A CREDIBILITY FRAMEWORK FOR GAUGING FIRE CLASSIFICATION EXPERIENCE

BY

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PART ONE — BACKGROUND AND PHILOSOPHY OF FIRE CREDIBILITIES

The need for "credibility" judgments in fire insurance is inescapable. However, it is not necessary, nor is it a common custom, always to express credibility evaluations in mathematical language. In his daily work, the underwriter soon acquires the habit of accepting certain evidence as credible and dismissing others as untrustworthy. Now, these personal evaluations will vary not only from underwriter to underwriter; but even the same man may, at different times, employ different standards in similar situations because of purely subjective conditionings on each of the particular occasions. Probably no one will be amazed at this familiar observation, and few will find the underwriter's vacillations on credibility in any way reprehensible as long as his fund of common sense and knowledge of the business allows the company a profitable operation.

However, this purely subjective evaluation of credibility becomes unworkable when overall loss experience must be appraised from time to time for rating or policy underwriting decisions as contrasted with the underwriter's every day risk decisions. In his habitual review of risk offerings, the underwriter's faulty evaluations of credibility in a small number of situations will not necessarily mean unprofitable operations. But an incorrect decision on rate level or underwriting policy because of a misreading of credibility requirements can have serious repercussions on a company's results.

At the national level, there seems to be no inclination for the fire insurance industry to recognize officially any standards of credibility. It is true that some company executives have occasionally protested against proposed fire classification revisions on the plea that loss experience on such a statistical system would have no credibility. But to my knowledge, these verbal admonitions have never been followed with any mathematical or other logical demonstrations; and seldom, if at all, have the supervisory authorities taken serious issue with these undocumented representations.

It is interesting to note that the New York Insurance Department in its 1951 rate revision negotiations with NYFIRO used the following credibility table without differentiation for all occupancy classifications.

5 yr. Premium ($1000)	*Credibility*	*5 yr. Premium ($1000)*	*Credibility*
Under 50	5%	1,800-2,500	60
50- 200	10	2,500-3,200	70
200- 450	20	3,200-4,000	80
450- 800	30	4,000-5,000	90
800-1,250	40	5,000 & over	1.00
1,250-1,800	50		

In the 1951 *Proceedings* of the Casualty Actuarial Society, it was observed that there was no mathematical support for these tabular data. Nor was any clue afforded as to what logic lay behind the figures —presupposing that the data were the consequence of some formal reasoning process.

It is probably safe to say that there is no such thing as a formal mathematical theory of fire credibilities. Even the literature on this subject is scanty — an understandable neglect in view of the familiar adequacies of fire rates in the past. But as the rates approach the break even point, the companies may display a livelier interest in discovering a predictable relationship between their pricing practices and the actual loss experience.

It is unlikely, however, that the various credibility standards developed for certain casualty coverages can be automatically transferred over to our fire insurance rating problems. We would hardly be justified in assuming identical parameters for both loss distributions, as we suspect that the average chance of loss as well as the spread of the losses about the average expectation would probably be much different for fire than for automobile or workmen's compensation experience. Nevertheless, the attack on the problem should be identical in both instances.

It is obviously not possible for us, nor are we inclined, to dismiss the question of fire credibilities as solely an academic problem for which there is no satisfactory solution. We are even less disposed to slight the mathematical approach as of secondary importance to an approximate language understanding of credibility. For although our ultimate decision may be a qualitative one, (i.e., to accept or to reject certain evidence), the development of standards is necessarily quantitative (i.e., mathematical).

It has been discovered in other lines of endeavor that satisfactory solutions are often found by reorienting the statement of a problem so that it may be resolved with available techniques rather than searching for some abstruse methodology which, even if found, would not be generally intelligible. We suspect that at least a measure of truth, if not always of respectability, can be predicated of the theory that a "correct" answer is sometimes achieved by staking out the area within which a solution will be acceptable and then turning to a workable method for developing this answer. Certainly, this type of approach can not be regarded as incompatible with the pursuit of an immateriality such as "credibility" or more popularly, "belief."

