SOME FURTHER NOTES ON ESTIMATING ULTIMATE INCURRED LOSSES IN AUTO LIABILITY INSURANCE

BY

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In a previous paper I described how auto liability incurred losses emerged in New York State on the average. One of the puzzling notes about that paper may be Equation (4) (see Volume XLV, 1958 *Proceedings* of the Casualty Actuarial Society):

$$\log_{10}y = 2.0674t^{-.80599}10^{-.24841t}$$

which has the extraordinary virtue of being able to describe how much of the total losses incurred has been paid as of a given time but does not describe the forces which contribute to the total amounts paid.

This paper is concerned with a summarization of the contributory forces which act to produce total loss cost. Those forces are the number of claims paid and the average size of claim payment. A substantial degree of success was achieved in determining the number of claims paid, the corollary seasonal forces and the construction of policy year by definition; somewhat less success was encountered in deriving the average paid claim cost.

With respect to the number of claims paid expressed as a function of time, it was first observed from Exhibit I of Mr. Tapley's paper (Pages 166-198, Volume XLIII, 1956 Proceedings of the Casualty Actuarial Society) that the emergence of number of claims paid tended to become smaller with the passage of time as measured from the time of accident. This suggested

1. that the "easier" claims are settled first and
2. that the number of claims paid during a particular time interval is functionally related to the number of claims outstanding at the beginning of that time interval.

It was also realized that the rate of emergence of countrywide claims for an insurance carrier such as the State Farm Mutual Insurance Company was probably quite different from the rate of emergence of New York claims for stock and mutual carriers in the aggregate.

Fortunately, some limited data for members and subscribers of the National Bureau of Casualty Underwriters and the Mutual Insurance Rating Bureau was available in the form of paid number of claims for policy years 1951 and 1952 reported as of December 31, 1952 and rereported as of March 31, 1953. By relating these number of claims to a later reporting of the incurred number of claims for these policy years a distribution of four values according to approximate time after the average accident was formulated. This distribution is as follows:

Age of Average Accident	*% of Claims Paid*
3.5 months (approx.)	28%
6.5 months (approx.)	45
12.0 months (approx.)	65
15.0 months (approx.)	74

It was found that the foregoing values are reasonably satisfied by a formula for paid increments comprised of 9% of the amount outstanding as of the beginning of each month. Stated differently it was observed that this is a problem in "force of decrement". The solution was found to be

$$N = 1 - e^{-1.08 t_A} \qquad \text{I}$$

where N = the cumulative number of claims paid and

t_A = the time in years measured from the time of accident.

This equation can be more precisely written as

$$N = 1 - e^{-1.08(t_R - t)} \qquad \text{II}$$

where measurement is taken between the time of review t_R and the time of accident t_A.

The accident year data exhibited by the State Farm Mutual Insurance Company is noteworthy for its evidence of seasonal distribution of accidents. There appears to be a relative dearth of claims in the first quarter of the year, a piling up of claims in the last quarter and almost an average distribution of claims in the two middle quarters. A little reflection convinces us that this is not unreasonable. Relatively more driving is done during the months when good weather and holidays prevail than during other months. With more driving there exists a greater exposure to accident.

These observations are reinforced by the record of personal injury accidents reported in New York State. For the year 1955, the distribution of accidents is, approximately:

1955 Quarter	*Personal Injuries*
1	21.08%
2	24.52
3	25.49
4	28.91
Total	100.00%

We are interested in ascertaining whether or not our equation for N fits certain observed values for policy year 1956 private passenger claims paid for members and subscribers of the National Bureau of Casualty Underwriters and the Mutual Insurance Rating Bureau. The number of claims paid during each quarter of 1956 may be expressed as percentages of the total number of claims paid as of December 31, 1956.

