MULTIPLE PERIL RATING PROBLEMS — SOME STATISTICAL CONSIDERATIONS*

BY

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1. INTRODUCTION

Few will fail to appreciate the misgivings with which a technician approaches a popular excitement like "Multiple Peril Rating." The very name, whatever its inadequacies semantically, can stir up such partialities that the rational approach is often overwhelmed in an arena of turbulent emotions. But this is not a milieu unprecedented for researchers. Early in the day of modern mathematics, Gauss withdrew from many of the then popular contentions to avoid the "clamor of the Boeotians." And from his Holland retreat, two centuries earlier, Descartes sadly observed that common sense was reputed a commodity of which even the most feeble felt they had no lack.

There is something pathetically childlike in the picture of the scholar railing against the busy world from his high chair of scorn. It would appear unseemly for technicians not to attempt some contribution to the industry's needs in the field of multiple peril insurance. Since it is only on the rarest occasion (if we can believe in its possibility at all) that any worthwhile contribution can ever be considered as the sole responsibility of one human's urge to truth, we shall expect that many of the thoughts expressed herein will seem to the reader only some imperfect image of his own ideas. And those other notions, if any, which appear strange in unwanted trappings may encompass valid concepts possibly of some practical value, once they have been analyzed and refined within the exchange of our professional society.

It is proposed that the Homeowner's policy will serve as our point of departure for exploring certain statistical aspects of multiple peril rating. Our search may occasionally lead into speculations a bit afield . . . but never, we hope, away from the essential problem.

2. PLACE OF STATISTICS IN MULTIPLE PERIL UNDERWRITING

Let us start out by carefully assessing what cannot be done. Now, the realm of the impossible may well be the narrowest of kingdoms —and its borders may still be contracting until the circle which would ring this principality ultimately will shrivel to an isolated point in an imaginary plane. But, the impossible has not yet disappeared as those whose faith outrun their reason will soon discover if they let their "likings" dictate their logic. Even in the insurance field there is an outer bound beyond which we should not let our fancy stray.

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* We should like to acknowledge our indebtedness to the late Dr. Henry D. Locke for his inspiration and guidance in the preparation of this paper.

Ours is fundamentally a "figure" business, although our results in certain segments of the industry have not yet attained the precision of the mathematical science. It has long been characteristic in fire insurance underwriting to seek out the risk which will not burn. In the search for this ideal structure, little analysis has been made of the basic consideration of rate with its concomitant postulate of "likelihood of loss." The traditional approach to fire insurance underwriting much resembles the medieval alchemist's quest for the Philosopher's Stone which would turn all things to gold. And yet, were his dream realized, the storied alchemist would be no better off than the unemployed fire insurance underwriters hedged in on all sides by an incombustible world.

Thus, our first demarcation of the impossible. Without losses there can be no insurance business which fundamentally is protection of policyholders against the consequences of loss. Insurance performs, both by providing an accumulation of funds to indemnify for accident occurrences and a prevention service to reduce the likelihood of such occurrences. In an economic sense, loss prevention is the productive service which insurance renders. As a corollary the premium rate, albeit conceivably a fallible approximation to the likelihood of loss, is in the final analysis of even greater importance to the underwriter than the physical characteristics of a particular risk. And the function of our system of statistics presumably is to yield the premium rate structure for the underwriter's use.

The second reality is a realization that fire and wind insurance demands a substantial number of exposures because the individual risk has only a significantly small probability of loss. Many have been the schemes for scaling credibilities. Actuaries have investigated the Bernoullian, the Lexis and the Poisson distributions. They have searched the many variations of the Pearson Curves and the Charlier systems. Sometimes it has seemed that the range or the median offered more promise than the mean and the deviations therefrom. But heated as may have been on occasions the defense for each of the various partialities, there has always been a basic understanding that the believability of the loss experience tended not to be independent of the sample size—and that the required exposures must in some way be related to the probability of loss. In interpreting the statistics, the knowledgeable underwriter must establish for himself a meaningful scale of credibilities (judgments).

3. CAN "INDIVISIBLE MULTIPLE PERIL" BE A USEFUL AND MEANINGFUL STATISTICAL CONCEPT?

It is said that the Homeowner's policy introduced glamor to the insurance business. Unquestionably, the merchandisers soon recognized the potentialities in concepts as nebulous as "multiple peril" and "packaged policy" and were quick to marshal popular slogans to support their cause. At a certain high plane, there was probably ample justification for enthusiasm. The production forces of the industry and the general public, as well, have evidently welcomed this development with a truly remarkable premium growth.

We should not be surprised that in an industry so dependent on "paper work" there would be many to grasp at a plan promising to cut "red tape" and "all those unnecessary details." Moreover, to the typical policyholder the one indivisible rate for all the coverages in the package seems refreshingly straightforward and clear. His is a transfer solely of dollars for protection. Whatever his potential curiosity as to the justification for the dollars charged, he is seldom eager enough to persist through the technical make-up of the insurance charges. And this is as it should be . . . for the policyholder to concentrate on the total dollar price and purposely neglect the troublesome problems of ascertaining the costs involved in the various elements of protection.