Now, this "credibility" or belief is essentially the degree of assurance that a person must have in order to do something. In fire insurance rating, it is the confidence we have in the loss experience (with reference only to its statistical implications) to which we should adjust rates or revise underwriting policies. Naturally the degree of assurance required before venturing upon any commitment will be a function of each individual's personality. Perhaps there would even be a wide variation in the demands of individual respondents. Nevertheless, there is likely a neighborhood in which the demands will converge. Within this area, we shall set our standard of credibility as the common level having the minimum departures from a unanimity of opinion.

Sometimes we have better luck with a problem by marking out, first of all, the range of possible solutions rather than concentrating our attention solely on the one "best" solution. We will not get very far in fire insurance credibilities by searching for that very point at which the experience becomes trustworthy with all experience based on any lesser number of observations being automatically rejected. We would rather try our fortunes on the possibility of describing a range of credibility values from "0%" to "100%". It is not expected that we would achieve a complete agreement at any point of the scale. But it is even less likely that many people would ask that our standards for 10% and 90% credibilities be reversed. And as we shade our credibilities through the various tones of grey on the way from black to white, we have a better chance of approximating the true values than by positing a standard at which confidence must be conceded by arithmetical fiat. Although it may be the most obvious of mathematical tricks, this theory of the "continuous function" enables us to explain phenomenon which otherwise would not be intelligible without laborious counting of discrete observations.

PART TWO — STANDARDS FOR FIRE CREDIBILITIES

Let us, therefore, preface our mathematical development by defining the two extremes of "insignificant" or "zero" credibility and its antithesis "Fully Significant" or "100%" credibility. It matters not that neither end actually exists. It will suffice that we recognize that the one is the extreme position from the other and that, if needs be, we can imagine an infinite sequence of values between. Just one more time, we can position these fiducial limits to reflect whatever degree of confidence a person may be in need of. The ideas are the same, and so too the theory and the development — only the figures will change.

Thus we shall define "Insignificant or Non-Credible" experience to be a summary of loss experience based on such a number of independent risks that with any lesser number of risks one could not, in two out of three instances, reasonably expect that the true loss ratio would be less than 10% above the indicated figure.

Although tortuous, this definition is not beyond our working it out. First of all, there is no explicit restriction on the time interval over which the experience is to be collected. In pure theory the number required for credibility need not be visualized as a factor of any particular extension in time. The actual loss ratio for the period reviewed is to be taken as one sample of the various possible loss ratios which could have been experienced within this identical time.

The statistical method, then, indicates the credibility of the developed experience considering it solely as a sample from the universe of all possible loss ratios which could have occurred under the influence of the identical inherent hazard to loss. The mathematics do not establish the representativeness of the particular time interval reviewed. It is up to the rater to say whether or not this particular time interval is sufficiently representative to be used to set his prices for future coverage.

It should be noted that our definition sets the upper limit to Non-Credibility. With any greater number of risks, we are not to consider the statistics as non-credible. But with any lesser number, the experience is to be completely rejected.

The need for "personal assurance", an aspect of credibility to which we have previously alluded, helps to set the "two out of three" and the "10% above indicated" standards appearing in the definition. Although other figures could have been used, these values are arbitrary only in the sense that one person will demand a greater degree of probability (i.e., assurance) than another, before doing something. Actually in our important decisions, most of us require a relatively favorable degree of certainty. Few people would jeopardize a substantial portion of their funds on only a 5% chance of a successful outcome. On the other hand, the cost (even including monetary costs) of absolute certainty would be prohibitive, and the effort to attain such assurance is needless.

