ACTUARIAL ASPECTS OF UNEMPLOYMENT INSURANCE

BY

NATHANIEL GAINES

INTRODUCTION

The following article is a condensed version of a more comprehensive paper. The uncondensed paper is available, in its entirety, in the library of the Casualty Actuarial Society.

Any paper on this subject would not be complete without acknowledging the contributions of the late H. J. Winslow and W. S. Woytinsky. The uncondensed paper contains more detailed references to their contributions in this field.

CHAPTER I

THE ACTUARIAL PROBLEM IN UNEMPLOYMENT INSURANCE

Unemployment insurance is a program which provides, in accordance with a definite formula, indemnity against wage loss resulting from involuntary unemployment. The best known examples of such programs up to now have been the 51 State unemployment insurance programs and the Federal Railroad Unemployment Insurance program. As a result of recent collective bargaining agreements, the field has been expanded considerably by the introduction of the guaranteed wage or supplementary unemployment benefit programs. Up to now, unemployment insurance programs were almost entirely government-operated. Now, private corporations are beginning to play a role in unemployment insurance analagous to the one they have had over the past years in the field of retirement pensions.

DESIGN OF AN UNEMPLOYMENT BENEFIT PROGRAM

The unemployment insurance programs in the United States, both governmentally and privately operated, have the following characteristics:

1. Benefits are generally payable to workers involuntarily unemployed.
2. In order to be eligible for unemployment benefits, the claimant must have demonstrated an "attachment" to the worker group covered by the program. Such attachment is generally considered to exist if certain specified requirements for eligibility are fulfilled such as the following:
   a. Specified minimum duration of employment prior to the involuntary termination; or
   b. Specified minimum earnings prior to termination of employment; or
   c. A combination of prior earnings and employment.
3. Benefits are paid on a regular basis, usually weekly.
4. The weekly benefit payment is a specified proportion of the claimant's average weekly earnings, generally with the proviso that such payments may not exceed a specified maximum amount.
5. Benefits are payable only for a limited duration, such as 20 or 26 weeks, or whatever figure may be specified.
6. A claimant may be required to serve a waiting period of one or more weeks during which he is not entitled to benefits.

Almost all State programs have such a waiting period requirement.

COST DETERMINANTS

Benefit expenditures are dependent both upon the benefit provisions contained in the unemployment insurance plan and economic conditions. The benefit provisions specify the amount of benefits payable for each week of insured unemployment, the number of weeks of entitlement to benefits, and the conditions necessary to qualify for benefits in the event of unemployment. Regardless of potential benefit entitlement, however, benefits are payable only upon the incidence of insured unemployment; the number of claimants applying for benefits and the number of weeks for which they apply under a given program vary with economic conditions.

Assuming rigid benefit provisions, two crucial items in the preparation of actuarial estimates in unemployment insurance are (1) the man-weeks of compensable unemployment per man-week of insured employment, and (2) the weekly payment per man-week of compensable unemployment. If values for the above items are known, costs can be calculated as follows:

B = total benefits disbursements

C = man-weeks of compensable unemployment

E = man-weeks of insured or covered employment

H = number of hours worked per man-week of insured or covered employment

R = weekly benefit payment per man-week of compensable unemployment

W = wages or earnings per man-week of covered employment

$\frac{B}{E}$ = benefit cost per man-week of covered employment

$\frac{B}{HE}$ = benefit cost per man-hour of covered employment

$\frac{B}{WE}$ = the ratio of benefit costs to covered earnings

If contributions for financing benefits under the plan are to be so many cents per hour worked, it would clearly be desirable to calculate $\frac{B}{HE}$. The ratio $\frac{B}{WE}$ would be convenient in cases where contributions are a percent of earnings.

The following equations follow from the definitions:

$$B = RC$$

$$\frac{B}{HE} = \frac{R}{H} \cdot \frac{C}{E}$$

$$\frac{B}{WE} = \frac{R}{W} \cdot \frac{C}{E}$$

Unusual technical problems are not encountered in determining the size of the average weekly benefit payment R, or the ratio of the average weekly benefit payment to the average number of hours per week $\frac{R}{H}$, or the ratio of the average weekly benefit amount to average weekly considered earnings, $\frac{R}{W}$. The problem of estimating $\frac{C}{E}$, man-weeks of compensable unemployment per man-week of covered employment, is peculiar to unemployment insurance and presents the basic difficulty in the preparation of actuarial estimates for this type of program. The difficulty in estimating the value of $\frac{C}{E}$, man-weeks of compensable unemployment per man-week of covered employment, stems from the lack of stability in the incidence of the unemployment risk.

As stated, an unemployment insurance program provides benefit entitlement for limited duration, and may also contain a waiting period requirement before benefits become payable. Hence, an otherwise eligible unemployed worker will receive the weekly unemployment benefit payments only if he has been unemployed long enough to have fulfilled the waiting-period requirements but not for so long as to have exhausted his entitlement to benefits. If there were no labor turnover over prolonged periods and no change in the level of unemployment, the unemployed group would be composed of the same persons in a continuing state of unemployment.[1] Under such conditions, all unemployed would have exhausted their benefit rights because of the length of their unemployment and there would be no benefit disbursements. Regardless of the level of unemployment — whether it be high or low — benefit disbursements for a static unemployed group would ultimately be zero.

It is possible for compensable unemployment and consequently benefit expenditures to be higher during periods of low unemploy-

[1] Assuming no withdrawals of unemployed workers from the labor force and no deaths among the unemployed.

ment than during high unemployment. This can occur if during periods of high unemployment, for example, unemployment is heavily concentrated in the longer duration-of-unemployment intervals, with only a small proportion of the total entitled to benefits; on the other hand, a high proportion of the unemployed may be receiving benefits during periods of low unemployment because of the high rates of labor turnover frequently experienced during such periods.

In our economy, persons are continually shifting from employment to unemployment status and back again, even when there is no change in the level of unemployment. Employed workers are continually being laid off and unemployed are being hired. Hence, even during stable periods of extremely low levels of unemployment, workers are exercising benefit entitlement and receiving benefits.