On the other hand, the technician responsible for rate adequacies cannot dismiss in cavalier fashion the fundamental link between protection and price. For him there can be no easy retreat behind the popular obscurities of "Multiple Peril Packages." He might well reflect that a package is supposedly a neat and trim contrivance for handling a small number of items . . . not a bulky crate into which is squeezed a multiple assortment of oddities. He himself does not fail to see that two is a multiple of one. At the same time he recognizes that, in the popular fancy, "Multiple Peril" has acquired an extension hardly to be confined within any limit short of an indefinitely large number of perils.

From years of training and experience, the technician fully appreciates that it is only by recording our experience according to a logical frame work that we are able to move forward from and, because of, the accumulated knowledge of the past. In his philosophical writings, J. S. Mill analyzes the terms or elements of classification systems according to their:

1. denotation . . . the extension or the scope of entities to which the term may be applied.
2. connotation . . . the qualities or characteristics, the possession of which implies the entities as proper members of the class or term.

It is observed that as the denotation or extension is increased, the connotation or specification is decreased. This appears a two-way rule. Possibly at one end position, the single note of existence is possessed by all things . . . and at the other extreme, a complete enumeration of characteristics reduces the class to a single member, specified in all its individual details.

As classification systems approach either extremes, it would seem that the statistics thereon become less meaningful and less useful. While we cannot expect the definitions of our "Multiple Peril" classifications to be launched with the precision of the logician's standards, it would seem that care might be exercised to avoid, if possible, egregious errors which cannot help but rob much of the meaning from our summaries of loss experience.

Possibly the technician should not squander a disproportionate share of his concern on the imprecision of "Multiple Peril" as an entity per se. There appears, at least at the present time, possibly an even greater danger in the constant reassemblings of the policy perils . . . now adding, now subtracting, now adding and substracting simultaneously. "In" yesterday, "Out" today, "Back" tomorrow with his troublesome brother . . . such is the prospect for "Childe Coverage" in the topsy-turvy land of "Multiple Peril." The situation has now or will shortly reach the point where evaluation of loss experience becomes most confusing.

Everyone must surely know that statistics are useless and even devoid of meaning without a significant degree of stability. In all scientific endeavors (and the research statistician's field is no exception) our definitions of classifications must maintain a basic consistency in use and in time. Deliberately to superimpose switches in classification definitions upon the normal uncertainties surrounding our langage understandings is to invite a degree of chaos that no prudent technician would care to contemplate.

Thus in summary of this section: the concept "Multiple Peril" is sufficiently ambiguous, in itself, to warrant the strongest representations for a logical determination of coverage definitions . . . with no less regard for the insurance industry's innate need for continuity of consistent statistics than for the legitimate demands of the buying public.

4. INTERPLAY OF EXPERIENCE AND RATES BETWEEN MULTIPLE PERIL AND INDIVIDUAL POLICIES

For a number of years now, fire underwriters have been warning of a sharp upturn in loss ratios on dwelling fire coverage (i.e., when written on the traditional . . . individual policy basis) with the increasing popularity of the Homeowner's policy. A study of the trend in fire classification loss ratios will testify to the accuracy of this prediction. But the sequence of the events does not necessarily demonstrate an underlying causality, since we tend to shy away from "post hoc ergo propter hoc" arguments.

By implication the underwriter might have us believe that the purchaser of a Multiple Peril dwelling policy is a better fire risk than the the other homeowners. While there may be no ready loss cost data statistically significant to substantiate this conclusion, one would not be at loss to find easy rationalizations of a most persuasive tenor.

On the other hand, we estimate that there was an 11.4% decline in dwelling rates over the 5 years (1953-57), the period covered by the latest available industry data. This figure is based on our company's geographical distribution of business, and insurers with different premium mixes will undoubtedly come up with different answers. However, we suspect that few technicians will fail to agree that there was a significant erosion in fire dwelling rates over the studied period. Admittedly there were various attempts to match these fire rate reductions with decreases in Homeowner's premium charges. But it might be argued that this action has no bearing on dwelling fire loss ratios, per se, and anyway, few raters would attempt to adjust the fire portion from the overall Homeowner's rate with any substantial degree of confidence.

It is also important to consider that the ratio of insurance to value may be significantly less for dwelling fire than for Homeowner's policies. "Under insurance" could be thought a function of the time interval from the original purchase of the policy. On such a theory we would expect a significantly smaller ratio of insurance to value on the fire dwelling policy, a contract of long standing compared to the Homeowner's which started only in the middle 1950s. And with time, as the argument would go, the Homeowner's too will begin to suffer more and more from "under-insurance" . . . and, in this regard at least, be no better than the dwelling fire policy.

The statistician can listen to such arguments with a Horatian "unice securus" . . . sublimely indifferent to their underwriting justifications. He does not, however, fail to appreciate the importance to his company and to his own fortunes of the underwriter's ability to select better risks than provided for in average rates. And he would help by pointing out the dangers of unsupported generalizations and statistical systems founded thereon.

