 1.0 and V_m^2 equal to zero, these being the minimum possible values of these parameters since they correspond to the condition that every accident consists of a single claim.

Although the values used for Em and V_m^2 have a marked effect on V_L^2, they have little effect on the ratio of the value of V_L^2 developed with a $5,000 claim limit to the value of V_L^2 developed with a $10,000 claim limit.

Calculation of Relative Credibility

The theoretical justification for basing credibility on V_L has been mentioned above in connection with the relation of this statistic to probability. A brief discussion of this question from the point of view of controlling the contribution of indicated rates to the mean square error of formula rates is given in Exhibit G.

In accordance with these concepts, the comparative credibilities of 10/20 and 5/10 experience have been estimated from the ratio of the value of V_L with a $5,000 claim limit to the value of V_L with a $10,000 claim limit. This ratio is .84 with $E\text{m} = 1.0$, $V_m^2 = 0$, and .90 with $E\text{m} = 1.7$, $V_m^2 = 1.0$.

As mentioned earlier, the values of V_c, hence of V_L, computed for a $5,000 claim limit, reflect the same underlying accident limitations as the values computed for a $10,000 claim limit ($20,000 or more), rather than an accident limitation half as great as the accident limitation associated with a $10,000 claim limit. Therefore values of V_L for a $5,000 claim limit as well as the ratios just given must be regarded as biased upward. In view of this bias, the credibility of 10/20 experience should be somewhat less than 85%, perhaps 80%, as great as the credibility of 5/10 experience.

If we express these results in terms of the number of claims, we find that 10/20 experience would require at least 40% more claims for full credibility to retain the same statistical reliability as 5/10 experience. It will be noted that the increase in claim requirements is more than proportional to the decrease in credibility. This is because of the inverse square relationship between claims and credibility.

EXHIBIT A
Sheet 1

COMPARISON OF PROBABILITY DISTRIBUTIONS OF CLAIMS AND OF LOSSES WITH $5,000 AND $10,000 CLAIM LIMITS

I. Basic statistics on individual claims

	$5,000 Limit	*$10,000 Limit*
(a) Mean	$ 732	$ 827
(b) Coefficient of variation	1.48	1.86
(c) Standard deviation	1,080	1,540

II. Tabulation for selected ranges above and below the mean *in units of standard deviation,* based on individual claims

	Dollar Range		*Fraction of Distribution Within Range*		
No. of σ's	*$5,000 Limit*	*$10,000 Limit*	*$5,000 Limit*	*$10,000 Limit*	*Normal Curve*
.1	624 – 840	673 – 981	.087	.106	.080
.2	516 – 948	519 – 1,135	.181	.223	.159
.5	192 – 1,272	5 – 1,597	.531	.757	.383
1.0	0 – 1,812	0 – 2,367	.900	.920	.683
1.5	0 – 2,352	0 – 3,137	.921	.945	.866
2.0	0 – 2,892	0 – 3,907	.940	.959	.955
2.71	0 – 3,659	0 – 5,000	.954	.970	.993
3.95 {	0 – 4,999.99	0 – 6,910	.970	.980	.99995
	0 – 5,000		1.000		
5.96 {		0 – 9,999.99		.987	
		0 – 10,000		1.000	

III. Tabulation for selected ranges of *percentages of the mean* above and below the mean, based on individual claims

	Dollar Range		*Fraction of Distribution Within Range*	
Percent	*$5,000 Limit*	*$10,000 Limit*	*$5,000 Limit*	*$10,000 Limit*
10%	659 – 805	744 – 910	.060	.034
20	587 – 878	662 – 992	.120	.112
50	366 – 1,098	414 – 1,241	.323	.312
100	0 – 1,464	0 – 1,654	.875	.890
200	0 – 2,196	0 – 2,481	.912	.922
300	0 – 2,928	0 – 3,308	.940	.947
400	0 – 3,659	0 – 4,135	.954	.961
500	0 – 4,392	0 – 4,962	.964	.969
583 {	0 – 4,999.99	0 – 5,648	.970	.974
	0 – 5,000		1.000	
1109 {		0 – 9,999.99		.987
		0 – 10,000		1.000

EXHIBIT A
Sheet 2

The points to be noted are that: (1) In units of standard deviation the more skewed claim distribution developed with a $10,000 claim limit shows more concentration around the mean until the $5,000 limit is reached, while beyond that limit the $10,000 claim limit permits 3% of claims to take on larger values than $5,000. (2) In terms of percentage deviation from the mean the $10,000 claim limit shows more concentration around the mean only from ± 100% of the mean to the $5,000 limit.