If the incidence of unemployment were relatively stable, subject only to gradual change because of secular factors in the economy, the preparation of actuarial estimates in unemployment insurance would not involve special problems. Estimates could be prepared from data on loss ratios, adjusted for changes in wage levels and benefit formulas.

However, unemployment varies with business conditions. Because of lack of stability in the incidence of the unemployment risk, special problems are encountered in the preparation of actuarial estimates in unemployment insurance.

ACTUARIAL SOUNDNESS IN UNEMPLOYMENT INSURANCE

In general, actuarial soundness implies an orderly arrangement for financing obligations under a benefit program. Precise formulations of what constitutes actuarial soundness have been adequately developed elsewhere, so that there is no need for further discussion here.

Without ascribing regularity or periodicity to the so-called "cyclical" fluctuations in the economy, it is essential in planning an unemployment insurance system to recognize that unemployment will rise and fall. If benefits are to be financed on a level-premium basis, surplus funds must be accumulated during favorable years when unemployment benefit expenditures are low, and used during periods of rising unemployment to supplement the regular contributions. The actuarial problem in unemployment insurance is to determine the rate of contribution which will provide adequate funds over periods of low and high unemployment.

CHAPTER II

ANALYSIS OF UNEMPLOYMENT BENEFIT COSTS

The main problem in deriving level cost rates in unemployment insurance is to determine the additional cost to be paid during peak years of business activity to provide reserves to meet the added expenditures during years of rising unemployment. Consequently, costs of unemployment insurance must be estimated on the basis of economic assumptions wherein recognition is given to the danger of a rise in unemployment.

LONG-"RANGE" COSTS OF UNEMPLOYMENT INSURANCE

The distinction between "long run" and "short run" costs of unemployment insurance is relative. Because of uncertainties with regard to future economic developments, the outlook with respect to unemployment insurance costs can change radically over a relatively short time interval. For example, costs could be increased sharply during periods of high unemployment over what they would otherwise have been if employers should decide to rotate jobs among unemployed workers in such a way as to maximize the outlay in unemployment benefits. In this discussion, costs over the long run are the estimated costs over a "cycle" of business activity.

The term "business cycle," as used here, is not intended to imply that there is regularity or periodicity in the variations in business activity or unemployment levels. As used here, it is only intended to represent a pattern of business activity which includes periods of increasing and decreasing unemployment.

LONG-RANGE IMPACT OF CYCLICAL UNEMPLOYMENT

Even during peak years of business activity there is unemployment, which is generated by seasonal, technological and frictional factors in the economy. When employment declines, the "cyclical" layoffs in covered industries occur among workers hitherto in relatively stable employment, with sufficient background of earnings and employment to be eligible for benefits. Benefit costs will rise sharply in the initial stages of a downturn.

Benefit expenditures should eventually decline even if business conditions do not improve,[2] since a large proportion of the unemployed workers will exhaust benefit rights and will not have the opportunity to renew benefit entitlement because of the scarcity of job opportunities. Moreover, if business conditions improve, the most likely to be hired first will be those most recently laid off who will be the most likely ones not to have exhausted their benefit rights. In periods of relatively high unemployment, after sufficient time has passed for the "depression" claimants to exhaust their benefit entitle-

[2] Implicit in this statement is the assumption that the employer will not cooperate with the worker to institute job rotation, whereby the proportion of unemployment in compensable status is deliberately maximized or augmented over what it would otherwise have been.

ment, unemployment may be regarded for purposes of actuarial estimating as being composed of the following two groups:

(1) "Long-duration" unemployment composed of workers with practically no chance of receiving unemployment benefits.

(2) "Turnover" group composed of seasonal intermittent and frictional unemployment.

As a first approximation, therefore, compensable unemployment in an average week of a business cycle will be the sum of the following two items:

(1) The volume of compensable unemployment generated on the average during a week of peak business activity; and

(2) The total number of compensable weeks of unemployment incurred because of the "cyclical" rise in unemployment, averaged out over the total number of weeks assumed to be covered by the business cycle pattern.

In general, long-range cost estimates in unemployment insurance may be regarded as the cost rate during peak years of business activity loaded for additional losses due to "cyclical" declines in business activity.

DISTRIBUTION OF UNEMPLOYMENT BY DURATION

During peak years of business activity, unemployed workers are out of work for relatively short duration.

Table I shows the distribution of unemployed workers by duration of unemployment during each of the calendar years 1947 through 1951, when the unemployment rate varied from 3.0 to 5.5 percent. The proportion unemployed for more than 26 weeks varied from 5.6 to 11.4 percent. The 11.4 percent of the unemployed out of work for more than 26 weeks occurred in 1950 following a mild rise in unemployment during 1949 and early 1950. In all of these five years, the proportion out of work for more than a year was negligible.

TABLE I

PERCENTAGE DISTRIBUTION OF UNEMPLOYMENT IN THE UNITED STATES BY DURATION OF UNEMPLOYMENT DURING AN AVERAGE WEEK OF EACH CALENDAR YEAR, 1947-51

Duration of unemployment (in weeks)	*Percent of unemployment*				
	1947	*1948*	*1949*	*1950*	*1951*
1 or less	9.9	11.0	7.8	8.0	12.4
2	14.4	16.2	12.9	12.7	16.0
3	12.8	14.2	12.8	10.7	13.7
4	11.6	11.2	11.2	10.2	11.3
5 to 6	9.5	10.1	9.1	8.8	9.0
7 to 10	14.4	14.4	16.3	15.2	13.4
11 to 14	9.0	7.9	9.7	9.6	8.1
15 to 26	10.9	9.4	12.6	13.5	8.8
over 26	7.7	5.6	7.5	11.4	7.3
Unemployment as a percent of civilian labor force	3.6	3.4	5.5	5.0	3.0

Source: U.S. Bureau of the Census: *Current Population Reports*, Series P-50 Nos. 13, 19, 31, and 40.

Data on distribution of unemployment by duration during years of high unemployment are available from area surveys made during the depression years of the 1930's.