To statistical theory, it makes no difference whether the risk "Homeowner" is better than the risk "Dwelling Fire", or vice versa. In either case there is a logical necessity for the establishment of separate classes and subclasses . . . only if the elements which go to make up the class Homeowner are significantly different, and have a natural bond and/or barrier which distinguishes them from the Dwelling Fire risks.

Contrariwise, the statistical design and the underwriting distinctions established thereon can fail if these two conditions are not met. If there are only chance variations in pure premiums, the rates based thereon will gyrate haphazardly one to the other. Consequently, there could be traffic "in" or "out" of the class Homeowner's depending upon the rate relativities prevailing at the moment. As to the second consideration, even if there were a significant difference between the constituent elements of the original classes, the statistical plan would become progressively inoperative . . . if the risks in one class could switch to the other class solely as personal considerations dictated, without any significant modification in the characteristics of the elements constituting the class.

In summary of this section: the technician will probably be unconcerned in theory as to the effect of the withdrawal of the "better" risks from the fire dwelling to the Homeowner's policy as long as there are statistics to indicate the necessity for the resulting higher rates. On the other hand, he would be much concerned if the classification system were vulnerable to the arbitrary determinations of the very elements which were supposed to make up the respective classes. His misgivings would even increase, if he had some indication that the relative movement of the subsidiary coverages, package to individual policies, were responsive primarily to chance variations.

But time may prove that there are both distinguishing notes and compelling reasons for membership in one as contrasted with the other class. Moreover, if some of the component coverages of the Homeowner's fluctuate loss-wise solely by chance vis-a-vis the corresponding individual policies, it may be that the force of the random variation would not "swallow-up" the inherent difference in loss cost between the two modes of affording protection to the respective policyholders. No thoughtful person would assume that chance might whip the loss patterns for each sub-coverage of the policy along according to the same time schedule. It is sometimes reassuring to reflect that bad fortune itself is fickle.

5. BASIC STATISTICAL MEASURES FOR MULTIPLE PERIL POLICIES

The basic statistical measure for the Homeowner's policy is the ratio of the dollar losses to the dollar premiums. This loss ratio technique was lifted with hardly a change from the fire insurance field. The plan for a subsidiary measure based on the number of policies was short-lived. There were many to argue that the policy count is an unsatisfactory measure of exposure. Moreover, the "dollars of premium" standard is automatically processed through the accounting and statistical routines of the company. It is subject to ready and relatively inexpensive verifications. The concept is understood and easily handled by the many non-technicians with whom the insurance industry must work. While this "loss ratio" approach proved unserviceable for many important casualty lines, it has long been used in the fire field to no detriment of the companies or of the composite of their policyholders.

The ultimate reality of the insurance venture must be the dollars "taken in" compared to the dollars "paid out". For economic enterprises, outgo cannot exceed income indefinitely. If one can gauge the total dollar losses, he can tell what the overall minimum premium must be. It is conceivable that in a static economy, one might run an insurance operation solely on losses with no advertence to exposures. If, however, there are significant variations in exposure in time; or if there is a demand for exact equities among classes of risks, an exposure measure becomes a useful and perhaps an indispensable tool.

The notion of exposure is traditional in the development of mathematical probabilities. Without a knowledge of the possible numbers of happenings in the Chevalier de Méré's gaming exercises, Pascal and Fermat could never have conducted the original research into mathematical probabilities. From these first beginnings, elaborate techniques have been devised; fundamental concepts have been investigated and refined; and further modifications in methodology have often resulted from the added precision and extensions in the basic notions. Also in this continuing evolution of the science which underlies the insurance business, no thoughful students have denied the reality of the "event" or the necessity of an "exposure element" which must precede the event. Admittedly, modern statistical theory regards the possible numbers in the universe as an unknowable entity . . . but it is customary to posit a ratio of favorable to possible happenings . . . and then in experimentation observe the actual number of occurrences out of the total number of trials (i.e. exposures).

The familiar event (a counting of units out of total possible occurrences . . . devoid of any mark other than that of "happening") is obviously very simple compared with demands for an exposure count in insurance statistics. For the Homeowner's policy, the exposure should attempt to gauge as a minimum the composite of the number of units susceptible to loss, the sum of the values on these risks, and the length of time for which the risks are exposed.

Of the number of fine papers (listed in the appended bibliography), two treated in considerable detail on the inadequacies of the earned premium as an exposure measure in the fire insurance field. These observations likely apply *a fortiori* to the Homeowner's policy. Even on a policy which affords a more or less single uniform coverage like the fire insurance contract, the premium exposure measure is fraught with limitations. It is not easy to turn written to earned premiums on a classification basis. And to adjust for rate changes and annual payment plans, the corrections become most difficult, even for the accomplished actuary. In passing, we must mention that the premium becomes all tangled up with term credits, special rating plans, deviations . . . all of which tend to invalidate the premium as a usable standard for exposure, despite its eminent qualifications as an accounting tool.

