AN ACTUARIAL NOTE ON THE CREDIBILITY OF EXPERIENCE OF A SINGLE PRIVATE PASSENGER CAR

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The experience of the Canadian merit rating plan¹ for private passenger cars provides a means of evaluating the experience rating credibility of the experience of one car. The Canadian experience includes the experience of virtually every insurance company operating in Canada and is collated by the Statistical Agency (Canadian Underwriters' Association—Statistical Department) acting under instructions from the Superintendent of Insurance.

Merit ratings in Canada depend on the number of full years since the insured's most recent accident or since the insured became licensed. The ratings of A, X, Y and B correspond to three or more, two, one, and no years since the most recent accident or since licensing.² A + X would be the experience for two or more accident-free years and A + X + Y would be the experience for one or more accident-free years. Table 1 presents the data upon which this study is based. Earned premiums are converted to a common rate basis by use of the relationship in the rate structure that A:X:Y:B = 65:80:90:100. Other calculations in the table are self-explanatory. The authors have chosen to calculate Relative Claim Frequency on the basis of premium rather than car years. This avoids the maldistribution created by having higher claim frequency territories produce more X, Y, and B risks and also produce higher territorial premiums.

The experience rating formula commonly used may be expressed in the form:

Modification = ZR + (1 - Z) where Z = credibility and R = the ratio of the actual losses to the expected losses.

If the modification is made equal to the subsequent experience of experience-rated risks relative to the average experience of all risks, and if R is made equal to the past experience on which the experience rating is based relative to the average of all risks, then the formula can be solved for the credibility. Where R = 0 as it is for accident-free risks, the credibility equals 1 — Modification. Referring to Table 1 and setting the Modification equal to the "Relative Claim Frequency", the credibilities obtained for a private passenger car for experience pe-

¹ See also "The Canadian Merit Rating Plan for Individual Automobile Risks," Herbert E. Wittick, P. C. A. S. XLV, pg. 214.

² Class 1A Select was introduced effective September 1, 1959 and uses a fiveyear period, but such risks are still a part of Class 1A in data used in the paper.

riods of one, two, or three years are shown in Table 2. For example, in Class 1A the Modification = .920 which gives Credibility = .080 as shown in Table 2 for a three-year period. As another example, in Class 5, A + X + Y, the Modification = .962 which gives Credibility = .038 as shown in Table 2 for a one-year period.

Table 2 also shows the average claim frequency of each class and the ratio of the three-year credibility to the annual claim frequency. If the variation of individual insureds' chances for an accident were the same within each class, the credibility (for experience rating) would be expected to vary approximately in proportion to the average claim frequency.³ Classes 2, 3, 4 and 5 are more narrowly defined than Class 1, and the fact that the ratios in the last column of Table 2 for these classes are less than the ratio for Class 1 confirms the expectation that there is less variation of individual hazards in those classes. This also illustrates that credibility for experience rating depends not only on the volume of data in the experience period but also on the amount of variation of individual hazards within the class.

Table 3 shows the credibility of a two or three-year period in relation to the credibility for one year. If an individual insured's chance for an accident remained constant from one year to the next and if there were no risks leaving the class or no new risks entering the class, the credibilities for experience periods of one, two and three years would be expected to vary approximately in proportion to the number of years.⁴ It should be remembered that experience rating is a procedure to find the deviation of an individual risk from the average risk and is different from class rate-making, which is a procedure to find the average and where an increase in the volume of the experience increases the reliability of the indication only in proportion to the square root of the volume. The fact that the relative credibilities in Table 3 for two and three years are much less than 2.00 and 3.00 is partially caused by risks entering and leaving the class. But it can be fully accounted for only if an individual insured's chance for an accident changes from time to time within a year and from one year to the next, or if the risk distribution of individual insureds has a marked skewness reflecting varying degrees of accident proneness.

If Class 1B risks have an average of 1.044 accidents in the year prior to the rating⁵ the credibility for 1B risks for a one-year experience period is found to be:

Modification =
$$ZR + (1 - Z)$$

1.476 = $Z \frac{1.044}{.087} + 1 - Z$
 $Z = .043$

³ See Appendix I.