With losses (random aggregates of claims) developed in successive experience periods under the same conditions of hazard, the comparison would be qualitatively similar but the characteristics of the two distributions would become less and less distinct as they converged toward the normal distribution with increasing numbers of claims expected in each experience period. If the probability of a variate falling in a given range of one distribution is sometimes greater and sometimes less (depending on the size of the range) than the probability of the variate of the second distribution falling within the range, then the hypothesis of Postulate II, page 236, is satisfied. [It may be remarked that increased volume required under 10/20 limits to yield a C.V. equal to the C.V. for a given volume under 5/10 limits would partially offset the greater kurtosis and skewness of the 10/20 claim distribution. It would not completely offset them, however, because the effect of the claims over $5,000 on the higher moments of the distribution is necessarily greater than their effect on the C.V.]

EXHIBIT B
Sheet 1

DISTRIBUTION OF CLAIMS BY SIZE
NEW YORK STATE
PRIVATE PASSENGER BODILY INJURY INSURANCE

Stock and Mutual Carriers Combined

Accident Year 1956 or 12 Months in 1956-1957

(1) Size of Claim At Least	(1) Size of Claim Less Than	Number of Claims (2) Within Interval	Number of Claims (3) Accumulated Up	Proportion of Claims (4) Within Interval	Proportion of Claims (5) Accumulated Down	(6) Losses Within Interval	(7) Average Claim Cost Within Interval
	25	4,820		.0547	.0547	45,395	9.42
25	50	5,548	83,272	.0630	.1177	197,983	35.69
50	100	7,396	77,724	.0840	.2017	486,057	65.72
100	250	16,239	70,328	.1843	.3860	2,542,915	156.59
250	500	18,311	54,089	.2079	.5939	6,315,379	344.90
500	1,000	17,932	35,778	.2035	.7974	11,939,730	665.83
1,000	2,000	9,444	17,846	.1072	.9046	12,548,756	1,329.
2,000	3,000	3,267	8,402	.0371	.9417	7,787,658	2,384.
3,000	4,000	1,589	5,135	.0180	.9597	5,421,409	3,412.
4,000	5,000	872	3,546	.0099	.9696	3,856,389	4,422.
5,000	6,000	551	2,674	.0063	.9759	2,975,000	5,400.
6,000	7,000	370	2,123	.0042	.9801	2,368,000	6,400.
7,000	8,000	273	1,753	.0031	.9832	2,020,000	7,400.
8,000	9,000	203	1,480	.0023	.9855	1,705,000	8,400.
9,000	10,000	158	1,277	.0018	.9873	1,485,000	9,400.
Sub-Total		86,973		.9873			
Over 10,000		1,119	1,119	.0127			
Grand Total		88,092		1.0000			

See Sheet 2 *for explanation of the calculation of* V_c^2.

Derivation of column entries:

Col. (2) As reported up to 5,000 size of claim. Computed by differencing col. (3) beyond 5,000.

(3) Computed by accumulating col. (2) up to 5,000; beyond 5,000, entries equal (5) $\times \Sigma$ (2) $= 88{,}092 \times$ (5).

(4) Computed by differencing col. (5).

(5) Up to 5,000, entries equal the downward accumulation of (2) divided by Σ (2). Beyond 5,000, entries are as computed on Exhibit E.

(6) As reported up to 5,000 size of claim. Beyond 5,000, entries equal (2) $\times$ (7).

(7) (6) $\div$ (2) up to 5,000 size of claim. Beyond 5,000, entries are selected at 400 above the lower limit of the interval in consideration of the positions of the averages within the 3,000-4,000 and 4,000-5,000 interval.