A survey of unemployment for each of the years 1929-33 was sponsored in the city of Buffalo, New York, by the Buffalo Foundation in cooperation with the State Department of Labor. Students of the State Teachers' College in Buffalo and of the University of Buffalo made house-to-house calls for the purpose of determining what proportion of those able and available for work were without jobs. The enumerations were made on the same date in November of each year and in the same areas, in order to obtain maximum comparability over the years. The duration distributions were computed separately for male and female workers the first three years, but only for males in 1932 and 1933. Table II shows the unemployment rates among males and the corresponding duration distributions derived from the survey.

As shown by the data in Table II, the proportion of unemployment in long duration intervals rose sharply with increasing unemployment, and continued to rise even when the unemployment was no longer rising, indicating that hiring chances might be better among workers unemployed for short durations. In 1933, the proportion of unemployed males out of work for more than a year is shown to have been as high as 68 percent.

TABLE II

PERCENTAGE DISTRIBUTION OF UNEMPLOYED MEN BY DURATION OF UNEMPLOYMENT IN BUFFALO, N. Y., 1929 to 1933

Duration of Unemployment	*1929*	*1930*	*1931*	*1932*	*1933*
Under 2 weeks	15.8	4.3	2.6	1.4	2.7
2 to 3 weeks	22.2	7.9	5.0	2.7	5.2
4 to 9 weeks	30.4	21.0	12.7	6.3	10.1
10 to 19 weeks	12.3	17.9	13.4	7.8	5.7
20 to 29 weeks	6.2	14.3	11.7	10.7	4.4
30 to 39 weeks	3.1	7.9	6.4	5.9	2.3
40 to 51 weeks	0.7	5.6	5.2	5.1	1.4
52 weeks and over	9.3	21.1	43.0	60.1	68.2
Unemployment as a percent of labor force:	6.2	17.2	24.3	32.6	28.7

Source: *Monthly Labor Review,* March 1934, page 526.

Table III contains similar data for the city of Philadelphia compiled for each year of the period 1931-7 except 1934. The Philadelphia experience is consistent with what was found in Buffalo.

TABLE III

PERCENT DISTRIBUTION OF UNEMPLOYMENT BY DURATION OF UNEMPLOYMENT IN PHILADELPHIA, 1931-7*

Duration of Unemployment	*1931*	*1932*	*1933*	*1935*	*1936*	*1937*
Under 2 months	24.9	18.8	11.9	6.5	14.3	21.2
3 to 5 months	26.9	17.3	9.4	11.5	10.7	10.6
6 to 8 months	14.8	10.1	8.7	7.6	6.9	5.9
9 to 11 months	13.4	18.1	15.1	9.0	7.7	4.4
Total, under one year	80.0	64.3	45.2	34.6	39.6	42.1
Unemployment as a percent of labor force	25.7	42.1	46.0	33.0	30.2	24.4

*Except 1934

Source: "Recent Trends in Employment and Unemployment in Philadelphia", by Gladys L. Palmer; distributions for males and females were combined.

SUMMARY

The rate of benefit expenditures in unemployment insurance is not a simple function of the unemployment rate. It also depends upon the variations in the duration-distributions of unemployment with changing economic conditions.

In the next chapter a theory is developed for constructing mathematical models to study variations in duration-distribution of unemployment and their effect on unemployment benefit costs.

CHAPTER III
MATHEMATICAL MODELS

Experience with unemployment insurance under the State programs after the end of World War II provides an empirical basis for estimating rates of unemployment benefit expenditures during periods of low unemployment. Similarly, individual companies could utilize the statistics obtainable from their records for the postwar years of operations to obtain similar cost estimates for years of low unemployment. By the use of mathematical models, the additional benefit costs from assumed rises in unemployment can be reflected.

The mathematical models are used to determine the variations in the duration-distribution of unemployment.

BASIC LABOR FORCE MODEL

In order to develop manageable mathematical concepts, it is necessary to oversimplify the dynamics of the labor market by postulating rigid mechanistic models. As a starting point, a labor force with the following characteristics may be considered:

1. The covered labor force is constant in size and composition, i.e. there are no new entrants into and no withdrawals out of the labor force.
2. All workers are subject to the same probabilities of being hired or laid off; no account is taken of superior skills, attachment to expanding industries, personal connections, sex, age, or any other factor which would create disparities among workers with respect to their abilities to find jobs or to retain their current positions.
3. The hiring and layoff probabilities are constant over a specified period of time such as a month or a year.

4. Hirings and layoffs occur continuously over the specified time interval.

The dynamics of this labor force model over an interval of time may be described by the following variables:

L = labor force

U = unemployment

E = employment

h = probability of a worker unemployed at the beginning of the interval being hired at least once sometime before the end of the interval

f = probability of a worker employed at the beginning of the interval being separated at least once before the end of the interval

$\hat{h}$ = an approximation of h from empirical data

$\hat{f}$ = an approximation of f from empirical data

The formulation of this type of model is an initial step in the estimating. Consideration may then be given to adjustments for bridging the gap between the simplified model and the realities of the labor market.

HIRING AND FIRING PROBABILITIES

For the labor force model postulated above, the following relationships follow from the definitions of the terms:

(1) $\frac{U_o - U_t}{U_o} = h$ where U_o represents the unemployed workers at the beginning of an interval, and U_t the workers in the original U_o continuously unemployed up to the point $t = 1$.

(2) $\frac{E_o - E_t}{E_o} = f$ where E_t represents the workers continuously employed from the beginning of the interval to the point $t = 1$.

(3) $U_o - U_t = hU_o$

(4) $E_o - E_t = fE_o$

If the total number of accessions were equal to $U_o - U_t$ and the total number of separations to $E_o - E_t$, then the monthly hiring and firing probabilities could be readily calculated from data on total accessions and total separations. However, the separations and accessions totals are not the same as $E_o - E_t$ and $U_o - U_t$, respectively, as the latter expressions were defined in (1) and (2). Even if accessions and separations were adjusted to exclude shifts from job to job and new entrants into the labor market, they would include hiring of workers not unemployed at the beginning of the month, and layoffs among workers not employed at the beginning of the month. This is due to the fact that workers may be hired or fired more than once over the course of a month.