Consequently, we have set up our statistical requirements for fire credibilities so that the play of chance losses will not typically move the loss ratio more than 10% above the "true" loss ratio (i.e., inherent hazard of the particular universe). We can, if it is desired, reduce the allowable chance swing from 10% to 5% or 1% about the "true" average — but the narrower the desired control band, the greater the number of risks for credibility (i.e., at each level of the credibility scale). Likewise, the degree of assurance, the "two out of three instances" of our definition, can be increased to "three out of four" or "nine out of ten" or even more rigorous fiducial limits. But again, the greater the degree of certainty required, the greater the number of risks for each of the various credibility values.

You will note that our credibility standard is geared to a restriction in the swing of the loss ratio on solely the *top* side of the "true" figure. The possible play of the variation is unrestrained on the side *under* the central point. It is true that commonly the control limit is established as an equal range *both* above and below the mean position.

Such an added restriction could have been imposed in this problem. But again, the greater the limitations the greater the number of risks for credibility. Although the exposition is worked out in terms of the values outlined above, credibility tables can readily be developed for varying "fiducial limits" and "average departures from true values".

Now that we are familiar with the terms, let us proceed without further comment to define "Fully Significant" or "100%" credibility. Then we may proceed to examine with some care the backgrounds of our statistical thinking.

"Fully Significant or 100% Credible" experience is a summary of loss experience based on such a number of independent risks that in fewer than 3 in 100 instances, one would expect that the true loss ratio would be more than 10% above the indicated figure.

It will be noted that although we have used here the same standard for the allowable departures from the indicated loss ratio, the fiducial limits have been made much more rigorous. The previous "two out of three" break point for the "zero credibility" was deemed a sufficient "assurance" level only for the least possible value for credibility. And for the other extreme of "Full Credibility", the relatively severe "more than ninety-seven out of one hundred" standard was selected. The manner in which the credibility values are to be graduated between these two positions will be reviewed in a subsequent section.

PART THREE — MATHEMATICAL THEORY OF FIRE CREDIBILITIES

Although the idea may be anathema to underwriters and loss prevention engineers, our credibility tables are based on the premise that fire losses are inevitable. Every class (occupancy, construction, geographical) is viewed as possessing a certain inherent hazard to loss. But the loss potentials of these various classifications are not uniformly active within any specified time interval. Why and under what circumstances, any single unit's inherent hazard to loss jumps from the solely potential state into a real existence is not our concern here. It is enough that each class have its own characteristic loss potential.

We do not even have to know aforehand the value of the inherent hazard of the class. From the observation of prior happenings we establish its most likely average. And actually little harm is done if the "true" value does not exactly coincide with our approximation thereto. With an estimate to the probability of a loss (i.e., inherent hazard) we can build up a range within which the occurrence values will typically swing about its true value.

For example, a class with a 1% inherent hazard to loss will not likely produce exactly 10 losses on 1000 exposures for every period reviewed. In one case there may be no loss occurring; whereas in another there may be 20. Generally, the observations will tend to cluster about the true inherent hazard of 10 losses per 1000 exposures, and the departures from this average may be treated as responsive to a describable statistical pattern.

Let us tie down this term "inherent hazard" a little closer to our fire insurance statistical problem. This expression immediately suggests the "likelihood of loss". But such a concept would be only an imperfect representation of "inherent hazard" in fire insurance. Since over 75% of all fire losses account for less than 5% of the total payments, the rater will have but incidental interest in the total number of losses. The controlling element in fire insurance is the chance of a medium size or severe loss in view of the fact that, excluding the dwelling classification, well over half of all payments are traceable to losses over $10,000 each. Therefore in the subsequent development we shall intend by "inherent hazard" the likelihood of suffering a fire loss other than a trivial loss.

As previously noted, we propose that each fire classification has its own individual potentiality for non-trivial fire losses. This tendency to loss is not uniformly realized over each successive time interval, but rather makes its appearance in a seemingly haphazard fashion — but actually capable of being described and anticipated according to a precise statistical model. This model is constructed upon the fundamental mathematical logic which lies behind all those exercises in coin tossing. The chance of averaging 3 or fewer heads in 5 tosses of ten coins can be predicated by the so-called Binomial Theorem. We can also measure the expected spread of the results about the mean position. Actually our credibility standard is set not directly on the measure of the inherent hazard, but rather upon the expected spread of the results about this average value.