Our first step in developing such a comparison was to find a curve which fits the quarterly distribution of personal injuries. Since the distribution of accidents for the first half of the policy year may be expressed as the accident year weighted by the time t, summed up over the time interval, we constructed a polynominal of the following form

$$Y = At^3 + Bt^2 + Ct \qquad \text{III}$$

By factoring out t, which corresponds to the policy year weight, we were left with a second degree equation. The constants for this latter equation were obtained by setting the summation for the first quarter, the fourth quarter and the entire year equal to the respective values in the table of personal injuries. It was found that

$$Y = .48t^3 + .018t^2 + .831t \qquad \text{IV}$$

We are now able to write the general expression for the number of claims paid on accidents incurred to time t of the policy year and observed at time t_R. That expression is:

$$F(t) = \int_0^t (.48t^3 + .018t^2 + .831t)\,[1 - e^{-1.08(t_R - t)}]dt \qquad \text{V}$$

When t lies between 0 and 1 the solution is:

$$\begin{aligned} F(t) &= .12t^4 + .006t^3 + .4155t^2 \\ &\quad - e^{-1.08(t_R - t)}\,[.44444t^3 - 1.21790t^2 + 2.80075\,(1.08t - 1)] \\ &\quad - 2.80075e^{-1.08t_R} \qquad \text{VI} \end{aligned}$$

The foregoing expression gives the percentage of claims paid as of the observed time of review t_R, from the beginning of the policy year to any time through December 31 of that same year. For purposes of comparison a table of derived values is shown below together with the policy year 1956 observed percentage of claims paid at monthly intervals for National Bureau member and subscriber companies, policy year 1956. In both instances the total number of claims paid through December 31 has been used as a base:

Policy Year Comparison of Number of Claims Paid
January through December 31, 1956 = 1.0000

	No. Claims Paid	
January 1, 1956 thru	*Observed*	*Calculated*
January, 1956	.0005	.0006
February, 1956	.0040	.0048
March, 1956	.0138	.0159
April, 1956	.0352	.0372
May, 1956	.0759	.0719
June, 1956	.1247	.1235
July, 1956	.1992	.1953
August, 1956	.2977	.2911
September, 1956	.4177	.4149
October, 1956	.5885	.5496
November, 1956	.7734	.7384
December, 1956	1.0000	1.0000

A comparison of cumulative quarterly totals of policy year 1956 for members and subscribers of the National Bureau of Casualty Underwriters and the Mutual Insurance Rating Bureau is shown below together with the comparable values calculated from the definite integral, equation V:

Members and Subscribers of NBCU and MIRB
Policy Year 1956 Private Passenger Auto Liability Experience
Comparison of Number of Claims Paid
January through December 31, 1956 = 1.0000

	No. Claims Paid	
January 1, 1956	*Observed*	*Calculated*
March, 1956	.016	.0159
June, 1956	.134	.1235
September, 1956	.433	.4149
December, 1956	1.000	1.0000

A comparable expression for the emergence of the number of paid claims resulting from accidents occurring during the second half of the policy period may be derived. That expression is given by the following:

$$G(t) = \int_1^t (2\text{-}t)\ [.48\ (t\text{-}1)^2 + .018\ (t\text{-}1) + .831]\ [1\text{-}e^{-1.08(t_R-t)}]dt \qquad \text{III}$$

Its solution is

$$\begin{aligned} G(t) &= .12t^4 + .634t^3 - 1.58889t^2 + 2.586t \\ &\quad - e^{-1.08(t_R-t)}\ [-.44444t^3 + 2.99568t^2 - 8.48921t + 10.25483] \\ &\quad - 1.5115 + 4.31686e^{-1.08(t_R-1)} \qquad \text{VIII} \end{aligned}$$

By utilizing equations VI and VIII and selecting t_R as March 31 of the appropriate year the theoretical percentage of claims paid may be found.

The observed private passenger figures for members and subscribers of the National Bureau of Casualty Underwriters and the Mutual Insurance Rating Bureau covering policy years 1951 and 1952 are compared with the theoretical values:

Members and Subscribers of NBCU and MIRB
Policy Years 1951 and 1952 Private Passenger Auto Liability Experience

Comparison of Number of Claims Paid
1952 Incurred as of 12 months = 1.00
1951 Incurred as of 24 months = 1.00

		No. of Claims Paid	
Policy Year	*Valued As of*	*Observed*	*Calculated*
1952 (12 Months)	12 Months	28%	26%
1952 (12 Months)	15 Months	45	43
1951 (24 Months)	24 Months	65	62
1951 (24 Months)	27 Months	74	71

Having established that the theoretical equations for expressing the policy year emergence of number of claims paid fits the actual observations, it is now possible to derive the number of claims paid at any time, t_R. An exhibit of such values is shown in Table A. Also, the paid amounts in Exhibit VII of the previous paper is reproduced here as Table B.