Let S = total number of separations over the interval

A = total number of accessions over the interval

$$\frac{A}{U_o} > h$$

$$\frac{S}{E_o} > f$$

This problem might be resolved by a simple approximation. If hiring occurred only among workers unemployed at the beginning of the month, then the probability of an unemployed worker being hired within a month would be $\frac{A}{U_o}$. However, workers separated during the month compete for the available jobs with those unemployed at the beginning. Consequently, the accessions must be related to a quantity greater than U_o in order to reflect the fact that the newly separated workers apply for jobs and in some instances obtain them before the end of the month. One might use $U_o + S$ as the group exposed to hiring during the month and let $\hat{h} = \frac{A}{U_o + S}$. It might be reasoned that, if separations occur evenly over the month, a worker separated during the month will be exposed to hiring for only half of the month on the average. If this reasoning is correct, then the exposure quantity for the month will be $U_o + \frac{S}{2}$ and an approximation of the hiring probability will be computed from $\hat{h} = \frac{A}{U_o + S/2}$. The same reasoning can be followed to obtain an approximation for the firing probability, $\hat{f}$.

An alternative approach for deriving approximate values of the hiring and firing probabilities would be to fit a continuous probability density function to empirical data. An unemployed group at the beginning of a specified time interval, U_o, may be treated as a cohort subject to continuous diminution with the passage of time because of hires, with U_t representing the volume continuously unemployed from the beginning of the interval to the point t. Thus, U_t will be a function of t:

$$U_t = U\,(t)$$

If $U\,(t)$ is a continuous function of t with the first derivative existing at each point of the interval, the slope of the curve at each point will be negative and equal to $\frac{dU_t}{dt}$ and the number in the initial cohort, U_o, unemployed from the beginning of the interval to a point t will be $\int_t^\infty \frac{dU_t}{dt}\,dt$. It follows that:

$$(5)\quad \frac{dU_t}{dt} = -rU_t, \text{ where } r = \left|\frac{1}{U_t}\frac{dU_t}{dt}\right|.$$

$$\int_t^\infty \frac{dU_t}{dt} = -\int_t^\infty rU_t \cdot dt$$

Thus, $(-rU_t)$ is a continuous curve, and the area under the curve may represent the number of workers continuously unemployed or the number hired in an interval of time. When multiplied by a constant $\frac{1}{U_o}$, this curve becomes a probability density function, and the total area under it is unity. In similar fashion, a cohort of employment at the beginning of a time interval may be considered, with $s = \left|\frac{1}{E_t}\frac{dE_t}{dt}\right|$ and $(-sE_t)$ a probability density function when multiplied by the constant $\frac{1}{E_o}$.

Instead of evaluating $-\int rU_t dt$ directly, it is more convenient to begin with equation (5) and work with differential equations. Thus,

(6) $dU_t = -rU_t dt$ where rdt represents the probability of a worker U_t being hired within an infinitesimal interval; r is a constant because of the assumption of constant hiring probability in the model.

$$\frac{dU_t}{U_t} = -\,rdt$$

$$U_t = Ke^{-rt};\ k = U_o$$

(7) $$U_t = U_o e^{-rt}$$

$$U_o - U_t = U_o\,(1 - e^{-rt})$$

(8) $$\frac{U_o - U_t}{U_o} = 1 - e^{-rt}$$

The symbol K shown above, is the constant of integration, and e is the base of the natural logarithms.

Similarly, it can be shown for the employment cohort, that

(9) $$dE_t = -sE_t\,dt$$

(10) $$\frac{E_o - E_t}{E_o} = 1 - e^{-st}$$

Over a unit of time, when $t = 1$,

(11) $$\frac{U_o - U_t}{U_o} = 1 - e^{-r} = h$$

(12) $$\frac{E_o - E_t}{E_o} = 1 - e^{-s} = f$$

Thus, it has been shown that the hiring probability as defined in (1) is a function of r and the firing probability as defined in (2) is a function of s. The values of r and s can be approximated empirically, as will be shown in the following discussion.

Consider a convenient time interval of, say, four weeks, and denote it by unity. Let t be any point in this interval such that $0 \leqq t \leqq 1$. Also, let us assume that accessions and separations occur continuously and evenly over the time-interval.

$S =$ the number of separations per unit time-interval

$A =$ the number of accessions per unit time-interval

$S \cdot \triangle t =$ the number of separations in a sub-interval, $\triangle t$ in length

$A \cdot \triangle t =$ the number of accessions in a sub-interval, $\triangle t$ in length

In an interval from the point zero to the point t_1, which is assumed to be $\triangle t$ in length, the number of separations is $S \cdot \triangle t$ and the number of accessions, $A \cdot \triangle t$. Considering only intervals bounded by the point zero at one extreme, as $\triangle t$ becomes smaller, $S \cdot \triangle t$ tends to include a continuously increasing proportion of workers employed at the beginning of the interval; similarly, $A \cdot \triangle t$ tends to include a continuously increasing proportion of workers unemployed at the beginning of the interval.

$\lim_{\Delta t \to 0} \frac{S}{E_o} \Delta t =$ the probability of a worker employed at the beginning of an interval being separated in an infinitesimal time-interval after the beginning.

$\lim_{\Delta t \to 0} \frac{A}{U_o} \Delta t =$ the probability of a worker unemployed at the beginning of a time interval being hired within an infinitesimal interval after the beginning.

Since hiring and firing probabilities are assumed to be constant over the time-interval, $0 \leqq t \leqq 1$ it follows that

$U_t \cdot \frac{A}{U_o} dt \doteq$ the number of workers unemployed at point t who will be hired within an infinitesimal sub-interval.

$E_t \cdot \frac{S}{E_o} dt \doteq$ the number of workers employed at point t who will be separated within an infinitesimal sub-interval.