For any small number of samples, the Binomial Distribution of rare events is apt to be quite non-symmetrical; that is, the curve representing the distribution of losses will be humped toward either the lower or the upper end of the scale. Such a situation may first seem somewhat of an annoyance statistically; but fortunately as the number of samples is increased, the curve representing the distribution of even rare events approaches the normal or symmetrical form. This fact is indicated algebraically by the demonstration that the Normal Curve has a Beta One (B_1) of zero and a Beta Two (B_2) of three which also is the limiting position of these ratios for the Binomial Distribution as the number of samples "n" approaches infinity.

$$B_1 = \frac{(q-p)^2}{npq}$$

where p = chance of loss $q = 1 - p$

$$B_2 = 3 + \frac{(1-6pq)}{pqn}$$

As you recall, we have in our development visualized the actual loss ratio for any defined extension in time as only one of an infinite number of possible occurrences which could have taken place under the same inherent hazard to loss in the identical time interval. Consequently, we have set up our problem so that our "n" approaches infinity as a limit.

PART FOUR — DEVELOPMENT OF FORMULAS

In the Binomial Distribution the *arithmetic average* (m) is given by: $m = np$ where:

n = number of observations in sample.
p = chance that the event will occur.
$(1-p)$ or q = chance that event will not occur.

The spread of values about the average (m) is measured by the *standard deviation* (s) which is equal to the square root of the sum of the squares of the deviations from the average.

$$s = \sqrt{npq}$$

Our credibility standard was geared to a maximum tolerance of 10% above the indicated loss ratio. Now since our measure is expressed in terms of a maximum allowable increase in loss ratio, we have cancelled out the rate as a function in our solution. And our credibility criterion thus becomes solely the number of risks needed so that the losses will typically not exceed 110% of their expected value.

We have discussed heretofore the proposal that each class has its own inherent hazard to loss (i.e., non-trivial losses). We have not insisted that these losses (non-trivial losses) be segregated by size groups, each of which is to be graduated by its own probability of loss. Rather we prefer to establish a relative likelihood of occurrence for a non-trivial loss, as an entity per se. We are aware that there is no precise value corresponding to this mathematical abstraction. But we know that the probability even of the most frequent "non-trivial losses" is of such a low order of probability, that to attempt to graduate the probabilities of the less frequent "non-trivial" losses could well be a needless gesture.

Therefore, we are to think of the loss ratio as the result of the occurrence of a predetermined number of non-trivial losses corresponding to the inherent loss characteristic of the class plus additional "non-trivial" losses due solely to the operation of chance. These chance losses are, by our standard, not to be so frequent as to increase the losses (i.e., loss ratio) by 10%. The expected number of non-trivial losses is given by our "m" (i.e., np) and the allowable chance deviation is set at a maximum of 10%.

Now, let us recall that in setting our upper limit for "Insignificant or Non-Credibility" we geared our 10% deviation to an assurance level i.e., fiducial limit) of "two out of three times". We know that in the Normal Curve (i.e., the limiting position of the Binomial as "n" approaches infinity) that about 30% of all occurrences are beyond a point corresponding to one-half a standard deviation above the arithmetic mean. Consequently, slightly more than two-thirds of the observations will lie to the left (i.e., the lower portion of the scale) of this point. And therefore the chances are two to one, or two out of three, that at this point the losses (or the loss ratio) will not exceed the average or expected number by more than 10%.