⁴ See Appendix I.

⁵ See Appendix II.

This gives an interesting confirmation to the credibility of .046 produced by considering the combined A + X + Y group.

Tables 1, 2 and 3 are based on accident frequency in order to reduce chance fluctuations caused by variations in the size of claims. However, we noticed that B risks had an average claim cost consistently higher than average and A risks consistently lower. This tends to increase the credibility. Table 4 shows for Class 1, which has enough volume to make the average claim cost reliable, the same data as is presented in Tables 1, 2 and 3 except that losses are used instead of number of claims.

In summary, we feel that the Canadian merit rating data for private passenger cars leads to the following conclusions:

- (1) The experience for one car for one year has significant and measurable credibility for experience rating.
- (2) In a highly refined private passenger rating classification system which reflects inherent hazard, there would not be much accuracy in an individual risk merit rating plan, but where a wide range of hazard is encompassed within a classification, credibility is much larger.
- (3) If we are given one year's experience and add a second year we increase the credibility roughly two-fifths. Given two years' experience, a third year will increase the credibility by one-sixth of its two-year value.

TABLE 1

Canada excluding Saskatchewan

Policy Years 1957 & 1958 as of June 30, 1959

Private Passenger Automobile Liability-Non-Farmers

Merit	Earned	Earned Prem. at Present	No. of Claims	Claim Freq. per \$1000	Relative Claim	
Rating	Car Years	$B \ Rates$	Incurred	of Prem.	Freq.	
Class 1 — Pleasure — no male operator under 25						
A	2,757,520	159,108,000	217,151	1.365	.920	
X Y	130,706	7,910,000	13,792	1.744	1.175	
	163,544	9,862,000	19,346	1.962	1.322	
B	273,944	17,226,000	37,730	2.190	1.476	
Total	3,325,714	194,106,000	288,019	1.484	1.000	
A + X	2,888,226	167,018,000	230,943	1,383	.932	
$\mathbf{A} + \mathbf{X} + \mathbf{Y}$	3,051,770	176,880,000	250,289	1.415	.954	
Class 2 — Ple	asure — Non-p	rincipal male oper	ator under 2:	5		
A	130,535	11,840,000	14,506	1.225	.932	
X	7,233	712,000	1,001	1.406	1.070	
Y	9,726	944,000	1,430	1.515	1.153	
В	21,504	1,992,000	3,421	1.717	1.307	
Total	168,998	15,488,000	20,358	1.314	1.000	
A + X	137,768	12,552,000	15,507	1.235	.940	
A + X + Y	147,494	13,496,000	16,937	1.255	.955	
Class 3 — Bu	siness use					
Α	247,424	25,846,000	31,964	1.237	.920	
x	15,868	1,783,000	2,695	1.511	1.123	
Y	20,369	2,281,000	3,546	1.555	1.156	
В	37,666	4,129,000	7,565	1.832	1.362	
Total	321,327	34,039,000	45,770	1.345	1.000	
A + X	263,292	27,629,000	34,659	1.254	.932	
A + X + Y	283,661	29,910,000	38,205	1.277	.949	
Class 4 Unmarried owner or principal operator under 25						
Α	156,871	18,450,000	22.884	1.240	.901	
X	17,707	2,130,000	3,054	1.434	1.041	
Y	21,089	2,523,000	3,618	1.434	1.041	
В	56,730	6,608,000	11,345	1.717	1.247	
Total	252,397	29,711,000	40.901	1.377	1.000	
A + X	174,578	20,580,000	25,938	1.260	.915	
$\mathbf{A} + \mathbf{X} + \mathbf{Y}$	195,667	23,103,000	29,556	1.279	.929	
Class 5 — Married owner or principal operator under 25						
A	64,130	5,349,000	6,560	1.226	.941	
X	4,039	345,000	487	1.412	1.084	
Ŷ	4,869	413,000	613	1.484	1.139	
B	8,601	761,000	1,291	1.696	1.302	
Total	81,639	6,868,000	8,951	1.303	1.000	
A + X	68,169	5,694,000	7,047	1.238	.950	
A + X + Y	73,038	6,107,000	7,660	1.254	.962	