EXHIBIT B
Sheet 2

The formula used for computing V_c^2 for each claim limitation is $V_c^2 = [(\Sigma FC^2)/N - (EC)^2] \div (EC)^2$ where V_c^2 is the squared coefficient of variation, C represents claim cost, EC is the mean of the claim costs, F is the number of claims in each interval, and N is the total number of claims. Each C value used is the average claim cost in the interval. To compute V_c^2 for a specified claim limitation, all claim sizes greater than the limitation were assigned the value of the limitation. For example, let us refer to Sheet 1 to illustrate the computation of V_c^2 for a $250 claim limitation. To obtain EC, the items items in column (6) were summed for claim sizes less than $250. The corresponding item in column (3), 54089, representing the number of claims whose size is greater than $250, was multiplied by $250 and added. This result was divided by the total number of claims, 88,092, to obtain EC. To compute the value of $\frac{\Sigma FC^2}{N}$, the items in column (7), average claim cost, were squared, multiplied by the corresponding items in column (2), claim frequency in interval, and added for all claim sizes less than $250. The corresponding item in column (3) was multiplied by $(250)^2$ and added. This result was divided by the total number of claims, 88,092. For the limits of $5,000 and $10,000, .04 and .03 respectively were added to the values so calculated to offset the reduction in variance introduced by grouping. The final values were rounded to 2.2 and 3.5 respectively.

EXHIBIT C

Derivation of Formula for Relative Variance, or Squared Coefficient of Variation, of Losses as a Function of the Number of Claims

Definition of Symbols

L = losses
N = number of claims
n = number of accidents
m = number of claims per accident
$\bar{m}$ = average number of claims per accident = N/n
C = cost per claim
$\bar{C}$ = average cost per claim = L/N
S = standard deviation of subscript variable
E = expected value of variable following
V = coefficient of variation of subscript variable, e.g.,
$V_u = S_u/Eu$. V^2 is the relative variance.

Since losses are a sum of claims

(1) $$L = C_1 + C_2 + \ldots + C_N$$

(2) $$= N\bar{C}$$

(3) $$EL = ENEC$$

if average claim cost and the number of claims are independent in their random fluctuations, as may ordinarily be expected in automobile liability insurance.

(4) $$EL^2 = E(C_1 + \ldots + C_N)^2$$

(5) $$= E(C_1^2 + \ldots + C_N^2 + \Sigma C_i C_j);$$

there being $N(N - 1)$ cross products with $i \neq j$.
To the extent that each claim is statistically independent of the others we are justified in taking the sum of the cross products as $N(N - 1)(EC)^2$.

Then for any particular value of N

(6) $$EL^2 = NEC^2 + (N^2 - N)(EC)^2$$

and over all N

(7) $$EL^2 = ENEC^2 + (EN^2 - EN)(EC)^2$$

(8) $$= EN[(EC)^2 + S_C^2] + [(EN)^2 + S_N^2 - EN](EC)^2$$

(9) $$S_L^2 = EN(EC)^2 + ENS_C^2 + (EC)^2(S_N^2 - EN)$$

since $S_L^2 = EL^2 - (EL)^2$ as a consequence of its definition as $E(L - EL)^2$, the value of EL being taken from Eq. (3).

EXHIBIT C *(Continued)*

Then on division by the value of $(EL)^2$ we have

(10) $\quad V_L^2 = 1/EN + V_C^2/EN + V_N^2 - 1/EN = V_C^2/EN + V_N^2$

But since $N = n\overline{m}$, if the number of accidents and the average number of claims per accident are statistically independent a similar argument with N, n and m standing in the places of L, N and C respectively leads to

(11) $\quad V_N^2 = V_m^2/En + V_n^2$

If the number of accidents, though not necessarily of claims, has a Poisson probability distribution we can substitute in Eq. (10)

(12) $\quad V_L^2 = V_C^2/EN + V_m^2/En + 1/En$

And since we have taken n and $\overline{m}$ to be statistically independent, $EN = EnEm$ and we can write

(13) $$V_L^2 = \frac{V_C^2 + Em(1 + V_m^2)}{EN}$$

For single-claim accidents $Em = 1$ and $V_m^2 = 0$, in which case

(14) $\quad V_L^2 = (V_C^2 + 1)/EN$

the last equation being in agreement with Mr. Arthur Bailey (P.C.A.S. Vol. XXIX, page 60)*.