Or, in symbols:

for $\left(+\frac{x}{s}\right)$ above np, the area under the normal curve to the right of this point equals $(1-0.69146)$ or 30% approximate.

$$x = 10\% \text{ of average or } x = \frac{np}{10}$$

$$s = \sqrt{npq}$$

$\frac{x}{s} = \frac{1}{2}$ or $\sqrt{npq} = \frac{2np}{10}$ or $n = 25\,\frac{q}{p}$ and since $q = (1-p)$, $n = 25\left(\frac{1}{p} - 1\right)$
or letting $\frac{1}{p} = k, n = 25(k-1)$

Now if "p" the chance of loss is 1% the experience cannot be considered "Non-Credible" if the number of risks exceeds 2475 (i.e., 25×99). Consequently we can express "zero credibility" limits as a variable of "p" the chance or the inherent hazard to loss. To translate these criteria to premium dollar figures we would multiply the number of risks times an average rate and policy size for each classification.

The procedure for "Fully Significant" or "100%" credibility is identical to the above approach. However, our 10% loss ratio tolerance is now geared to the more rigorous (i.e., 97 out of 100) assurance level. At 2s above np, the area under the normal curve to the right of this point equals $(1-0.97725)$ or 2.3%.

$$x = 10\% \text{ of average, or } x = \frac{np}{10} \quad s = \sqrt{npq}$$

$$\frac{x}{s} = 2 \text{ or } x = 2s \text{ or } \frac{np}{10} = 2\sqrt{npq}$$

$$n = 400\,\frac{q}{p} \text{ or } n = 400\,(k-1) \text{ where } k = \frac{1}{p}$$

Consequently, if "p" the chance of loss is 1%, the data would comply with the requirements for "Fully Credible" with 39,600 (i.e., 400×99). Again we can express "full credibility" requirements in terms of "p" the chance or inherent hazard to loss. And these standards can be expressed in terms of equivalent premium dollars by extending the number of risks by the average rate and policy size for each classification.

PART FIVE — CONSTRUCTION OF FIRE INSURANCE CREDIBILITY TABLES

With the development of the two equations for "zero" and "full" credibility, we are in a position to set these limiting standards in terms of the inherent hazard (chance of non-trivial loss). There are various methods by which the credibilities can be graduated from 100% down to 0%. On casualty lines the credibility is characteristically introduced at a decreasing rate with increasing exposures. This approach makes sense for those lines wherein there is a frequency of small and medium size losses which have a predominating influence on the total loss payments.

In this respect, the theory may not exactly fit the fire insurance field. But by excluding trivial losses, we might, with greater justification, think of these residual fire losses as being scaled similarly to the casualty loss pattern, but only at a higher level of loss cost per occurrence. Consequently, we have adopted a modified $\frac{p}{p+k}$ formula

with $Z = \dfrac{N-C}{N-C+A}$ below the Focal Point of the graduation curve.

In the above equation N is the number of risks required for credibility (Z). Of the two constants, C is determined so that the curve will start at the statistical norm for zero credibility, while A is a constant such that the point of 67% credibility in linear interpolation would coincide with the corresponding 67% value from the above equation. Above the Focal Point the credibility values have been taken from the straight line joining the points 25 (k-1) and 400 (k-1). The graduations are developed in a supplementary section.

It may be a more rewarding effort to assign the major fire occupancy classification groups to inherent hazard values by some rough statistical estimates from summary data, than to attempt to measure this factor directly. Mainly as a trial to illustrate the approach, out of a relatively small sample of 14,500 mercantile policies in earned annual exposure, 585 losses were suffered, or a frequency ratio of .039.

Over a longer period, of 5306 mercantile losses, 409 exceeded $5,000 each or a severity ratio of .077. Thus the estimated chance of suffering a mercantile loss over $5,000 is the product of:

1. that a loss will occur = .039
2. that if it occurs, the loss will exceed $5,000 = .077

Thus the chance of a non-trivial loss (i.e., inherent hazard) of the mercantile classes is .039 × .077 = .0030, or approximately 0.3%.