TABLE 2

Credibility			Claim Frequency	Ratio 3 year cred. to annual	
Class	1 year	2 years	s years	per car year	claim frequency
1	.046	.068	.080	.087	.920
2	.045	.060	.068	.120	.567
3	.051	.068	.080	.142	.563
4	.071	.085	.099	.162	.611
5	.038	.050	.059	.110	.536

TABLE 3

	RELA	ATIVE CREDIBIL	ITY
Class	1 year	2 years	3 years
1	1.00	1.48	1.74
2	1.00	1.33	1.51
3	1.00	1.33	1.57
4	1.00	1.20	1.39
5	1.00	1.32	1.55

TABLE 4

Canada excluding Saskatchewan

Policy Years 1957 & 1958 as of June 30, 1959

Private Passenger Automobile Liability--Non-Farmers

	Earned	l Premiums			
Merit	at .	Present	Incurred		Relative
Rating	B	Rates	Losses	Loss Ratio	Loss Ratio
Class 1-Ple	easure	no male ope	erator under 25	5	
Α	159.	108.000	63.191.000	.397	.911
X	7,	910,000	4,055,000	.513	1.177
Y	9,	862,000	5,552,000	.563	1.291
В	17,	226,000	11,809,000	.686	1.573
Total	194,	106,000	84,607,000	.436	1.000
A + X		018,000	67,246,000	.403	.924
A + X + Y	176,	880,000	72,798,000	.412	.945
	Credibility				
	Class	1 year	2 years	3 years	
	1	.055	.076	.089	
	Relative Credibility				
	Class	1 year	2 years	3 years	
	1	1.000	1.38	1.62	

APPENDIX I

To illustrate that the credibilities would vary approximately in proportion to the number of years* for the first few years and for typical frequencies, consider a model in which 100,000 risks have an inherent hazard, as measured by their true claim frequency, of .05, 100,000 risks have a claim frequency of .10 and 50,000 risks have a frequency of .20. The number of persons claim-free for the past t years assuming a Poisson approximation to the distribution is as follows:

Frequency	t == 0	t = 1	t = 2	t = 3		
.05	100,000	95,123	90,484	86,071		
.10	100,000	90,484	81,873	74,082		
.20	50,000	40,937	33,516	27,441		
Total	250,000	226,544	205,873	187,594		
The number of claims in the subsequent year will be:						
Frequency	t = 0	t = 1	t = 2	t = 3		
.05	5,000	4,756	4,524	4,304		
.10	10,000	9,048	8,187	7,408		
.20	10,000	8,187	6,703	5,488		
Total	25,000	21,991	19,414	17,200		
Claim frequency of						
total group	.10000	.09707	.09430	.09169		
Relative to $t = 0$	1.0000	.9707	.9430	.9169		
Credibility		.0293	.0570	.0831		
Relative credibility		1.000	1.945	2.836		

APPENDIX II

Class 1B risks are known to have had one or more claims in the past year. Using the Poisson distribution as an approximation to the risk distribution (another curve which we have used in practice fits more exactly, but for theoretical considerations such as these, the Poisson is a good approximation), we observe that the number of persons having no claim last year is Ne^{-m}, where m is the claim frequency of the class and N is the radix or total number of persons in the population under consideration. Therefore, N (1-e^{-m}) persons produce the one or more claims with which we are concerned. The number of claims produced by the entire group is Nm. Hence the average number of claims produced by those risks which have one or more claims is Nm/N (1-e^{-m}).

In our specific problem, the Class 1 claim frequency is .087 per car which means that risks that had one or more claims last year (and are Class 1B this year) had an average of $.087/(1-e^{-.087}) = 1.044$ claims.

^{*} This illustration may be used equally as well to demonstrate that the credibilities vary approximately in proportion to the average annual frequency because in the Poisson distribution an increase in the annual frequency has the same effect as an increase in the length of time.