* It is evident that the approximation given in (1.5), page 58 of the writer's paper, "Graduation of Excess Ratios by the Method of Moments", (P.C.A.S. Vol. XLIV) could have been made exact by omission of the term V_a^2/m^2, that expression being cancelled out by the dropped quantity mentioned in Note †, page 57 of that paper, derived from the small negative correlation between n^2 and $\bar{a}^2$. The writer is indebted to Mr. Robert Bailey, as a result of whose insistence that this term is extraneous, the correlation was recognized. The latter has found that Eq. (14) above is also consistent with his own calculations as well as with those of R. E. Beard, "Analytical Expressions of the Risk Involved in General Insurance", Transactions of the XVth International Congress of Actuaries, 1957, Vol. II, page 233.

EXHIBIT D

CALCULATION OF ESTIMATED VALUE OF V_c^2 WITH A $10,000 CLAIM LIMIT, BY EXTRAPOLATION WITH LIMITING SECANTS

Claim Limitation $= \hat{c}$ *(1)*	*Square of Coefficient of Variation* $= V_c^2$ *(2)*	*Log(2)* $=$ *Log* $\hat{c}$ *(3)*	*Log[(3) — 2]* $=$ *Log(Log* $\hat{c}$ *— 2) (4)*	*[Log(2)] + 2 =* *Log* V_c^2 *+ 2 (5)*	$\frac{\Delta 5}{\Delta 3}$ *(6)*	$\frac{\Delta 5}{\Delta 4}$ *(7)*
$ 25	.0223	1.39794		.34830		
					.923	
50	.0423	1,69897		.62634		
					1.021	
100	.0858	2.00000	— ∞	.93349		
					.882	0
250	.1926	2,39794	—.40018	1.28466		
					.857	1.055
500	.3489	2.69897	—.15554	1.54270		
					.816	1.580
1,000	.6144	3.00000	0	1.78845		
					.802	2.112
2,000	1.071	3.30103	.11429	2.02979		
					.784	2.505
3,000	1.472	3.47712	.16942	2.16791		
					.741	2.627
4,000	1.822	3.60206	.20468	2.26055		
					.7146	2.715
5,000	2.137	3.69897	.23019	2.32980		
6,000						
7,000						
9,000						
10,000	3.51*	4.00000	.30103	2.5449		
10,000	3.33ø	4.00000	.30103	2.5221		
10,000	3.42 (Median Estimate)					
	3.45 (Median Estimate + Adjustment for Grouping)					

* Extrapolated from concave-downward curve (column (3) is independent variable)

3.51 = Antilog (2.5449 — 2)

2.5449 = 2.32980 + .7146 (4.00000 — 3.69897)

ø Extrapolated from concave-upward curve (column (4) is independent variable)

3.33 = Antilog (2.5221 — 2)

2.5221 = 2.32980 + 2.715 (.30103 — .23019)

EXHIBIT E

EXTRAPOLATION OF CLAIM DISTRIBUTION FROM \$5,000 LIMIT TO \$10,000 LIMIT BY TRANSFORMATION OF THE VARIATE

Maximum Claim Size (Thousands of Dollars) $=\hat{c}$ *(1)*	*Transformation of* $\hat{c}$ $=u$ *(2)*	*Cumulative Distribution (Fraction)* $=F(c)$ *(3)*	*Standard Normal Variate* $=t$ *(4)*	*Cumulative Distribution (Number) 88,092* $\times$ *(3) (5)*	*Range Distribution (6)*
3	.57788	.9417	1.5692	82,957	
					1,589
4	.69085	.9597	1.7475	84,546	
					872
5	.77172	.9696	1.8750	85,418	
					551
6	.83478	.9759	1.975	85,969	
					370
7	.88601	.9801	2.056	86,339	
					273
8	.92941	.9832	2.124	86,612	
					203
9	.96695	.9855	2.183	86,815	
					158
10	1.00000	.9873	2.235	86,973	

Col. (2) $= .5\{\log(1) + \log[1 + \log(1)]\}$

Col. (3) is taken from Exhibit B for $\hat{c} = 3$, 4 and 5. For $\hat{c}$ beyond 5, values are taken from the normal curve to correspond to Col. (4).