Let us now construct a sample credibility table by fire major classification groups on the basis of the following averages:

Fire Classification	*Inherent Hazard*	*Annual Rate*	*Ave. Policy*	*Ave. Premium*
Mercantile Contents	.003	.80	15,000	120
Manufacturing	.002	.75	40,000	300
Dwellings	.005	.20	12,500	25

Credibility Table

Credibility	*Dwellings*	*Mercantile Contents*	*Manufacturing*
10	$ 193,000	$ 1,549,000	$ 5,819,000
20	280,000	2,241,000	8,421,000
30	391,000	3,130,000	11,770,000
40	539,000	4,316,000	16,224,000
50	746,000	5,976,000	22,455,000
60	1,057,000	8,466,000	31,811,000
70	1,430,000	11,454,000	43,039,000
80	1,617,000	12,948,000	48,653,000
90	1,803,000	14,442,000	54,267,000
100	1,990,000	15,936,000	59,880,000

PART SIX — CRITICAL APPRAISAL OF THEORY

Before any comment on the statistics, it might be desirable to question some aspects of the theory advanced in the previous argument. Even granting that a reasonable defensible mathematical expression could be found to measure "credibility", a person might doubt that any advantage would thereby accrue to management. Fundamentally, any mathematical or other schematic approach to problems limits the range of judgment. Of course, there are situations wherein such restrictions are not only inescapable but are actually desirable. We all recognize that certain basic relationships must be taken for granted, if we are to avoid the chaos of a constant experimentation to find out what has already been long known. A reasonable man cannot afford to ponder each detail of his daily living. But it would be equally unwise for anyone to so condition his mind that he responded with a mechanical-like reflex in all situations.

Now, various statistical tests can be used to identify significant differences in a series of data. As an example, these methods would indicate that the loss ratio on Class A is really better than on Class B. But the tests do not hold conversely. Just because the formulas do not indicate that "A's" loss ratio is significantly different from "B's", one cannot infer that the classes are essentially similar. In other words, the two classes may be really different, but mathematics cannot be used to prove it.

This corollary from the statistician's so called "Null Hypothesis" bears out a long standing belief of management. There is no rule or equation which will automatically solve our problems. Each situation must be thought out on its own merit in its own particular environment. There are instances wherein a person with intimate understanding of the underwriting facts will know that one type of risk is to be preferred to another, regardless of what the mathematics may say. Any research analyst who would slight the significance of the underwriting "know-how" is obviously unfamiliar with the insurance field. The successful underwriting manager is too busy guiding his men to select the profitable types of risks to bother with credibility tables which may, in his eyes, best be used as a crutch for the unsuccessful to explain their failures.

Possibly one might view the concept of the "non-trivial" fire loss as an abstraction of questionable validity. There can be no doubt, of course, that the preponderance of dollars paid is traceable to a relatively small number of losses. This observation is supported by the fact that about 75% of all losses by number constitute only 5% of all loss payments by amounts. But this characteristic distribution of fire losses does not, per se, prove the objective merit of the "non-trivial" fire loss. The very fact that fire losses can be demonstrated to follow a graduation from small through medium-sized to large means, in turn, that the large losses too must observe a graduation by size. There is no such thing as a single loss size which can be taken as typical of all non-trivial fire losses. As an alternative method, one might study the areas under the curve of fire losses by amount of loss. It is possible, of course, that the curve of actual fire loss distribution by amounts may be so skewed and so irregular (multi-modal) that it would not lend itself to statistical projections.

There is also some question on the merit of using the simple "Binomial Distribution" to develop fire credibilities. If the chance of event is remote (less than 5%) and the number of observations is small, the binomial distribution is very markedly skewed. In such an instance, the area under the curve is quite irregular and the distribution of the frequencies is a fairly inexact representation of the corresponding expectations under the normal curve.

Now it is true that, even with a very small "p" (chance of loss) the binomial approaches the normal curve at the limit as the size of the sample becomes infinitely large. But at the limit both the mean (np) and the standard deviation $\sqrt{npq}$ also approach infinity, and there is some doubt whether or not the theory is usable at this extreme position. Anyway, it appears somewhat fanciful to view the experience for any prescribed period as a sample of an infinite number of possible loss ratios which could have happened in the identical time interval due to the same inherent hazard to loss.