Losses by cause (provided by the Homeowner's statistical plan) to total earned premiums is a most deceptive measure. What should be allowed by cause, it is not easy to say. How many of the dollars collected should be reserved for windstorm losses and for how long a period is most baffling. It is a paltry contribution to the sum of insurance knowledge to record that for a given period, one fourth of the dollar volume of losses are chargeable to wind, a third to fire, 12% to crime, 10% to liability, and the balance to an assortment of changing coverages. Such knowledge is most academic, a technician might observe, if the rate is to be determined solely by the totality of losses. If one does not know or cannot admit what portion of the total rate must be reserved for windstorm losses, what good are analyses of losses by subsidiary coverages?

In rebuttal, one might observe that losses by each cause can be expressed in points contribution to the overall Homeowner's loss ratio. As an alternative, a company can sample its business by state . . . and using rates (discounted) for subsidiary coverages, project its probable premium breakdowns by major coverage components. Underwriters might use such experience projections in a first review of their results.

But now, if losses are to be related to exposed annual dollars, it may be another matter entirely. If a known portion of the basic ECE rate should be earmarked for windstorm losses (and only chance knows when) then maybe this segment of the necessary premium dollars can be set aside and not dissipated when supervisory authorities take the insurance companies at their word and adjust rates to a 54% loss ratio overall.

In summary of this section: for a Multiple Peril policy, the statistical system should allow for analysis by cause of loss. With the ever-present likelihood of changes in the package of coverages, it becomes imperative, rate-and underwriting-wise, to know the loss distributions at least according to major hazards. This analysis by cause of loss will, however, be of only limited value unless losses are measured against consistent exposure elements which will be free of the limitations cited for the standard accounting of earned premiums.

6. STATISTICAL PLANS FOR COLLECTING MULTIPLE PERIL EXPERIENCE BY CLASSIFICATION

Now granted first that a valid distinction can be maintained between the general Homeowner's and Non-Homeowner's risks and secondly that the statistical design affords an analysis of losses by subsidiary coverage causes against a meaningful and consistent exposure measure, we are then led to possible finer breakdowns of the genus Homeowner's according to various classification schemes. It is conceivable that these further investigations might lead to formal rate differentials by classification, but it is more likely that at least initially such knowledge will be reserved by the underwriter chiefly as another guide for risk selections.

The skillful underwriter much resembles a mathematical intuitionist beset with a restless curiosity. He constantly lives with his portfolio of business. He is forever speculating on the characteristics of his risks which produce losses. He will readily support and often initiates programs for testing his underwriting theories. The researcher can, in such an environment, make a significant contribution to his company's underwriting fortunes when dealing with a relatively new policy like the Homeowner's about which so little is as yet actually known.

We suspect that the present statistical plan under which the Homeowner's classification experience is published by the industry and filed by the companies may be somewhat too sketchy to satisfy for very long the more imaginative underwriter. If the Homeowner's were to be a small volume line, there would be no reason for any detailed analysis of risk characteristics. The underwriter might be satisfied to review his experience simply by policy form by certain geographical areas on a line like the PPF which will average industry-wide only some $50 million dollars in annual earned premiums, even twenty years after it was originally designed.

But on the Homeowner's, which within five years has spiralled to a volume greater than the entire Inland Marine coverage, of which the PPF is only a modest fraction, every underwriter is faced with a challenge he cannot afford to dismiss. It may not be enough for the underwriter to know the "B" Policy in a certain state has produced an adverse loss ratio . . . with few or no facts with regard to the identity or the characteristics of the risks producing the losses. While each underwriter will want to create his own system, the following is a partial list of possible variants for a subsidiary classification plan: occupation of the assured, income level, size of family, its standard of living, the assured's personal character, the size of the home, its age, its upkeep, the economic and social level of the community, its prospects for the future.

In summary of this section: the emergence of a formal classification plan to measure inherent risk characteristics and set appropriate rate schedules for the Homeowner's policy may still be far in the future. This fact in itself, plus the indication that the Homeowner's will be a major element in the personal lines field is sufficient incentive to start the skillful underwriter probing for facts. The actuary has, therefore, a real opportunity to help in the formulation of his company underwriting policy in this area . . . and particularly to guard against the creation of underwriting policies inconsistent with statistically significant findings.

7. CREDIBILITY AND STATISTICS

Under "Insurance Credibilities" is bedded an expectation of consistency. Those pre-notional images of ours of successive runs of non-irregular happenings merge subconsciously into a mental disposition wherein we instinctively seek to evaluate events in terms of various tests thought responsive to a "law of large numbers."

Now, no attempt will be made herein to apply to insurance credibilities the various interpretations of the logical foundations for probability. We shall note, *en passant,* only that the argument continues unabated at a most austere and recondite scholarship between the behaviorists championed by Von Mises, R. A. Fisher and earlier by Venn and the axiomatic theories of Carnap and Jeffreys. The first tend to view probability as an empirical concept to be applied only in cases wherein the relative frequency in an infinite sequence would approach a limiting value. The latter may possibly be typified as holding that probability in the number value to be assigned to the logical truth or analytic consistency of two given propositions . . . which may be designated as the statements of evidence and conclusions. Personally, the writer feels that the two schools contain basic elements which are not mutually contradictory at all levels of understanding.