By dividing the values in Table A into those contained in Table B for the appropriate period of time we can obtain values for the average paid claim cost expressed as a percentage of the average incurred claim cost. Further, by taking the amounts and the number of claims paid during a particular time interval it becomes possible to express the average paid claim cost during that time interval in relation to the final incurred average claim cost.

SUMMARY

We have seen that there are four main forces at work in the evaluation of total loss cost.

The first results from the familiar definition of the policy year for one year policies. The earned portion of the policy year is proportional to the time during the first 12 months of the policy year and is proportional to one minus the time during the second 12 months. This is equivalent to the parallelogram constructed on a time line which is sometimes used by the rating organizations in computing factors to adjust for rate level changes. It is also equivalent to the proportionate parts of the policy year earned which may be expressed as 1/24, 3/24, 5/24 . . . 23/24, 23/24, 21/24, ... 3/24, 1/24. Excepting as other forces may need to be considered the occurrence of losses should approximate the manner in which premium is earned.

The second force to be considered is the seasonal variation. Seasonal variation may come about in a variety of ways. Weather conditions are one element. Holidays are another and vacation schedules may be a third. Each of these contributing factors has its impact upon the extent of driving done during a particular calendar period. The net effect of these influences becomes evident in the accident records which may be compiled. Mr. Tapley's paper (pages 166-198, Volume XLIII, 1956 Proceedings of the Casualty Actuarial Society) indicates that such seasonal variation exists. A review of records of personal injury accidents in New York State likewise indicates substantial seasonal variation. These latter records indicate that approximately ½ of the annual reported accidents occur between April and September, equally distributed between the two quarters. Only 21% of the actual accidents are reported during the first quarter while 29% are reported during the last quarter of the year. Based upon this information it is possible to construct a second degree equation which represents the seasonal movement of reported accidents. The combination of this second degree equation with the first force representing the construction of the policy year furnishes reasonably close approximations to claims occurrences during the policy year. The combination, however, is best made in two parts, summing up separately for the first 12 months of the policy year and for the second 12 months of the policy year. In doing so it is necessary to use t-1 instead of t for the second half of the policy year.

The third element is the emergence of the number of claims paid expressed as a function of the time t. This is readily achieved by observing that the number of claims paid during the relatively small interval of time is proportional to the number of claims outstanding at the beginning of that time interval. The resulting equation is the natural logarithm base, e, to a power of t, with appropriate constants which express the rate at which claims are being paid. In this connection it is of course interesting to note that t is here measured from the time of occurrence. Since the time of occurrence is spread throughout the policy year it becomes necessary to make a transformation which will then enable all three forces to be combined along a common time line. This transformation is achieved by a substitution of $t_R - t$ for the time of accident, t_A where t_R is the time of review or observation while t is measured from the beginning of the policy year.

The fourth force is the size of average claim payment. Measured from the time of occurrence, those claims which are paid immediately are paid at an average cost well below the incurred average claim cost. From observation of all the available data it would appear that there is a minimum size of average claim even immediately after the occurrence of the claim. As time goes on the average size of claim payment increases. It appears to increase rapidly until it approaches the average incurred claim cost and then slows down for a time. After it has risen above the average incurred claim cost its size again begins to increase rapidly. This suggests some type of monotonic growth curve with a minimum value, an inflection point, and increasing throughout. Measured from the time of occurrence, the average paid claim cost increases with time. It is left to the reader to speculate on the effect which might result if a company made every effort to clear out its claims quickly.

Unfortunately very limited average claim cost data are available. Whatever is available is a hybrid of claims paid during a calendar period relating to claims whose occurrence is spread over the policy year. Attempts at formulating an expression which is consistent with the observations indicate that such an expression is complex. Alternative methods of solving this problem might be the subject of further study by other members of the society.