Hence

$$dU_t \doteq -U_t \left(\frac{A}{U_o}\right) dt$$

$$dE_t \doteq -E_t \left(\frac{S}{E_o}\right) dt$$

Thus, $\frac{A}{U_o}$ appears to be a logical approximation of r and $\frac{S}{E_o}$ an approximation of s.

The distinction between r and the hiring probability $(1 - e^{-r})$ may require clarification. Although $h(=1-e^{-r})$ and r are both ratios with U_o in the denominator, they represent different things. The hiring probability h, as defined in (1), has all the characteristics associated with the conventional probability concept. For example, it is always positive and cannot conceptually exceed unity. The quantity r on the other hand is a nominal hiring rate and although always positive, it may increase without limit. Similarly, s is a nominal firing rate, and may assume any positive value.

Consider a month with hiring rate h. Then

$1 - e^{-\frac{r}{2}} =$ the probability of workers unemployed at the beginning of the month being hired before the end of the first half of the month

$1 - e^{-\frac{r}{n}} =$ the probability of workers unemployed at the beginning of the month being hired before the end of the first $1/n$th part of the month

For an effective hiring probability of $1 - e^{-\frac{r}{n}}$ for $1/n$th part of the month, the corresponding nominal monthly hiring probability is

$$n\,(1 - e^{-\frac{r}{n}})$$

and

$$\underset{n \to \infty}{\text{Limit}}\; n(1 - e^{-\frac{r}{n}}) = r$$

In the special case where the hiring probability is certainty, we have

$$1 - e^{-r} = 1$$

$$e^{-r} = 0$$

$$e^{-\frac{r}{n}} = 0$$

$$1 - e^{-\frac{r}{n}} = 1$$

Hence, when the probability of a worker unemployed at the beginning being hired before the end of the interval is certainty, the probability of being hired before the end of any fraction of the year is also certainty in the particular model under consideration. The hiring probability $h = 1 - e^{-r}$ approaches certainty in this model only if r increases without limit, representing a situation wherein all unemployed are hired immediately after being laid off. In such a situation, if the volume of unemployment is assumed to be constant throughout interval and U_o is also the total number of hires in each infinitesimal interval and U_o is also the total number of hires in each infinitesimal interval.

It is clear that the hiring probability as defined in (1) will be certainty for a given period if all workers unemployed at the beginning of the period are hired before the end of it, and no separations occur during the period. Assuming that hires occur continuously, the nominal hiring rate for a sub-interval at the beginning of the year will not increase without limit as the length of the sub-interval approaches zero, yet the value of h is one. In this situation, however, the hiring probability is not constant throughout the interval as assumed in our model. Instead, the hiring probability for a sub-interval toward the end of the period is higher than for a similar sub-interval near the beginning.

A more realistic approach would be to recognize that h is not a constant and treat it as a function of t (time).

$$r = \phi\,(t)$$

$$dU_t = -U_t \cdot \phi\,(t)\,dt$$

$$(13) \quad U_t = U_o e^{-\int_t^t \phi(t)\,dt}$$

By fitting data to equation (13) adjustments could be made for the fact that a worker's chances for finding employment tend to decline with continuation of his unemployment status.

As discussed in a later section of this chapter, the results obtained from our model would have to be adjusted for the fact that unemployed workers are not a homogeneous group, particularly with respect to hiring probabilities. Equation (13) provides a basis for handling heterogeneity among unemployed workers.

The basic model can also be used to portray continuous shifting between employment and unemployment status.

Let U_t' represent unemployment at any point t in the interval and E_t' those employed at the point t. U_t' is composed of workers who may have been employed or unemployed at the beginning of the period. Since it is assumed in our model that all workers, whether initially employed or unemployed, have the same probability of being hired or of being fired in a neighborhood of every point in the interval, we have, for any point t,

$$(14) \quad dU_t' = -(rU_t' - sE_t')\,dt$$

$$(15) \quad dE_t' = -(sE_t' - rU_t')\,dt$$

Equation (14) is solved in the following manner.

$$L = U_t' + E_t', \text{ where } L \text{ is the constant labor force.}$$

$$dU_t' = -(rU_t' - sL + sU_t')\,dt = [-U_t'(r+s) + sL]\,dt$$

$$\frac{dU_t'}{-U_t'(r+s) + sL} = dt$$

$$\frac{-(r+s)\,dU_t'}{-U_t'(r+s) + sL} = -(r+s)\,dt$$

$$-U_t'(r+s) + sL = Ke^{-(r+s)t}$$

$$\text{At } t = o,\ K = -U_o(r+s) + sL$$

$$-U_t'(r+s) + sL = \Big[-U_o(r+s) + sL\Big]e^{-(r+s)t}$$

$$U_t'(r+s) = sL - \Big[sL - U_o(r+s)\Big]e^{-(r+s)t}$$

$$(16) \quad U_t' = \frac{sL}{r+s} - \left[\frac{sL}{r+s} - U_o\right]e^{-(r+s)t}$$

Equation (16) may be simplified by defining U and E as proportions of the labor force, in which case $L=1$. Hence,

$$(17)\quad U'_t = \frac{s}{r+s} - \left[\frac{s}{r+s} - U_o\right] e^{-(r+s)t}$$

and when $t=1$

$$(18)\quad U'_t = \frac{s}{r+s} - \left[\frac{s}{r+s} - U_o\right] (1-h)(1-f)$$

Equation (16) may also be transformed so as to indicate what segment of U'_t was employed at the beginning of the interval, and what segment was unemployed. Since the labor force, L, was assumed to remain constant in both size and composition, it must be composed, at any point t, only of workers who were either employed or unemployed at the beginning of the period. Hence, at any point t,

$$L = E_o + U_o$$

Substituting in equation (16)

$$U'_t = \frac{s}{r+s}(U_o + E_o) - \left[\frac{s}{r+s}(U_o + E_o) - U_o\right] e^{-(r+s)t}$$

$$= U_o\left[\frac{s}{r+s} - \left(\frac{s}{r+s} - 1\right) e^{-(r+s)t}\right] + E_o\left[\frac{s}{r+s} - \frac{se^{-(r+s)t}}{r+s}\right]$$

$$(19)\quad U'_t = U_o\,\frac{s + re^{-(r+s)t}}{r+s} + E_o\,\frac{s - se^{-(r+s)t}}{r+s}$$

Thus, equation (19) shows the entire group U'_t broken down into two mutually exclusive segments. Of those unemployed at the point t, the group of workers that were unemployed at the beginning of the interval is represented by $U_o\,\dfrac{s + re^{-(r+s)t}}{r+s}$. The group that was employed at the beginning of the interval is represented by $E_o\,\dfrac{s - se^{-(r+s)t}}{r+s}$.