But to the scholar as well as to the layman probability usually presupposes, in some fashion or other, an ordered randomness. On occasions the underlying pattern emerges only after years of study. Frequently, the merits of the statistical indications are widely accepted before the precise mathematical relationships are determined.

Fire loss frequencies (an important coverage under the Homeowner's policy) afford a very pertinent example of the ordered randomness underlying insurance probabilities. The writer and certain of his colleagues have attempted to express the relationship mathematically. Our efforts thus far are considered unsatisfactory in that the resulting equations were not readily handled algebraically, nor easily explained. But the statistics seemed to indicate a fundamental underwriting character of fire insurance risks. The following tables give the percentage distribution of our Fire PD losses by individual loss size over the 10 years 1949-1958 . . . for all classifications combined.

	(1)	*(2)*	*(3)*	*(4)*	*(5)*	*(6)*
	Number of Losses			*Amount of Loss Payments*		
Year	*Under $100*	*$100 to 9,999*	*$10,000 & Over*	*Under $100*	*$100 to 10,000*	*Over $10,000*
1949	.742	.251	.007	.072	.547	.381
1950	.735	.255	.010	.051	.418	.531
1951	.752	.241	.007	.070	.496	.434
1952	.757	.233	.010	.052	.400	.548
1953	.755	.237	.008	.051	.444	.505
1954	.751	.241	.008	.056	.460	.484
1955	.750	.244	.007	.057	.424	.519
1956	.723	.267	.010	.043	.360	.597
1957	.692	.301	.007	.050	.487	.463
1958	.696	.297	.007	.046	.472	.482
Mean	.735	.257	.008	.055	.452	.493

We would not expect that corresponding samples for other companies would reproduce the above tabular indications. However, we would be even more surprised at any radical variations. Seemingly, our competitors would soon be out of business if their results on our risk distribution should reverse our columns (1) and (3). This, in the nature of things, cannot happen . . . no more than the sun can fail to rise one morning . . . without the world ending.

We should like to cite a second experiment with the inherent, "ordered randomness" to be encountered in Fire & Allied Lines statistics. This time, the series can be shown to observe a usable and familiar mathematical equation. The series involves windstorm losses, specifically the number of hurricane losses each year reaching the continental United States. We shall, later in this section, return to these data in attempting to generalize on the possible credibility requirements for Multiple Peril coverage, specifically the Homeowner's policy.

The mathematical series is the Poisson exponential which has been documented in a number of fine papers in our Proceedings as a reasonably satisfactory representation of loss probabilities on certain important casualty lines. We have tested the goodness of fit with Karl Pearson's Chi-Square test developed at the turn of the present century and still employed widely in certain comparisons of experiment to expectation. It will suffice to indicate these familiar equations with reference in the bibliography hereto for those who may wish to recheck the mathematical derivations.

Poisson Exponential

m = mean of observations

r = observed number of successes

e = 2.72 approx.

$$P = \frac{e^{-m}.m^{r}}{\lfloor r}$$

Chi-Square (X^2) Distribution

$$T_{k-1}\ (x^2)d(x^2) = \frac{(x^2)^{\frac{k-3}{2}} . e^{\frac{-x^2}{2}}\ d\ (x^2)}{2^{\frac{k-1}{2}}\ \Gamma\left(\frac{k-1}{2}\right)}$$

$$\text{where } \Gamma\ (n) = \int_{o}^{\infty} y^{n-1}.\ e^{-y}\ dy$$

When $n>o$

Where (k-1) = degrees of freedom

$$x^2 = \frac{(f_o - f_e)^2}{f_e} \text{ and } \begin{array}{l} f_o = \text{observed frequency} \\ f_e = \text{expected frequency} \end{array}$$

The fit of the number of annual losses to the Poisson exponential is remarkable. The agreement of actual with expected may not only delight the theorist but even disconcert those with no faith in figures. The mathematics supporting the "Null Hypothesis" suggests that our findings (a x^2 of 3.61) lies between a "P" of .70 for 3.000 and a "P" of .50 for a x^2 of 4.351. There is no mathematical evidence indeed, according to the Pearson test, to discard the "Null Hypothesis" . . .

in this case that the number of annual windstorms reaching the Continental United States fits the Poisson distribution.