As for the choice of formula, the Binomial Distribution presupposes that the chance of loss (p) is constant from sample to sample within any set, and also from set to set. If "p" varies from sample to sample but is constant from set to set, we have a Poisson distribution. And if, conversely, the "p" is constant from sample to sample but varies from set to set, we have a third type, or Lexis distribution.

Although the means are same for all three distributions (np), the standard deviations (s) are different:

$$\text{Binomial } S_b^2 = npq$$

$$\text{Poisson } S_p^2 = npq - \sum_{i=1}^{n} (P_i - p)^2$$

$$\text{Lexis } S_L^2 = npq + \frac{n^2 - n}{r} \sum_{i=1}^{r} (P^i - p)^2$$

Consequently it appears to be a gratuitous assumption to treat fire losses as corresponding to the Binomial Distribution.

Considerable exploration has been made in Casualty insurance of the possibilities of the Poisson Exponential $p = \frac{e^{-m}\ m^r}{r!}$

This equation has been successfully employed in fields other than insurance to describe situations wherein the probability of the given event is very remote. For example, this method has been used to estimate the likelihood of multiple dialing of the same telephone number at exactly the same time. Since fires are a rare event, it would seem that the Poisson exponential would have been a good approach to this credibility problem.

These criticisms will be considered in the following section.

PART SEVEN — REPLY TO COMMENTARY ON THEORY

We should first like to consider the question of the statistical methods. The precise equation to be used is admittedly not the most fundamental aspect of our credibility problem. But if we can cover this phase in a few general observations, we will avoid the typical mathematical colloquy with its almost endless formulas.

It is to be granted that the Binomial Distribution is badly skewed and only an imperfect representation of the Normal Curve if the event is rare. (i.e., "p" is very small) and the number of observations is not large. However, our problem was set up so that the number would be very large, but not necessarily infinite. Under such conditions, the Binomial does approach the Normal Equation ($p = ce^{-k\,x^2}$) and our projections from this curve appear to be serviceable approximations.

We are not disposed to slight the caution that the occurrence of fire losses may not best be described through Binomial sampling (i.e., the chance of the event ("p") is constant from sample to sample and from set to set). It is possible that fire losses may be characterized by Poisson or Lexis sampling wherein the chance of the event ("p") is not constant. But once we investigate the possibility of a variation in our "p" values, we must logically persevere in our theory and express "p" not as a constant within any set or for any group of samples within sets but rather as the function of multiple factors. And in establishing our "p" value not as a constant but instead as an exercise in multiple correlation, we are burdened with a cumbersome and unsatisfactory artifice.

In regard to the suggestion that the Poisson exponential $\frac{e^{-m} m^r}{r!}$ be used as a basis for fire credibilities, a glance at the Poisson tables will show that for moderate and large "n" values the distribution of events observes a symmetrical pattern. And with the Poisson, we shall not obtain an answer of a less demanding order of magnitude than that indicated with the Binomial (i.e., "n" very large).

Basically one's reaction to this study will be influenced by his attitude to the idea of "credibility". If the reader considers "credibility" as a valid concept which may assume under varying conditions different values, he will favorably regard a theory which would propose to measure its quantitative characteristics. He, of course, may not agree with the precise values or formulas used herein, but on the basic facts that the incidence of loss is relatively small, sporadic in its chance application, and potentially affecting a very large number of units (i.e., risks) he must necessarily gravitate towards the various limiting mathematical processes treated herein. And, most important, he must conclude that but little mathematical credibility can be attached to detailed classification experience based on an obviously small number of risks.

On the other hand, this mathematical approach and its consequent conclusions will hardly persuade the reader who considers "credibility" as only a language attempt at a subjective conditioning which is so a part of personality that no communication of its quantitative character is possible. Such a person will instinctively use "credible" and "not credible" as opposite poles of conviction with no intermediary mental way stations. This resoluteness of mind is characteristic of the active temperament which gets things done — often with a heavy dependence on personal judgment. We have witnessed too many successes of the leadership and too many failures of the contemplative personality not to be impressed with this power of independent judgment. But these experiences have not yet taken from the writer the conviction that each excellence is effective only in its own field.