In (7), $U_t = U_o\, e^{-rt}$, represents only that portion of the unemployment at point t, which was unemployed at zero, and did not experience a spell of employment in the interval from zero to t. On the other hand,

$$U_o\,\frac{s + re^{-(r+s)t}}{r+s}$$

represents the workers in U_o, who are unemployed at t regardless of their status in the interim bounded by zero and t.

Equation (15) can be solved in a similar manner to obtain the volume of employment at the point t. Solving equation (15), it can be shown that

$$(20)\qquad E'_t = \frac{rL}{r+s} - \left[\frac{rL}{r+s} - E_o\right] e^{-(r+s)t}$$

$$(21)\qquad E'_t = U_o \frac{r - re^{-(r+s)t}}{r+s} + E_o \frac{r + se^{-(r+s)t}}{r+s}$$

DURATION MODELS

In order to estimate the volume of compensable unemployment in our postulated labor force model under assumed economic conditions, it is necessary to determine the distribution of unemployment by duration. If a duration distribution for a point in time is available, distributions for subsequent points can be derived by the application of hiring and firing probabilities.

One approach would be to select a suitable distribution obtained empirically from a one-time census or survey. Another would be to construct a hypothetical distribution under restrictive conditions. Such a hypothetical distribution can be constructed under the assumption of a constant level of unemployment and constant hiring and firing probabilities prevailing over a sufficiently long period.

So long as the volume of unemployment is constant, with no entrants into and withdrawals from the labor force, the accessions and separations must be in balance. For convenience, the four-week interval (lunar month) may be selected as the time-unit of duration. It has been shown that for assumed unemployment and turnover rates, approximations of the hiring and firing probabilities can be computed. Since $\hat{h}$ represents an estimate of the probability that a worker unemployed at the beginning of a lunar month will be hired before the end of it, $1 - \hat{h}$ is an estimate of the probability of such a worker not being hired during the lunar month. Hence, if U_o is the assumed constant level of unemployment at the beginning of a lunar month, $U_o(1 - \hat{h})$ represents the number unemployed at the beginning and still unemployed without interruption by the end of the lunar month. After a sufficient number of months with constant volume of unemployment have elapsed, $U_o(1 - \hat{h})$ represents the number continuously unemployed for four weeks or more, $U_o(1 - \hat{h})^2$ the number unemployed for at least eight weeks, $U_o(1 - \hat{h})^3$ for twelve weeks or more, and so forth. The duration-distribution of unemployment will then be as follows:

Duration of Continuous Unemployment (Lunar Months)[3]	*Number Unemployed*[4]
1 or more	$U_o(1-\hat{h})$
2 or more	$U_o(1-\hat{h})^2$
3 or more	$U_o(1-\hat{h})^3$
4 or more	$U_o(1-\hat{h})^4$
5 or more	$U_o(1-\hat{h})^5$
6 or more	$U_o(1-\hat{h})^6$

[3] A four-week period is called here "lunar month."

[4] The procedure described yields expected values subject to random variability.

Given constant unemployment and turnover rates, which are assumed to have been prevailing for a sufficiently long time, a duration distribution for the labor force model can be constructed by the method shown above. The probability of not being hired over a two-week interval, or half of a lunar month is $(1-\hat{h})^{1/2}$. Hence, the number unemployed for at least two weeks will be equal to $U_o(1-\hat{h})^{1/2}$, and for at least six weeks to $U_o(1-\hat{h})^{3/2}$. In this model, the volume of unemployment will not change if the accessions and separations are in balance. An initial distribution may be constructed this way, if a suitable one cannot be obtained empirically.

A transition can be made from the initial distribution to the duration distribution prevailing as a result of subsequent variations in the level of unemployment. This transition can be effected by the application of hiring probabilities, or, more precisely, probabilities of not being hired to the cell frequencies of the initial distribution. These subsequent distributions are dependent upon the values assumed for the unemployment and turnover rates in subsequent months.

For example, consider a labor force of one hundred thousand subject to the restrictions in our basic model, with unemployment rate five percent and separation and accession rates of three percent.

$$L = 100{,}000$$
$$U_o = 5{,}000$$
$$E_o = 95{,}000$$

$a = g = .03$, when a and g are the accession and separation rates, respectively, expressed as a percent of employment at the beginning of each lunar month. Then

$$A = aE_o = 2{,}850$$
$$S = gE_o = 2{,}850$$

$$\text{Let } \hat{h} = \frac{A}{U_o + \frac{1}{2}S} = 0.4436$$

$$1 - \hat{h} = 0.5564$$

The initial distribution in this model will be as follows:

Duration of Continuous Unemployment	*Number of Unemployed*
Total Unemployed	5,000
4 weeks or more	2,782
8 " " "	1,548
12 " " "	861
16 " " "	479
20 " " "	266
24 " " "	148
28 " " "	82

Beginning with this distribution, it is assumed that the number of unemployed in this hypothetical labor force of one hundred thousand will rise from five thousand at the beginning of the year to ten thousand by the end of the year. The computations involve three variables, the volume of unemployment at the beginning of the lunar month, U'_{x-1}[5], the number of accessions during the four-week period A_x, and the number of separations S_x. Since the labor force is assumed to be constant in both size and composition, independent values may be assigned only to two of the variables. The third will be uniquely determined thereby. Assuming that unemployment will increase by a uniform amount over each lunar month, it follows that the total increase of five thousand over the year will occur at the rate of 384.6 per month. Hence, the volume of employment and unemployment at the end of each four-week period of the first year during which this rise occurs will be as follows:

[5] The volume of unemployment at the beginning of the year has been denoted by U_o which is also the volume at the beginning of the first lunar month, U'_1 is the volume at the end the first or beginning of the second lunar month; U'_{x-1} is the volume at the end of month $(x-1)$ or beginning of month x and U'_x is the volume at end of month x; E'_x is the employment corresponding to U'_x.