Tropical Windstorms Reaching the U. S. (1915-56)
U. S. Department of Commerce—Weather Bureau

Number of Storms r	*Observed Frequency* f_o	rf_o	r^2f_o	P	f_e	f_o-f_e	$\frac{(f_o-f_e)^2}{f_e}$
0	5	0	0	.153	6	—1	0.17
1	11	11	11	.287	12	—1	.08
2	13	26	52	.269	12	+1	.08
3	10	30	90	.169	7	+3	1.28
4	3	12	48	.080	3	0	0
5 & More	0	0	0	.042	2	—2	2.00
Total	42	79	201	1.000	42	0	3.61

$$m = 79/42 = 1.88$$

$$\sigma = \sqrt{\frac{\Sigma r^2 f_o}{\Sigma f_o} - \left(\frac{\Sigma r f_o}{\Sigma f_o}\right)^2} = \sqrt{\frac{201}{42} - \left(\frac{79}{42}\right)^2} = 1.12$$

$$P = \frac{e^{-m} . m^r}{\underline{|r}} \begin{cases} r = o \\ P = .153 \end{cases}$$

$x^2 = 3.61$ Degrees of freedom (N-1) $= 5$

Probability of Null Hypothesis

.70 for $x^2 = 3.000$

.50 for $x^2 = 4.351$

In the following abstract, we show the results of applying the same tests (Poisson exponential and Chi-Square) to the number of Fire catastrophes of $2,500,000 in the United States 1914-1958. This time the fit is poor. The figures suggest that this particular series does not follow the Poisson exponential. A study some ten years earlier from different source data produced a better fit. It is thought that monetary inflation has possibly had a disturbing effect on our mathematical measure for severe fires. While raw data were not readily available, we might have expected a better fit if the frequencies had been corrected for changes over the years in the building cost series. The same observation can be made for the Liberty loss frequency data cited above.

Fire Catastrophes—Over $2,500,000

1914-1958

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Number of Losses r	*Observed Frequency* f_o	rf_o	r^2f_o	P	f_e	f_o-f_e	$\frac{(f_o-f_e)^2}{f_e}$
0	18	0	0	.252	11	+7	4.46
1	13	13	13	.348	15	—2	0.28
2	5	10	20	.240	11	—6	3.28
3	2	6	18	.110	5	—3	1.80
4	3	12	48	.037	2	+1	
5	3	15	75	.010	1	+2	5.35
6	1	6	36	.002	0	+1	
7 & Over	0	0	0	.001	0	0	
Total	45	62	210	1.000	45	0	15.17

mean $= {}^{62}\!/_{45} = 1.38$

$$\sigma = \sqrt{\frac{\Sigma r^2 f_o}{\Sigma f_o} - \left(\frac{\Sigma r f_o}{\Sigma f_o}\right)^2} = \sqrt{\frac{210}{45} - \left(\frac{62}{45}\right)^2} = 1.66$$

$$P = \frac{e^{-m} . m^r}{\underline{|r}} \begin{cases} r = 0 \\ P = .252 \end{cases}$$

$x^2 = 15.17$ Degrees of freedom (N-1) = 4

Probability of Null Hypothesis

.01 for $x^2 = 13.277$

.001 for $x^2 = 18.465$

It is thought that the previous statistics suggest that at least on two major coverages (Fire & Wind) of the Homeowner's policy, the underlying loss frequencies may be responsive to an "ordered randomness". However, these two series, as every underwriter knows, may not follow identical pure loss expectancies.

It may be argued that an ordered distribution of fire losses by size may, per se, imply some pattern in the ratio of any loss size greater than zero to the total exposures (i.e., zero plus non zero losses). "Pure expectancy" can be viewed as solely a finer graduation by loss size . . . simply the transition from zero to "*e*" loss size, where "*e*" may be thought the smallest possible loss size greater than zero.

But we are reluctant to translate the reasoning that any similarity in graduations for the middle and upper registers between Fire & Wind necessarily means a persistency of this relationship as the probability of loss for the respective series approach zero. We shall, therefore, make no further attempt to dissect the obvious (which may be right). Let us accept what every fire underwriter knows that while not responsive to the identical pure loss expectancy. Windstorm losses are thought to occur much less frequently per annual exposed risk than fire losses.

It is surmised that losses from other coverages of the Homeowner's policy may also be responsive to some ordered randomness. We have fair evidence in the case of crime losses. Others probably have more complete documentation for CPL losses. We suspect that neither the loss patterns nor the pure expectancies are identical for all component coverages of the Homeowner's policy.

A single credibility table has been proposed to evaluate the total Homeowner package experience . . . with no distinction for the various coverage components. The plan for a single credibility table for all coverage losses is not theoretically unsound if:

1. the frequency distribution and the pure loss expectancies tend not to vary significantly by coverage, or
2. coverages with significant variations constitute only a relatively trivial portion of the total package losses.

If these conditions are not substantially fulfilled, one might as well be prepared for strange and unacceptable rate indications. And the more frequently judgment must be used as a crutch to carry the burden of imperfect statistical indications, the greater the weakening of confidence in the tabular values. As an end position, the inaccurate table itself might prove the only obstacle to the exercise of sound judgment.

Possibly the theory can be presented more forcibly by what might prove to be an all too realistic prediction of the future. Let us suppose an East Coast state with $5,000,000 earned premiums over a five-year period, and there are many such. The state has been running a 50% loss ratio thru 1964; and in 1965 it is hit with a $25,000,000 hurricane. Obviously, the Homeowner is not going to accept an increase in his $200 premium to $2,000. He will swing back to individual policies—dropping the ECE if necessary . . . or at least paring it down to a minimum. In such a situation the companies would not follow the credibility table indications, and would rather propose a much lesser rate increase. Of course, the somewhat pathetic aspect of the story is the fact that the same faulty credibility table was the factor which afforded substantial rate credits (unwarranted as proved later) for the artificially favorable experience of prior years.