Col. (4) is taken from the normal curve to correspond to Col. (3) for $\hat{c} = 3$, 4 and 5. For $\hat{c}$ beyond 5, t values are determined from the relationships: $t = (u - \bar{u})/\sigma_u$; $\sigma_u = (u_4 - u_3)/(t_4 - t_3)$; $\bar{u} = u_3 - t_3\sigma_u = u_4 - t_4\sigma_u$. The value of t_5 given by $t_5 = (u - \bar{u})/\sigma_u$ checks with the value corresponding under the normal curve to Col. (3) and thus confirms the validity of the transformation in this region of the distribution.

EXHIBIT F

CALCULATION OF MEAN AND COEFFICIENT OF VARIATION OF THE NUMBER OF CLAIMS PER ACCIDENT

Source: 1958 Call for Automobile Liability Experience, Accident Year 1957, Private Passenger Bodily Injury, National Bureau Members and Subscribers, Accidents Producing Excess Losses

m No. of Claims	*f(m)* No. of Accidents
1	2,072
2	301
3	165
4	120
5	69
6	50
7	15
8	8
9	6
10	4
24	1
28	1
39	1
Total	2,813

$$\frac{\Sigma \mathrm{mf(m)}}{\Sigma \mathrm{f(m)}} = E\mathrm{m} = 1.652 = 1.7$$

$$\frac{\Sigma \mathrm{m^2f(m)}}{\Sigma \mathrm{f(m)}} = E\mathrm{m}^2 = 5.411$$

$$E\mathrm{m}^2 - (E\mathrm{m})^2 = 2.68 = \sigma^2_{\mathrm{m}}$$

$$\mathrm{V}^2_{\mathrm{m}} = \frac{\sigma^2_{\mathrm{m}}}{(E\mathrm{m})^2} = .98 = 1.0$$

EXHIBIT G

EFFECT OF CREDIBILITY WEIGHTING ON THE MEAN SQUARE ERROR IN FORMULA RATES

If credibility is proportional to $1/V_L$, the direct contribution of indicated rates to mean square error of formula rates remains fixed at the same amount as selected for 100% credibility. If a power of V_L less than the first is used in the denominator, the contribution of indicated rates to mean square error increases without limit as credibility approaches zero. On the other hand, if a power greater than the first is used, less information is taken from indicated rates than may be safely used; hence there is an unnecessary sacrifice of responsiveness. This is true because the direct contribution of credibility-weighted indicated rates to mean square error in formula rates is $z^2\sigma^2_L$ where z is credibility and σ^2_L is the mean square error of the indicated rate. If $z = k/\sigma_L$ then $z^2\sigma^2_L = k^2$ regardless of z while if $z = k/\sigma^a_L$, $a < 1$, then $z^2\sigma^2_L = k^2\sigma_L^{2(1-a)}$ which increases without limit as $z \rightarrow 0$ and $\sigma_L \rightarrow \infty$ correspondingly (See Note 2). On the other hand, if $z = k/\sigma^a_L$, $a > 1$, z will be less for any given volume, short of full credibility, than if $a = 1$ and the indication will receive less weight, hence yield less information, then with "a" equal to one, which we have already shown to be a safe procedure.

NOTE 1:

Use of $z = k/V_L$ rather than $z = k/\sigma_L$ is a practical strategem. Since $V_L = \sigma_L/EL$, the direct contribution of indicated rates to mean square error in formula rates is therefore $k^2(EL)^2$ rather than just k^2, but $k^2(EL)^2$ is also a fixed quantity.

NOTE 2:

Even where $a < 1$, in practice a finite upper limit is placed (on the contribution of indicated rates to the mean square error of formula rates) by the adoption of a table of discreet values for z, so that zero credibility applies where σ_L exceeds some finite limit. This procedure does not, however, justify the use of values of "a" lower than one because the contribution of indicated rates to mean square error of formula rates will be larger at the low end of the credibility scale than at the high end and there seems to be no a priori reason for accepting a larger contribution at one time than at another. Furthermore, a credibility table which cannot be extended downward as close to zero as we please without producing dangerously large mean square error in formula rates is mathematically inconsistent.