End of Month	*Unemployment*	*Employment*
0	5,000	95,000
1	5,385	94,615
2	5,769	94,231
3	6,154	93,846
4	6,538	93,462
5	6,923	93,077
6	7,308	92,692
7	7,692	92,308
8	8,077	91,923
9	8,461	91,539
10	8,846	91,154
11	9,231	90,769
12	9,615	90,385
13	10,000	90,000

Because of the characteristics postulated with respect to the labor force model, it follows that

(22) $U'_x - U'_{x-1} = S_x - A_x$

With the volume of unemployment at the beginning and end of each month known, additional information is still needed regarding either the accessions or separations during each month. Expressing accession and separation rates for a four-week period as a percent of the employment at the beginning of the period, it follows that

$$S_x = gE_{x-1}$$
$$A_x = aE_{x-1}$$

For illustrative purposes, let us assume that $a = 3$ percent over each month of the year. At the beginning of the year, we have

$$L = 100{,}000$$
$$U_o = 5{,}000$$
$$E_o = 95{,}000$$

Over the course of the first lunar month, the total accession rate is three percent, and the total number of hires (aE_o) is 2,850. Substituting in (22), we find that the total number of separations, (sE_o), is 3,234.6. If $\hat{h}_x$ represents an approximation of the hiring probability in lunar month x

$$\hat{h}_1 = \frac{aE_o}{U_o + {}^1/_2\, gE_o} = 0.4307$$

$$1 - \hat{h}_1 = 0.5693$$

U'_x = the number of unemployed at the end of lunar month x.

$U'_{x:y}$ = the unemployed at the end of lunar month x who have been continuously out of work for y weeks or more

$U'_{x:y}\,(1 - \hat{h}_{x+1})$ = the unemployed who have been out of work y weeks or more by the end of lunar month x and who are not able to find employment by the end of lunar month, $x + 1$.

$$U'_{x+1:y+4} = U'_{x:y}\,(1 - \hat{h}_{x+1})$$

Thus far in our illustration, the values of U_o, U'_1, $\hat{h}_1$ and $(1-\hat{h}_1)$ have been computed. A cumulative distribution of U_o by weeks of unemployment has also been obtained as the initial distribution, so that we have values of $U_{o:y}$. A distribution of U'_1 can be obtained from the following relationship:

$$U_{o:y}\,(1 - \hat{h}_1) = U'_{1:y+4}$$

The duration-distributions in the hypothetical labor force of one hundred thousand for the beginning and end of the first lunar month are then as follows:

Duration of Unemployment	*Beginning of First Month*	*End of First Month*
All unemployed	5,000	5,385
4 weeks or more	2,782	2,846
8 weeks or more	1,548	1,584
12 weeks or more	861	881
16 weeks or more	479	490
20 weeks or more	266	273
24 weeks or more	148	151
28 weeks or more	82	84

Each cell frequency for the beginning of the first lunar month (or end of lunar month zero) was multiplied by 0.5693, the value of $(1-\hat{h}_1)$ to derive a cell frequency for the end of the first lunar month. A diagonal line connects each frequency in the second column to the one in the first from which it was derived.

The process is continued, and similar distributions are computed for the end-point of each of the thirteen lunar months. For each calendar year, therefore, there are fourteen distributions, one corresponding to the beginning of the year and the remainder for the end of each of the thirteen lunar months. These distributions are related to discrete and equally-spaced points in time. The number of workers unemployed y weeks or more in an average week of lunar month x is equal to $^1/_2\,(U'_{x-1:y} + U'_{x:y})$, and the number of unemployed y weeks or more in an average week of a calendar year is equal to:

$$\frac{^1/_2\,(U_{o:y} + U'_{1:y}) + {^1/_2}\,(U'_{1:y} + U'_{2:y}) + \ldots + {^1/_2}\,(U'_{12:y} + U'_{13:y})}{13}$$

$$= {^1/_{13}}\left[\,{^1/_2}\,U_{o:y} + \sum_{i=1}^{12} U'_{i:y} + {^1/_2}\,U'_{13:y}\right]$$

A duration distribution for an average week of the calendar year in duration intervals of four weeks can be derived in this manner. Smaller duration intervals can be obtained by interpolation. These distributions can be used to determine the number of unemployed in a compensable duration-of-unemployment interval.

Consider, for example, a plan for payment of benefits after a waiting period of one week, with benefits payable for twenty weeks of unemployment. In order to receive at least one benefit payment, a claimant must have been out of work long enough to have served his waiting period and to have experienced at least one additional week of wage loss; i.e. the claimant must have been out of work at least two weeks before he can receive an unemployment benefit payment. If benefits are payable for twenty weeks of unemployment, a beneficiary will receive the last weekly payment to which he is entitled at the end of his twenty-first week of unemployment. During an average week of the year, therefore, the workers in compensable status would be those who have been out of work at least two but less than twenty-two weeks.[6]

Let $\overline{U}{:}x$ = the number of unemployed in an average week of the year, who have been continuously out of work for x or more weeks.

[6] Only full weeks of wage loss are considered here. In practice, benefits are payable in some instances for partial weeks of wage loss, and cost estimates would have to be adjusted to reflect such payments.

$\overline{C}$ = the number of unemployed workers in the compensable duration-of-unemployment interval in an average week of the year.