The proposed table sets 100% credibility at $5,000,000 earned premiums over 5 years. We have noted before certain limitations of earned premium as a standard for anything other than an accounting measure of dollars collected at the price levels prevailing at various periods in the past. Our Proceedings contain a number of comments on the propriety of reducing credibility values when the sum total of the annual exposures (i.e. earned premiums) have been collected over a lesser time (yearly) interval than the magic number "5". Since the table grants 80% credibility for $3.2 million earned premiums and 30% for 450,000 earned premium, one might imagine that the designers selected a square root formula (KZ^2=N) to graduate down from 100% credibility. In the normal course of events, we would seriously examine graduation problems only after the philosophy of and the standards for 100% credibility had been established on other than some arbitrary basis.

In summary of this section: credibility in the Fire and Allied Lines field is a most difficult problem, and we may be yet far away from its final solution. We would most earnestly recommend that the industry not commence its investigations of credibility from an entrenched, unalterable position. We must be ready to revise our rudimentary notions on credibility before disaster does it for us.

8. SOME STATISTICAL CONSIDERATIONS OF MULTIPLE CLAIMS PER POLICYHOLDER

At the present time, the interest in repeaters (i.e., an assured with a history of a number of individual losses over the Homeowner's policy period) is confined primarily to underwriting risk evaluations. Someday the emphasis may spread to possible rate differentials . . . if the auto merit rating plans work out successfully.

It would be idle to consider individual policyholder loss frequency as a ratable element for a dwelling fire or an EC contract. The loss expectancies are normally too small to impute any ratable significance to the experience of individual dwelling policyholders. However, when the residence fire and EC policy is joined to a CPL, to a Crime, and to a Miscellaneous Damage policy, the assured's loss record over the policy period may begin to acquire some significance.

Fire underwriters working with the Homeowner's policy are reexamining their ideas on multiple claims over a policy period. While still scrutinizing for the claims-conscious assured, they are aware that the policy affords a multiple of coverages . . . some, of course, with only a very low order probability. Underwriters realize that multiple claims must be interpreted in the light of the fact that the assured could well have had over a three- or five-year period a claim on several of the individual policies which have since been packaged together into the Homeowner's.

Our company research on Homeowner's loss frequency, while still in the exploratory stage, affords data of some possible value. The ratio of 12-months-ending losses paid to estimated annual exposure in policies has been edging upwards over the some 30 months under review. With an adjustment for suspense cases and unreported losses, we estimate that approximately 20 losses occur for every 100 policies exposed to loss over the 12-month period. Now this is the average result from policyholders with no, or one, or two, or up to "n" losses in the year.

On our first trial, we imagined our universe to consist of risks with an underlying constant probability to loss. From tabulations of some 11,000 paid losses on 60,000 exposed policies, we set up polynominal equations on the assumptions successively that our universe was limited to risks that had only one, then one and two, then one, two, three, then one to "n" independent losses (i.e., of constant probability).

Subsequently, we sampled our renewals for their loss frequencies over the expired policy period. The fit was not good. Our sample results were then set up against the expectations from the Poisson exponential with the same mean.

Number of Losses	*Sample*	*Poisson*
0	0.701	0.616
1	.184	.299
2	.066	.073
3	.036	.011
4 & Over*	.013*	.001
Total	1.000	1.000

The Sample to Poisson also evidences not a good fit according to the Chi-Square test. We are still carrying on the experiment. The number of our samples thus far has been small . . . and not yet as random as we plan for our final summaries. However, even at this relatively early stage, there is at least some suspicion of a variation in the results by offices. It could well be that some sections of the United States are more claims-disposed than others.

In summary of this section: it is observed that multiple claims either are not now or soon will not be considered a rarity. There is at least the possibility that the frequency distribution of multiple claims will not follow the Poisson or other statistical series. In other words, the repeater losses may prove not to be the product solely of pure chance, but rather may be due to significant characteristics of the particular assureds. If continuing research proves this to be true, the underwriters will be interested in locating as soon as possible the areas and risk characteristics (i.e., classifications) which evidence a tendency to greater loss frequencies than expected solely on the basis of mathematical probabilities.

9. GENERAL SUMMARY

To recap the various sections of this paper:

A. The traditional attitudes in the fire insurance business towards losses, statistics, and rates are being reassessed and adapted to cope with the challenges presented by the Multiple Peril policy.

* Combined since the occurrences in the higher frequency classes become very few because of the limited number of samples collected thus far in this research.

B. The fundamental concept, Multiple Peril, is sufficiently ambiguous that the greatest forethought and care must be exercised in developing contracts and devising the statistical plans under which the loss experience is to be collected and reviewed.