For example, an underwriter, after reviewing a tabulation of insignificant experience, may conclude that Class #A is a profitable field to cultivate — and he may be right. His correct conclusion could be due to an intimate (but non-statistical) knowledge of the loss character and the general rate level of the class. Or, his success may stem from his being one of those rare individuals whom Fortune, that lord of chance, never allows to make a mistake. But this success is not due to his reading, by some mystic power, significance in a set of data which possesses no mathematical credibility!

SUPPLEMENT

Graduation Work Papers

Full and Zero Credibility set from area under Limit of Binomial Curve as m ∞

Various Focal Points investigated.

The Focal Points are expressed in varying fractions of the range from zero to Full Credibility.

$$\text{Graduation Formula } Z = \frac{N - C}{N - C + A}$$

Where

- Z = Credibility
- N = Number of Risks
- A = Constant for each "P"
- C = Constant in order to start curve at statistical norm for Zero Credibility: C = 25 (k-1)

Our first effort is to test above curve for each "P" (i.e., inherent hazard to loss) and varying Focal Points.

Number of risks for Zero Credibility $= N_o = 25$ (k-1)
Number of risks for Full Credibility $= N_f = 400$ (k-1)
Graduation Range $= N_f - N_o = 375$ (k-1)
Where K = 1/P and P = chance of Non-Trivial Loss.
Focal Point of Graduation $= N_g = G(375)$ (k-1) + C.
Where $O < G < 1$
If Focal Point = 90%; Ng = (.90) (375) (k-1) + 25 (k-1)
$N_g = 363$ (k-1)

	$N_g=90\%$		$N_g=80\%$		$N_g=66\frac{2}{3}\%$	
P	*N*	*A*	*N*	*A*	*N*	*A*
.010	35,937	3,718	32,175	7,425	27,225	12,375
.005	72,237	7,474	64,675	14,925	54,725	24,875
.003	120,516	12,468	107,900	24,900	91,300	41,500
.002	181,137	18,740	162,175	37,425	137,225	62,375
.001	362,637	37,518	324,675	74,925	274,725	124,875

Tables of "N" — For Various Focal Points — For "P" = .003

Z	$N_g=90\%$	$N_g=80\%$	$N_g=66\frac{2}{3}$
.10	9,685	11,063	12,911
.20	11,416	14,525	18,675
.30	13,642	18,982	26,086
.40	16,611	24,908	35,967
.50	20,767	33,200	49,800
.60	27,000	45,650	70,550
.70	37,390	66,392	105,133
.80	58,170	107,900	174,300
.90	120,500	232,400	381,800

Graduating Credibility over entire range according to Formula

$$Z = \frac{N - C}{N - C + A} \text{:Focal Point} = 66\tfrac{2}{3}\%.$$

Number of Risks for Varying "P's"
Focal Point $= 66\frac{2}{3}\%$

Z	*.005*	*.003*	*.002*	*.001*
.10	7,736	12,911	19,399	38,836
.20	11,194	18,675	28,069	56,194
.30	15,646	26,086	39,234	78,546
.40	21,567	35,967	54,079	108,267
.50	29,850	49,800	74,850	149,850
.60	42,287	70,550	106,037	212,287
.70	63,008	105,133	157,996	316,308
.80	104,475	174,300	261,975	524,475
.90	228,850	381,800	573,850	1,148,850

If values above the Focal Point ($66\frac{2}{3}\%$) are taken from the straight line which passes through the points 25 (k-1) and 400 (k-1), then the Upper Limits of the above table become

Z	*.005*	*.003*	*.002*	*.001*
.70	57,216	95,450	143,465	287,215
.80	64,680	107,900	162,178	324,678
.90	72,144	120,350	180,890	362,141
1.00	79,600	132,800	199,600	399,600