$\overline{C}'$ = an estimate of $\overline{C}$

If the unemployment insurance program provides twenty weeks of benefit entitlement after a waiting period of one week,

$$\overline{C}' = \overline{U}{:}2 - \overline{U}{:}22$$

Under the assumed conditions depicted in the labor force model, it is possible to study the cost of an unemployment insurance plan just as economic relationships are studied under assumed conditions and by apriori reasoning. It would be desirable to bridge the chasm between the over-simplified labor market conditions depicted in this model and the actual labor market environment. However, such a transition has proved to be very difficult.

REALISTIC LABOR MARKET CONDITIONS

Estimates of the volume of compensable unemployment derived on the basis of hypothetical models must be adjusted for the following factors:

1. Heterogeneity in the labor force.
2. Incidence of multiple spells of unemployment in a benefit year.
3. Continuous variation in the composition of the labor force.
4. Miscellaneous administrative factors not depicted in the models.

These items represent the major differences for actuarial purposes between the hypothetical models and the actual labor market.

HETEROGENEITY IN THE LABOR FORCE

The labor force is not a homogeneous entity, and there is considerable variation among workers with regard to hiring and firing probabilities. During the depression phase of a business cycle, for example, there is a substantial proportion of the unemployed with practically no chance of finding jobs; this segment of unemployment is known as the "hard core." This hard core of unemployment exists even though continued hirings and layoffs occur in other segments of the labor force.

Some insight into the impact of heterogeneity on unemployment insurance costs may be derived by a study of the hypothetical labor force models. For example, it can be demonstrated mathematically that for a specified average level of unemployment differences in unemployment rate between two segments of the labor force will result in a lower volume of compensable unemployment than if the unemployment were evenly distributed over the labor force.[7]

[7] See Appendix Note C, *Principles of Cost Estimates in Unemployment Insurance*, by W. S. Woytinsky.

The hypothetical models can be used to illustrate the impact of varying degrees of heterogeneity on the level of compensable unemployment. For example, the volume of compensable unemployment in a homogeneous labor force with unemployment rate of 12.5 percent may be contrasted with a labor force subdivided into four segments of equal size with unemployment rates of five, ten, fifteen and twenty percent, respectively. Similarly, variations in turnover rate among these segments may be considered. A segment with hiring probability equal to zero would represent a hard core of unemployment.

The emergence of a hard core of unemployment during the depression phase of a business cycle has an important impact on unemployment benefit expenditures over a business cycle. One method of reflecting the effects of the hard core is as follows:

Assuming that unemployment is composed of two groups—turnover and hard core—

$$L = U'_t + E'_t$$

$U' = N'_t + \triangle$, where

$N' =$ volume of turnover unemployment at the point t

$N_t =$ the segment of N'_t composed of workers continuously unemployed from the point $t = 0$

$\triangle =$ the volume of hard-core unemployment

The hard core is generally assumed to be constant in size and composition over a unit time-interval such as a four-week period (lunar month) and each of the two groups—hard-core and turnover unemployment—is assumed to be homogeneous with respect to hiring probability. Instead of equations (6) and (14) we have

$$(23) \quad dN_t = -rN_t\,dt$$

$$(24) \quad dN'_t = -(rN'_t - sE')\,dt$$

These equations are solved in the same way as (6) and (14). The derived hiring and firing probabilities will differ to the extent that unemployment subject to hiring is diminished by the exclusion of the hard core, $\triangle$. Thus, in approximating the hiring probabilities, we have

$$r \doteq \frac{A}{N_o}, \text{ or}$$

$$\hat{h} \doteq \frac{A}{N_o + \frac{1}{2} S} \text{ or } \frac{A}{N_o + S}$$

In constructing a duration distribution, the calculations are the same as shown for a homogeneous labor force (except for adjustments in the hiring probabilities), if the magnitude and composition of the hard core remains constant. However, it would be desirable to take account of shifts into and out of the hard core.

For this purpose, the proportion of unemployment in the hard-core group may be treated as a function of the unemployment rate and the phase of the business cycle—i.e. the declining phase and the recovery phase. By means of such a functional relationship, we would be able to estimate the proportion of unemployment that should be in hard-core status under given conditions. The proportion of unemployment in hard-core status for a specified level of unemployment should be higher during depression and recovery phases than during the prosperity and declining phases. This is an area requiring further empirical study.

CHANGING COMPOSITION OF THE LABOR FORCE

A typical labor force is one that is continually changing in both size and composition. Withdrawals from the labor force occur because of superannuation, disability, death, retirement and numerous personal reasons. At the same time, decisions are being made by people outside the labor force either to seek employment or to accept job offers. Although the bulk of new entrants consists of those becoming of age, part consists of individuals who had previously withdrawn from the labor market and decided to reenter.

The hypothetical models depict a labor force constant in both size and composition. In these models, job vacancies are filled only by the hiring of workers from the available pool of unemployment, and every separation results in the transfer of a worker from the status of employment to that of unemployment. This type of model could conceivably describe a pool of workers possessing a rare skill, who are attached to a plant, occupation or industry, and are unable to accept employment in any other type of activity. In general, however, not all separations result in unemployment. Aside from voluntary quits to accept other jobs immediately, separations due to death, retirement or disability result in withdrawals from the labor force and not in unemployment. Also, not all job vacancies are filled by persons currently in unemployment status. Some openings are filled by persons entering the labor market for the first time or reentering after a long absence, others by persons shifting from one job to another. In an unemployment insurance program covering only part of the labor force it is also significant that some of the covered job openings may be filled by workers separated from jobs not covered by the unemployment insurance plan while some of the workers separated from covered jobs enter non-covered employment.

Some of the above factors may be partially reflected in the labor force models. For example, the turnover rates should be reduced in order to eliminate the hires and fires caused by voluntary shifts from job to job. Possibly, a continuous work-history study over a sufficiently long period of time would yield satisfactory information for adjusting the actuarial estimates for the effects of changes in the composition of the labor force.

SUMMARY

There are relevant items, such as multiple spells of unemployment and administrative factors which are important, but cannot be treated adequately in a brief presentation.

Further experience with unemployment insurance will undoubtedly lead to the development of a more comprehensive theory and also to practical solution to problems confronting us at the present time.