C. There is some danger of traffic "in" and "out" of the coverages "Multiple Peril" and "Individual Policies" on solely arbitrary rather than on logical determinations . . . however, we cannot be sure that the class "Multiple Peril" will not hold together with sufficient consistency to operate a reasonably scientific insurance venture . . . time alone may be the final arbiter.

D. It may be necessary for both underwriting and rating purposes to find a substitute exposure measure for the present collected earned premiums.

E. Underwriters will evidence an ever-increasing interest in probing their experience beyond the present simple classification system under which the Homeowner's experience is currently being filed.

F. The current plans for Homeowner's credibility can be considered as most rudimentary and experimental . . . subject to reappraisal on an early occasion.

G. The possibility of multiple claims under the Homeowner's will acquire increasing importance underwriting—and even rating—wise in the years to come.

As noted in the introduction to this paper, technicians have a responsibility to point up and discuss the implications of administrative, sales, and underwriting actions in the Multiple Peril area. No technician can ever be assured that he will be "right" in his analysis of any given problem. It is well never to forget that the great D'Alembert incorrectly assigned two-thirds as the probability of a head at least once in two successive throws with a homogeneous coin in his article for Diderot's Encyclopedia.

The men who have developed Multiple Peril insurance must be respected for their enthusiasm and their determination . . . and nothing in this paper is to be construed in any manner derogatory of their inspiration. But over and above all aspirations and accomplishments of executives and individual technicians stands the necessity of knowledge which, in time, will wear away all tinsel and gloss. It is thought that in matters of insurance, as in scholarly disciplines generally, basic understandings are best advanced through the exchange of ideas by those whose prime interest transcends all special pleading.

HOMEOWNERS

CREDIBILITY TABLE

Experience Premium	Credibility Factor	Experience Premium	Credibility Factor	Experience Premium	Credibility Factor
$ 500	.01	$ 578,000	.34	$2,178,000	.66
2,000	.02	612,500	.35	2,244,500	.67
4,500	.03	648,000	.36	2,312,000	.68
8,000	.04	684,500	.37	2,380,500	.69
12,500	.05	722,000	.38	2,450,000	.70
18,000	.06	760,500	.39	2,520,500	.71
24,500	.07	800,000	.40	2,592,000	.72
32,000	.08	840,500	.41	2,664,500	.73
40,500	.09	882,000	.42	2,738,000	.74
50,000	.10	924,500	.43	2,812,500	.75
60,500	.11	968,000	.44	2,888,000	.76
72,000	.12	1,012,500	.45	2,964,500	.77
84,500	.13	1,058,000	.46	3,042,000	.78
98,000	.14	1,104,500	.47	3,120,500	.79
112,500	.15	1,152,000	.48	3,200,000	.80
128,000	.16	1,200,500	.49	3,280,500	.81
144,500	.17	1,250,000	.50	3,362,000	.82
162,000	.18	1,300,500	.51	3,444,500	.83
180,500	.19	1,352,000	.52	3,528,000	.84
200,000	.20	1,404,500	.53	3,612,500	.85
220,500	.21	1,458,000	.54	3,698,000	.86
242,000	.22	1,512,500	.55	3,784,500	.87
264,500	.23	1,568,000	.56	3,872,000	.88
288,000	.24	1,624,500	.57	3,960,500	.89
312,500	.25	1,682,000	.58	4.050.000	.90
338,000	.26	1,740,500	.59	4,140,500	.91
364,500	.27	1,800,000	.60	4,232,000	.92
392,000	.28	1,860,500	.61	4,324,500	.93
420,500	.29	1,922,000	.62	4,418,000	.94
450,000	.30	1,984,500	.63	4,512,500	.95
480,500	.31	2,048,000	.64	4,608,000	.96
512,000	.32	2,112,500	.65	4,704,500	.97
544,500	.33			4,802,000	.98
				4,900,500	.99
				5,000,000	1.00

BIBLIOGRAPHY

Problems of Fire Insurance Rate Making — L. H. Longley — Cook PCAS Vol. XXXVIII.
A Casualty Man Looks at Fire Insurance Rate Making—M. H. McConnell PCAS Vol. XXXVIII.
The Multiple-Line Principle—G. F. Michelbacher PCAS Vol. XLII.
Rate Revision Adjustment Factors—L. J. Simon PCAS Vol. XLV.
Ratemaking for Fire Insurance—J. J. Magrath PCAS Vol. XLV.
Presidential Address, "The Seat of Wisdom"—Page 14—D. M. Pruitt PCAS Vol. XLV.
Notes on Some Actuarial Problems of Property Insurance—L. H. Longley—Cook PCAS Vol. XLVI.
Logical Foundations of Probability—Rudolf Carnap—Uni. of Chicago Press 1950.
Mathematics of Statistics—Part Two—J. F. Kenny—D. Van Nostrand Co. 1947.
Advanced Theory of Statistics Vol. 1—M. G. Kendall—C. Griffin & Co. 1947.
An Introduction to the Theory of Statistics—G. U. Yule and M. G. Kendall—C. Griffin & Co. 1950.