When these four forces are combined to form the policy year, close approximations to the observed policy year payments is seen. In the process a new element has been introduced, namely t_R. This conforms to the policy year construction made by the rating organizations which requires that loss experience for a policy year be reported as of March 31. It is found that not only does the final result approximate the financial data of the Insurance Expense Exhibit, but it also closely approximates the ratemaking data reported as of March 31. This latter observation reinforces the thought that financial data can be effectively used as a supplement to normal ratemaking data.

TABLE A
Proportion of Total Claims Paid, P
As of Specified Time, t_R
[From Equations VI and VIII]

t_R	P	t_R	P	t_R	P
1 mo.	.0001	1 yr. 9 mos.	.5137	5 yrs.	.9853
2 mos.	.0007	1 yr. 10 mos.	.5533	5 yrs. 6 mos.	.9915
3 mos.	.0022	2 yrs.	.6260	6 yrs.	.9950
4 mos.	.0052	2 yrs. 2 mos.	.6876	6 yrs. 6 mos.	.9971
5 mos.	.0101	2 yrs. 4 mos.	.7390	7 yrs.	.9983
6 mos.	.0173	2 yrs. 6 mos.	.7820	7 yrs. 6 mos.	.9990
7 mos.	.0273	2 yrs. 8 mos.	.8179	8 yrs.	.9994
8 mos.	.0407	2 yrs. 10 mos.	.8479	8 yrs. 6 mos.	.9997
9 mos.	.0580	3 yrs.	.8730	9 yrs.	.9998
10 mos.	.0798	3 yrs. 2 mos.	.8939	10 yrs.	.9999
11 mos.	.1068	3 yrs. 4 mos.	.9114	11 yrs.	1.0000
1 yr.	.1398	3 yrs. 6 mos.	.9260	12 yrs.	1.0000
1 yr. 2 mos.	.2171	3 yrs. 8 mos.	.9382	13 yrs.	1.0000
1 yr. 4 mos.	.3010	3 yrs. 10 mos.	.9484	14 yrs.	1.0000
1 yr. 6 mos.	.3874	4 yrs.	.9569	15 yrs.	1.0000
1 yr. 8 mos.	.4726	4 yrs. 6 mos.	.9749	16 yrs.	1.0000

TABLE B
Proportion of Total Amounts Paid, y
As of Specified Time, t
(From $\log_{10} y = 2.0674 t^{-.80599} 10^{-.248411t}$)

t	y	t	y	t	y
1 mo.	.0000	1 yr. 9 mos.	.3281	5 yrs.	.9282
2 mos.	.0000	1 yr. 10 mos.	.3592	5 yrs. 6 mos.	.9495
3 mos.	.0000	2 yrs.	.4201	6 yrs.	.9643
4 mos.	.0001	2 yrs. 2 mos.	.4776	6 yrs. 6 mos.	.9747
5 mos.	.0005	2 yrs. 4 mos.	.5309	7 yrs.	.9821
6 mos.	.0019	2 yrs. 6 mos.	.5802	7 yrs. 6 mos.	.9872
7 mos.	.0052	2 yrs. 8 mos.	.6252	8 yrs.	.9909
8 mos.	.0110	2 yrs. 10 mos.	.6658	8 yrs. 6 mos.	.9935
9 mos	.0201	3 yrs.	.7025	9 yrs.	.9953
10 mos.	.0326	3 yrs. 2 mos.	.7355	10 yrs.	.9976
11 mos.	.0487	3 yrs. 4 mos.	.7648	11 yrs.	.9987
1 yr.	.0681	3 yrs. 6 mos.	.7912	12 yrs.	.9993
1 yr. 2 mos.	.1158	3 yrs. 8 mos.	.8146	13 yrs.	.9997
1 yr. 4 mos.	.1718	3 yrs. 10 mos.	.8353	14 yrs.	.9998
1 yr. 6 mos.	.2332	4 yrs.	.8538	15 yrs.	.9999
1 yr. 8 mos.	.2966	4 yrs. 6 mos.	.8977	16 yrs.	1.0000